Stability of solution of higher order nonlinear schrödinger equation for ion-acoustic wave in plasma

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Abstract

An increasing interest for mathematicians are shown towards the researching of nonlinear systems especially for stability of a nonlinear system like controlling the quad helicopter, high-speed drones. Analogously, in the last several decades , physicists had revealed much consideration in the areas of working out exact solutions of schrödinger equation since this equation is highly governed the dynamics of optical fibers for telecommunication. Due to a blend of these two fields arose a few interesting problems including optical pulse propagation in nonlinear optical communication, and instabilities of plasma. Conducting researches on stability of different waves propagated in plasma is significant since instabilities of plasma results in failure of resolving problems of controlling thermonuclear fusion. Similarly, such stability analyzing method could also be applied in other physics filed like behavior of optical solitons based on varying conditions. This paper starts with derivation of in-homogeneous higher order nonlinear schrödinger equation (IHNLSE) with basic continuity, momentum equations for ions and electrons as well as Poisson equation. Then, with method of Bäcklund transformation and Ablowitz-Kaup-Newell-Segur system, three family solutions of INNLSE will be obtained. In order to measure stability for each solution, Lyapunov stability adopted as a powerful tool to complete this work. Notably, with cautious selection of initial conditions, stability of many solutions can be apparently investigated without constructing of Lyapunov function.

Keywords: nonlinear Schrödinger equation, Plasma, Lyapunov stability, soliton.

1 Derivation of IHNLSE

1.1 Introduction

Understanding of nonlinear phenomenon especially modulational instabilities of solitions is necessary since its dynamics will influence the optical fiber communication system. In the last decade Korte-de-Vries equation[1] had been applied into nonlinear phenomenon which enables researchers to elaborate formation of soliton and its propagation both theoretically and experimentally. Experiments show that slow modulation of ion-acoustic wave in plasma could result in formation of envelope solitons and this phenomenon is capable of explaining by nonlinear schrödinger equation (NLSE). Actually NLSE not only lay a foundation for optics study but also it could also explain a wave propagate through nonlinear medium.

Sallahuddin[2] in his work has already studied the nonentities of ion-acoustic wave packets and he mentioned that it is valuable to study the stability of solitary wave solution. Stabilization of wave solution for solition is the threshold of controlling it and improving performance if optical fiber communication.The perfect equation for explaining behavior of solitons appear to be the nonlinear schrödinger equation and the next subsection will contained the content of how to derive higher order NLSE.

1.2 Higher order NLSE

With model devised by Gogoi R[3] *et al*, the basic equations for ions and electrons:

$$For\ ions,\quad \frac{\partial n_i}{\partial t}+\frac{\partial (n_i u_i)}{\partial t}=0, \frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}+\frac{\partial \phi}{\partial x}=0 \tag{1}$$

$$For\ electrons,\quad \frac{\partial n_e}{\partial t}+\frac{\partial (n_e u_e)}{\partial t}=0, \frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}=\frac{1}{Q}\left(\frac{\partial \phi}{\partial x}-\frac{1}{n_e}\frac{\partial n_e}{\partial x}\right) \tag{2}$$

$$Poisson,\quad \frac{\partial^2 \phi}{\partial x^2}+n_i-n_e=0 \tag{3}$$

where u_i, u_e indicates the flow variables of ions and electrons respectively, n_i, n_e are ions and electrons density, ϕ is the electrical potential. Now, introducing the slow variables $\xi=\epsilon(x-\lambda t), \tau=\epsilon^2 gt$, ϵ is a small parameter measure nonliterary while $\lambda\geq 0, g$ are some scale parameters. Then express dependent variables from eq (1) to (3) with slow parameter and fast variable x, t:

$$n_i=\sum_{j=1}^{\infty}\epsilon^j n_{ij}(x,t,\xi,\tau), n_e=\sum_{j=1}^{\infty}\epsilon^j n_{ej}(x,t,\xi,\tau), u_i=\sum_{j=1}^{\infty}\epsilon^j u_{ij}(x,t,\xi,\tau) \tag{4}$$

$$u_e=\sum_{j=1}^{\infty}\epsilon^j u_{ej}(x,t,\xi,\tau), g=1+\sum_{j=1}^{\infty}\epsilon^j g_j, \phi=\sum_{j=1}^{\infty}\phi_j(z,t,\xi,\tau) \tag{5}$$

coefficients function $n_{ij}, u_{ij}, u_{ej}, \phi_j$ will be determined from the solution of the field equations. From above equations, standard nonlinear schrödinger equation derived by

