13/02/2018
λ-CALCUL

1 CM

Suite codage de structure.

Codage des booleens :

$true \stackrel{=}{_{def}} \lambda x y . x$

$false \stackrel{=}{_{def}} \lambda x y . y$

fil conducteur : Comment s'en servir ?

Reponse pour booleen : Le test

-> Si cond alors U sinon V.

↳ comment coder ça : on efface tout sauf les mots clés.

↳ cond U V

↳ $b\ U\ V$

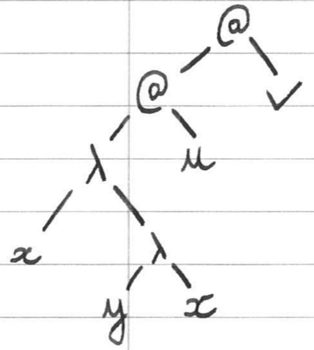
$true\ U\ V$

$(\lambda x y . x)\ U\ V$

$((\lambda x . \lambda y . x)\ U)\ V$

$(\lambda y . U)\ V$

U



$false\ U\ V$

$(\lambda x y . y)\ U\ V$

$(\lambda y . y)\ V$

V

not b : si b alors false sinon true

b false true

$b\ (\lambda x y . x)(\lambda x y . y)$

↳ exemple

true false true.

$(\lambda x y . x)$ false true.

false.

} impossible en typage simple.

$not = \lambda b . b$ false true

ou

$not = \lambda b . \lambda x y . ($si b alors y sinon $x)$

$= \lambda b . \lambda x y . b y x$

} possible en typage simple.

typage simple.

$bool \stackrel{=}{_{def}} T \to T \to T$

$not : bool \to bool \to (T \to T \to T) \to (T \to T \to T)$

$= \lambda b^{T \to T \to T} . \lambda x^T y^T . \underbrace{b\ y\ x}_{T}$

$$\dfrac{\dfrac{\vdash b : T \to T \to T \qquad y : T}{b\,y : T \to T} \qquad x : T}{b\,y\,x : T}$$

<u>and</u> $= \lambda b_1^{bool} . \lambda b_2^{bool} . \lambda x^T y^T .$

si b alors (si b_2 alors x sinon y)
 sinon y

$b_1 (b_2\ x\,y)\ y$

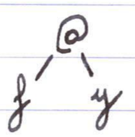
$\lambda b_1 . \lambda b_2 .$ si $b_{\cancel{2}1}$ alors b_2
 sinon b_1

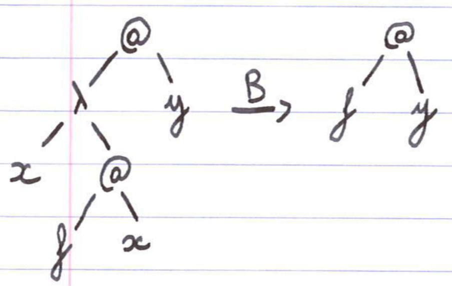
$\lambda b_1 b_2 . b_1 b_2 b_1$
$\lambda b c . b c b$

<u>or</u> $= \lambda b_2 \lambda b_2 . \lambda x\,y . b_1\ x\ (b_2\,x\,y)$

$\lambda b_1 b_2 . b_1\ b_1\ b_2$
$\lambda b_1 . b_1\ b_1$
$\lambda x . x\,x$

<u>eta</u> $f : \lambda x . f\,x$





<u>or</u> <u>true</u> <u>false</u>

$((\lambda b . b b)(\lambda x y . x))(\lambda x y . y)$

$((\lambda x y . x)(\lambda x y . x))(\lambda x y . y)$

$(\lambda x y . x) =$ true

ou

$(\lambda b . b b)$ true false
true true false
true

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2

CM **Codage des entiers (de church)**

$$0 \stackrel{def}{=} \lambda f x . x$$
$$1 \stackrel{def}{=} \lambda f x . f x$$
$$2 \stackrel{def}{=} \lambda f x . f(f x)$$
$$\vdots$$

On doit coder

- $0 \stackrel{def}{=} \lambda f x . x$
- le successeur. $\stackrel{def}{=} \lambda n^{nat} . \lambda f^{T\to T} x^{T} . n f x$
- l'addition. $\stackrel{def}{=} \lambda n^{nat} . \lambda m^{nat} . \lambda f^{T\to T} x^{T} . n f . (m f x)$
- la multiplication.

$\hookrightarrow \lambda n^{nat} . \lambda m^{nat} . \lambda f . n(m f)$

- exp $= \lambda n m . m n$
- autoexp $= \lambda n . n n$

$$\text{succ } 2 = (\lambda x . \lambda f x . f(n f c))2$$
$$= \lambda f x . f(2 f x)$$
$$= \lambda f x . f((\lambda f' x' . f'(f' x')) f x)$$
$$= \lambda f x . f((\lambda x' . f(f x')) x)$$
$$= \lambda f x . f(f(f x))$$

$$\text{test } 0 \stackrel{def}{=} \lambda n^{nat} . \lambda x^{T} y^{T} . n(\lambda z . y)(x)$$