Homework 3
Fall 2022: CS 260 Design and Analysis of Algorithms

Instructions: This homework is due on **Monday, December 5, 2022** in class. You can work in groups and should submit the homework handwritten on A4-size papers. Write names of group members clearly on top of the first page as well as the total number of pages. You may have queries related to this homework which should be directed to the TAs.

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1. **(20 points)** Is it possible to find an algorithm which, for a given program (in C++, for example), will recognize if this program satisfies the following conditions (answer Yes or No):

   (a) For any input from $\{1, 2, 3\}$, the considered program finishes its work after at most one hour and outputs a result.

   (b) For the input 0, the considered program finishes its work and outputs a result.

   (c) For the input 0, the considered program never finishes its work.

   (d) The length of a program is at most 1000 bytes, and the program does not use the operations of subtraction and division.

2. **(30 points)** Show that if f is a function of one variable that is recursive, nondecreasing, and unbounded, then its range is an infinite recursive set.

3. **(30 points)** Show that every infinite recursive set is the range of a nondecreasing unbounded recursive function of one variable.

4. **(20 points)** Let L_1, L_2, L_3 be subsets of the set $\omega = \{0, 1, 2, \ldots\}$ of nonnegative integers such that:

   - For all $i \neq j$, $L_i \cap L_j = \emptyset$, i.e., no number is common in any two subsets.
   - $L_1 \cup L_2 \cup L_3 = \omega$, i.e, every number from ω is in one of the subsets.
   - Each of the subset L_i, for $i = 1, 2, 3$, is recursively enumerable.

   Prove that each of the subset L_i, for $i = 1, 2, 3$, is recursive.