Kautani[4] (et al):

$$i\frac{\partial\Psi}{\partial\tau} + p\frac{\partial^2\Psi}{\partial\xi^2} + q|\Psi|^2\Psi = 0 \tag{6}$$

where q, p are dispersive and nonlinear coefficients can be expressed as below:

$$p = \left[\frac{1}{\omega^2} + \frac{Q}{4k^2} + \frac{1-4\omega^2}{4k^2(1-\omega^2)}\right]\left[2\left(\frac{k}{1-\omega^2} - \frac{1}{\omega k} + \frac{Q}{\omega}\right)\right] \tag{7}$$

$$\begin{aligned} q = \Bigg[&\frac{1}{\omega^2(1+Q)-4^2}\left(24Q\omega^2 + 4\omega^2 - 6\omega Q + 8\omega^2 Q^2 + \frac{\omega^4}{1-\omega^2}(8+Q)\right) \\ &\frac{9}{2} + 2Q - \frac{1}{2\omega^2} - 4Q(1-\omega^2) + \frac{2(1-\omega^2)^2}{3\omega^2}Q(Q-2) \\ &\frac{1-\omega^2}{\omega^2}\Big(\frac{3}{2}+Q\Big) - 4Qk^2\Bigg] \Bigg/ \Bigg[2\Big(\frac{k}{1-\omega^2} - \frac{1}{\omega k} + \frac{Q}{\omega}\Big)\Bigg] \end{aligned} \tag{8}$$

The above two coefficient functions are complicated. After some mathematical calculations and compared the derived equation with R Gogoi *et al*[5] paper, the IHNLSE obtained:

$$\begin{aligned} \Psi_z = {} & i(\alpha_1(z)\Psi_{tt} + \alpha_2(z)|\Psi|^2\Psi) + \alpha_3(z)\Psi_{ttt} \\ & \alpha_4(z)(|\Psi|^2\Psi)_t + \alpha_5(z)\Psi(|\Psi|^2)_t + \Gamma(z)\Psi \end{aligned} \tag{9}$$

Where $\Psi(x,t)$ means complex envelope of the electric field, z represents normalized propagation distance while t is the normalized retarded time, $\alpha_1(z), \alpha_2(z), \alpha_3(z), \alpha_4(z), \alpha_5(z)$ are distributed parameters and all of them are variables depend on distance z, related to group velocity dispersion (GVD), self-phase-modulation (SPM), third-order dispersion(TOD), self-steepening, and the delayed nonlinear response effect[6].
In a nutshell, a large number of mathematicians has already made some profound research in equation (9) for example Li *et al*[7] has already attained several families of exact analytical solution of this equation via the generalized sub-equation expansion method. For specified management conditions Yang *et al*[8] analyzed precise blended solution for dark soliton.
$\Gamma(z)$,in IHNLSE, denotes the amplification or absorption coefficient functions but in some cases[9] this term namely Kerr term are ignored to reduce the difficulty of finding exact analytical solution.Actually, equation (9) can be changed into many forms with varying coefficient functions $\alpha_i(z), i = 1 \cdots 5$. For instance, let $\alpha_3(z) = \alpha_4(z) = \alpha_5(z) = \Gamma(z) = 0$, the original equation will transformed into perturbed nonlinear Schrödinger equation which has been discussed in Tian *et al*[10].

2 Analytical solution of IHNLSE

According to AKNS system and Bäcklund transformation which accurately in analyzed in Lu *et al* paper[11], ion-acoustic wave Ψ have the following solution expression:

$$\begin{aligned}
&\Psi = \left[A_0(z) + A_1\frac{\delta cosh(\xi)+cos(\eta)}{cosh(\xi)+\delta cos(\eta)} + iB_1(z)\frac{\alpha sinh(\xi)+\beta sin(\eta)}{cosh(\xi)+\delta cos(\eta)}\right]e^{\Delta i}\\
&\xi = p_1(z)t + q_1(z),\\
&\eta = p_2(z)t + q_2(z),\\
&\Delta = k_2(z)t^2 + k_1(z)t + k_0(z)
\end{aligned} \tag{10}$$

