For description of any algorithm, please use text and do not use pseudocode

1. **(20 points)** Let $G = (V, E)$ be a connected undirected graph with nonnegative costs of edges and $e \in E$ be a unique edge in G with the minimum cost. Prove that each minimum spanning tree T of G contains e.

2. **(20 points)** Construct an optimal prefix-free code for the string

   $$TAGTAGTCGTAACTGTGT$$

   using Huffman algorithm.

3. **(20 points)** Compute the expected number of repetitions of steps **(3:)** and **(4:)** of the following randomized algorithm that returns a random permutation for a given array $A[1..n]$ of distinct integers. (The symbol := is the assignment operator and the symbol = is the equality operator.)

   Input: An array $A[1..n]$ of distinct integers.
   Output: A random permutation $C[1..n]$ of A.

   (**1:**) Set $B[1..n] := \{\text{FALSE}, \ldots, \text{FALSE}\}$

   (**2:**) Set $i := 1$

   (**3:**) Generate a random number j in $\{1, \ldots, n\}$

   (**4:**) If $B[j] = \text{FALSE}$ then do $C[i] := A[j]$, $i := i + 1$, $B[j] := \text{TRUE}$

   (**5:**) Repeat steps **(3:)** and **(4:)** until $i = n + 1$.

   (**6:**) Return C as a random permutation of A.

4. **(20 points)** Prove that the set cover problem U, F is NP-complete, even if for each element from U we have at most two subsets from the family F that cover this element.

5. **(20 points)** Design a linear-time algorithm to compute the cardinality of a *minimum vertex cover* in a tree.