Timetable Design Problem This problem has the following instance:

- A set H of work-periods.
- A set W of workers.
- A set T of tasks.
- For each $w \in W$, a set $A(w) \subseteq H$ for when w is able to work.
- For each $t \in T$, a set $A(t)$ for when each task t is available to be completed.
- For each pair $(w,t) \in W \times T$ a number $R(w,t) \in \mathbb{Z}_0^+$ referring to number of times a worker w has to work at task t.

The corresponding decision problem is:

Is there a timetable for completing all tasks? The answer is in the form of a function $f{:}\,W \times T \times H \to \{0,1\}$ where $f(w,t,h) = 1$ means that a worker w works on task t at time h. This function has to be subject ot the following constraints:

1. $f(w,t,h) = 1$ only if $h \in A(w) \cap A(t)$.
2. For each $h \in H$ and $w \in W$, there is at most one $t \in T$ for which $f(w,t,h) = 1$.
3. For each $h \in H$ and $t \in T$, there is at most $w \in W$ for which $f(w,t,h) = 1$.
4. For each pair $(w,t) \in W \times T$ there are exactly $R(w,t)$ values of h for which $f(w,t,h) = 1$.

This problem is known to be NP-complete. You could get rid of requirement 3. I am not 100% sure if this makes the problem easier or not. It might lead to trivial positive answers where every worker is working on 1 task. You might want have a number $M(t)$ for the maximum number of workers that can work on a task. I think with this additional criteria, we are back at the original problem since a task t can be split into tasks $t_1, \dots t_{M(t)}$. One thing this problem does not include is making sure every task is completed.

Basis Timetable Problem This problem has the following instance:

1. A set H of work-periods.
2. A set W of workers.
3. A set T of tasks.
4. For each $w \in W$, a set $A(w) \subseteq H$ for when w is able to work.
5. For each $t \in T$, a set $A(t)$ for when each task t is available to be completed.
6. A number $L(w) \in \mathbb{Z}_0^+$ for each $w \in W$ for the maximum number of tasks w can work on.
7. A number $S(t) \in \mathbb{Z}_0^+$ for each $t \in T$ which is the maximum number of times to perform a task (i.e. different workers on the same task. These tasks are allowed to happen at different times unlike the comment for the previous problem I made.)
8. A function $A{:}\,W \times T \to \{\text{true, false}\}$ that determines if a worker can perform a task. The function is A for adept.

The corresponding decision problem is:

Is there a timetable for completing all the tasks? Is there a function $f: W \times T \times H \to \{0,1\}$ as defined above that satisfies the following constraints.

1. $f(w,t,h) = 1$ only if $h \in A(w) \cap A(t)$.
2. For each $h \in H$ and $w \in W$, there is at most one $t \in T$ for which $f(w,t,h) = 1$.
3. For each $w \in W, t \in T$ and $h \in H f(w,t,h) = 1$ only if $A(w,t) =$ true.
4. For each $w \in W, \sum_{t,h} f(w,t,h) \leq L(w)$.
5. For each $t \in T, \sum_{w,h} f(w,t,h) \leq S(t)$.

This problem is in P so there is a polynomial time algorithm that finds a solution to this problem. I believe it is constructive, I will have to read more.

Extended Timetable Problem This problem has the following instance:

- A set H of work-periods.
- A set W of workers.
- A set T of tasks.
- A set R of resources.
- For each $w \in W$, a set $A(w) \subseteq H$ for when w is able to work.
- For each $t \in T$, a set $A(t)$ for when each task t is available to be completed.
- For each $r \in R$, a set $A(r) \subseteq H$ for when r is available.
- A number $L(w) \in \mathbb{Z}_0^+$ for each $w \in W$ for the maximum number of tasks w can work on.
- A number $S(t) \in \mathbb{Z}_0^+$ for each $t \in T$ which is the maximum number of times to perform a task
- A number $U(r) \in \mathbb{Z}_0^+$ for each $r \in R$ which is the maximum number of tasks a resource can complete. (i.e., a supercomputer may only be able to run 1 job at a time)
- A function $A: W \times T \to \{\text{true, false}\}$ that determines if a worker can perform a task. The function is A for adept.
- A function $RS: T \times R \to \{\text{true, false}\}$ for resource suitability where $RS(t,r)$ is true if a resource r can be used for task t.

This problem has the associated question:

Is there a timetable for completing all the tasks? Is there a function $f: W \times T \times H \times R \to \{0,1\}$ where $f(w,t,h,r) = 1$ meaning worker w completes task t at time h with resource $r$subject to the following constraints.

1. $f(w,t,h,r) = 1$ only if $h \in A(w) \cap A(t) \cap A(r)$.
2. For each pair $(w,h) \in W \times H$, there is at most one pair $(t,r) \in T \times R$ for which $f(w,t,h,r) = 1$.

3. For each pair $(r, h) \in R \times H$ there is at most one pair $(w, t) \in W \times T$ for which $f(w, t, h, r) = 1$.

4. For each $w \in W, t \in T, r \in R$ and $h \in H$, $f(w, t, h, r) = 1$ only if $A(w, t) = \text{true}$.

5. For each $w \in W, t \in T, r \in R$ and $h \in H$, $f(w, t, h, r) = 1$ only if $RS(t, r) = \text{true}$.

6. For each $w \in W$, $\sum_{t,h,r} f(w, t, h, r) \leq L(w)$.

7. For each $t \in T$, $\sum_{w,h,r} f(w, t, h, r) \leq S(t)$.

8. For each $r \in R$, $\sum_{w,t,h} f(w, t, h, r) \leq U(t)$.

This problem is NP-complete.

These definitions came from [Lov10].

References

[Lov10] April L Lovelace, *On the complexity of scheduling university courses* (2010).