For above equation system, one can easily notice that $A_0(z), A_1(z), B_1(z), p_1(z), p_2(z), q_1(z), q_2(z), k_0(z), k_1(z)$, and $k_2(z)$ are real number functions of distance z which to be determined. α, β, δ are three real constants. If one assume $\eta = \delta = k_2(z) = 0$, then the Riccati equation applied in Hung *et al* paper[6] can be applied. Now substitute expressions showed in Eq(10) into the original Eq (9), the IHNLSE will become ordinary differential equation (ODEs). Solving this this sophisticated ODE system three families of exact analytical solution obtained:

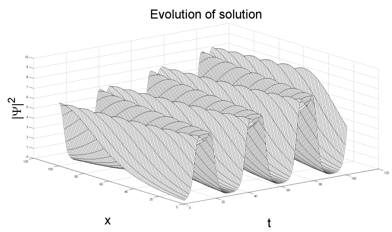
2.1 Family 1

$$\begin{aligned}
&\Psi = B_1(z)\beta\left[-\frac{\delta}{2M(1-\delta^2)^{\frac{1}{2}}} + \frac{M}{(1-\delta^2)^{\frac{1}{2}}}\frac{\delta cosh(\xi)+cos(\eta)}{cosh(\xi)+\delta cos(\eta)} + \frac{isin(\eta)}{cosh(\xi)+\delta cos(\eta)}\right]e^{\Delta i}\\
&a_2 = \frac{2C_5^2 a_1}{B_1^2(z)\beta^2}, a_4 = \frac{6C_5^2 a_3}{B_1^2(z)\beta^2}, a_5 = -\frac{6C_5^2 a_3}{B_1^2(z)\beta^2}, \Gamma(z) = lnB_1(z)_z\\
&\xi = C_5 t + \frac{C_5}{2(1-\delta^2)}\int\left[4C_4(\delta^2-1)a_1 + a_3(\delta^2(C_5^2+6C_4^2) - 6C_4^2 + 2C - 5^2)\right]\,dz + C_3\\
&\eta = \frac{MC_5^2}{(1-\delta^2)^{\frac{1}{2}}}\int(a_1 + 3a_3C_4)\,dz + C_1\\
&\Delta = C_4 t + \frac{1}{2(1-\delta^2)^{\frac{1}{2}}}\int\left[a_3(2\delta^2C_4^3 + 3\delta^2C_5^2C_4 - 2C_4^3) + a_1(2\delta^2C_4^2 + \delta^2C_5^2 - 2C_4^2)\right]\,dz + C_2
\end{aligned} \tag{11}$$

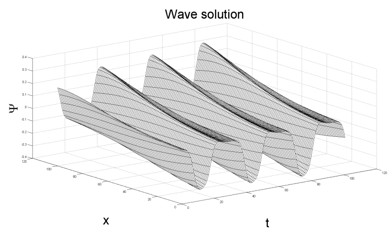
Where $M = \pm 1, \delta \in (-1,1), \beta \neq 0$ and $C_i, i = 1,\cdots,5$ are arbitrary constants, $a1(z), a2(z), \Gamma(z)$ are arbitrary nonzero functions of the normalized distance z. In the previous work the solitary wave solution of higher order non-Kerr non-linearity have not been investigated. Without any restriction of δ, one can let $\delta = 0$, If did so, Eq (11) definitely cancel out terms that included δ. Thus, coefficient ξ will delete two terms which make integral easier with respect to time while η do not change since there is no δ term contained in this expression. For Δ, integral terms is the most complicated term since this term consists of two sub-terms and both of them included δ Then the above solutions system becomes:

$$
\begin{aligned}
\Psi &= M\beta B_1(z)\,\mathrm{sech}(\xi)e^{i(\Delta+M\eta)} \\
\xi &= C_5 t + C_5\int\left[(a_1+3a_3C_4)-2C_4a_1\right]\,dz + C_1 \\
\eta &= MC_5^2\int(a_1+3a_3C_4)\,dz + C_1 \\
\Delta &= C_4 t - C_4^2\int(a_1+a_3C_4)\,dz + C_2
\end{aligned}
\tag{12}
$$

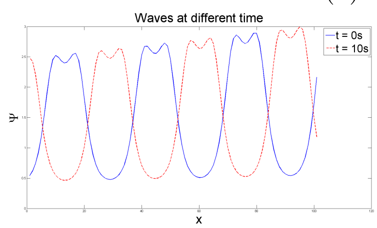
So for Eq (11), Using Mathmatica to simulate the solution, evolution of solution, and wave stability, following figure obtained with given conditions: $\delta = 0.2, \beta = 3, C_1 = 0.2, C_2 = 0.3, C_3 = 0.4, C_4 = 0.2, C_5 = 15, M = 1$ and $B_1 = e^{0.01z}, a_1 = sin(z), a_3 = cos(z)$



(a) Solution evolution



(b) Wave solution



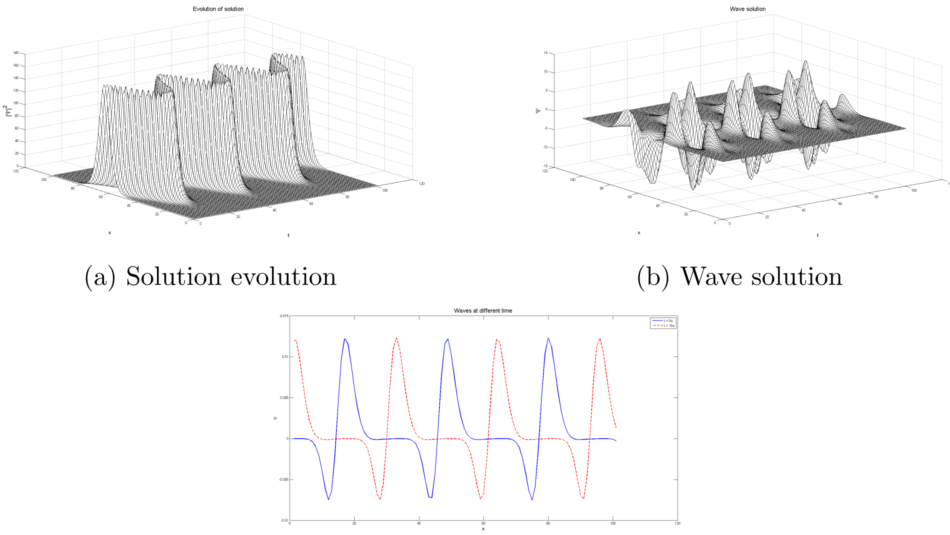
(c) Wave Stability

Figure 1: Family 1 solution($\delta \neq 0$)

Figure 1 shows three properties of wave including wave solution, wave stability and evolution of solution. It is easy to read from figure (c) that ion-acoustic wave is not stable under specified condition since amplitude of wave increases as time grows.

Likewise, one can also to tell from figure (a) that the amplitude of bright solitons augments as the propagation distance enlarges due to the amplification $\Gamma(z) = 0.01$ while the time shift and the velocity of the solitary are changing. Lyapunov stability will not considered here since in this example, the stability of ion-acoustic wave can be differentiated simply.

If anyone let $\delta = 0$, a concise solution of will be attained. Assume initial condition: $\beta = 13, C_1 = C_2 = C_3 = 0, C_4 = C_5 = 2, M = 1$ and $B_1 = 2sech(0.01z), a_1 = cos(z), a_3 = sin(z)$. As a special solution of family 1, one may states that this is system is also unstable. However, from figure (c) showed in solution of $\delta = 0$, this model is stable or whereas it is interesting to observe this model is not asymptotically stable since this model is shifted between upper and lower limits.



(a) Solution evolution

(b) Wave solution

(c) Wave Stability

Figure 2: Family 1 solution($\delta = 0$)

2.2 Family 2

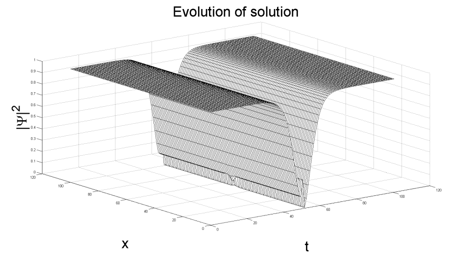
For family solution 2, it can be classified into four cases in order to simplify this paper only the first two cases will be discussed here.

Case 1:

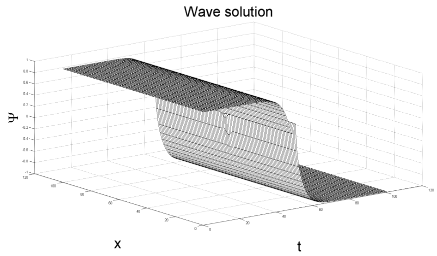
$$\begin{aligned}
\Psi &= \left[-A_1[z]\delta + \frac{M\beta B_1(z)}{\delta^2 - 1} \frac{\delta cosh(\xi) + cos(\eta)}{cosh(\xi) + \delta cos(\eta)} + \frac{iB_1 \beta sin(\eta)}{cosh(\xi) + \delta cos(\eta)} \right] \\
\xi &= C_6 t + C_3 \\
\eta &= C_4 t + C_1 \\
\Delta &= C_5 t + C_2
\end{aligned} \tag{13}$$

Take a look at first case of family solution 2, it seems to be much easier than the first one since three intermediate coefficients functions are merely functions of time which means these coefficients are independent of space.
With given conditions: $\delta = 2, \beta = 1.5, C_1 = C_2 = C_3 = 1, C_4 = 3, C_5 = 2, C_6 = 5, M = 1$ and $A_1(z) = 0.5tanh(0.8z), B_1(z) = 0.5sech(9z)$, Figure 3 presents that such model is not only stable but also asymptotically stable or exponential stable.

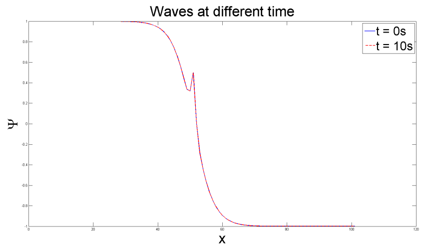
According to figure (c) wave stability, wave at latter times almost have the same shape of earlier time that is to say these two waves overlaps completely. On the other hand, from figure (b) it is not hard to recognize that wave will decay as time is large enough as time is small enough.



(a) Solution evolution



(b) Wave solution



(c) Wave Stability

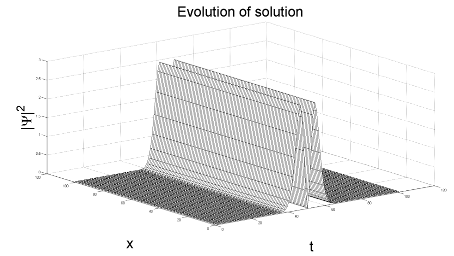
Figure 3: Family 2 solution(case 1)

Case 2:

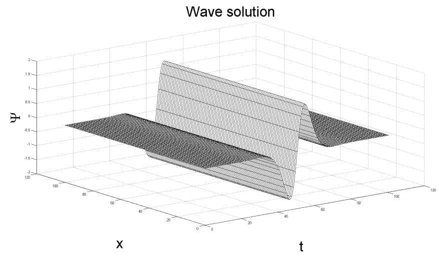
$$
\begin{aligned}
\Psi &= A_1(z)\left[E_7 + \frac{\delta cosh(\xi) + cos(\eta)}{cosh(\xi) + \delta cos(\eta)}\right] e^{i\Delta} \\
\xi &= E_6 t + E_1 \\
\eta &= E_5 t + E_2 \\
\Delta &= E_4 t + E_3
\end{aligned} \tag{14}
$$

Three intermediate variables just like case 1 since ξ, η, Δ are only depend on time has no relationship with spatial computation. Furthermore, the expression for wave function Ψ is much simpler than the solution Eq (13) and the second term contained in brackets is same to term showed in case 1.
for aforementioned selection of given conditions, this time a more heedful to opt initial conditions will be applied. Taking experience of aerodynamic non-linearity into consideration, choice of constants should be small which are more likely to achieve the stable status. Thus given conditions: $E_1 = 0.004, E_2 = 0.001, E_3 = 0.003, E_4 = 0.006, E_5 = E_6 = 0.003, E_7 = 8, \delta = 2$ and $A_1(z) = 0.4sin(z)sech(0.4\pi x)$
Like case 1, case 2 solution is also a stable one based on figure (c). One may wonder whether it is asymptotically stable. To answer this question, asymptotically stable

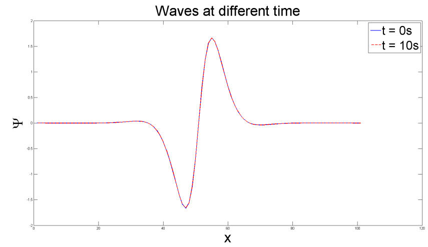
defines any system or behavior of object not only have to move in a boundary at any time but also this system also approaches to zero as time approaches infinity.



(a) Solution evolution



(b) Wave solution



(c) Wave Stability

Figure 4: Family 2 solution(case 2)

2.3 Family 3

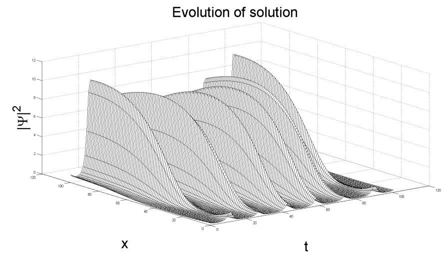
Similarly this third family solution is able to categorized into two cases and this subsection will explained both of them.

Case 1

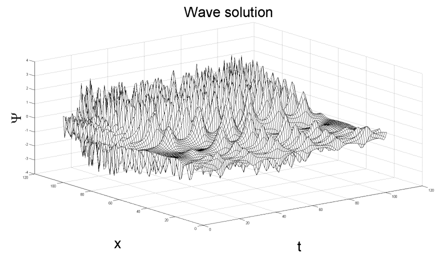
$$\Psi = B_1(z)\left[\frac{\alpha^2+\beta^2\delta^2}{2\delta M[(\alpha^2+\beta^2)(\delta^2-1)]^{1/2}}\right.$$
$$\frac{M(\alpha^2+\beta^2)^{1/2}}{\delta^2-1}\frac{\delta cosh(\xi)+cos(\eta)}{cosh(\xi)+\delta cos(\eta)}$$
$$\left. i\frac{\alpha sinh(\xi)+\beta sin(\eta)}{cosh(\xi)+\delta cos(\eta)}\right]e^{i\Delta}$$
$$\xi = \frac{\delta\beta C_4 k_2(z)}{a}t + \frac{\delta C_4 k_2(z)[C_4(\beta^2\delta^2-2\beta^2-\alpha^2)+2MC_5\beta(\alpha^2+\beta^2)^{1/2}(\delta^2-1)^{1/2}]}{4\alpha M(\alpha^2+\beta^2)^{1/2}(\delta^2-1)^{1/2}}$$

$$\eta = k_2(z)C_4t + \frac{C_4k_2z[C_4(\beta^2\delta^2 + 2\beta\delta^2 - \alpha^2\beta) + 2MC_5(\alpha^2+\beta^2)^{1/2}(\delta^2-1)^{1/2}]}{4\alpha M(\alpha^2+\beta^2)^{1/2}(\delta^2-1)^{1/2}}$$
$$\Delta = k_2(z)t^2C_5k_2(z)t + \frac{k_2(z)[C_4(\alpha^2+\beta^2+\delta^2)^2 + 2\alpha^2C_5^2(\alpha^2+\beta^2)^{1/2}(\delta^2-1)^{1/2}]}{8\alpha^2(\alpha^2+\beta^2)(\delta^2-1)} \tag{15}$$

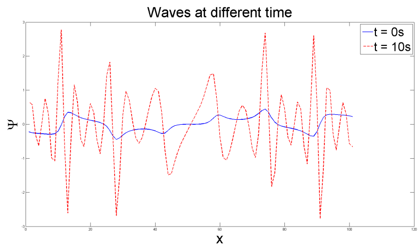
With given condition: $C_1 = 0.2, C_2 = 1, C_3 = 1, C_4 = 0.3, C_5 = 8, M = 1, \delta = 0.8, \alpha = 0.4, \beta = 0.3$ and $B_1(z) = \dfrac{-0.4tanh(2z)}{1-0.8sin^2(z)}, k2(z) = 0.7sech(0.1z)$



(a) Solution evolution



(b) Wave solution



(c) Wave Stability

Figure 5: Family 3 solution(case 1)

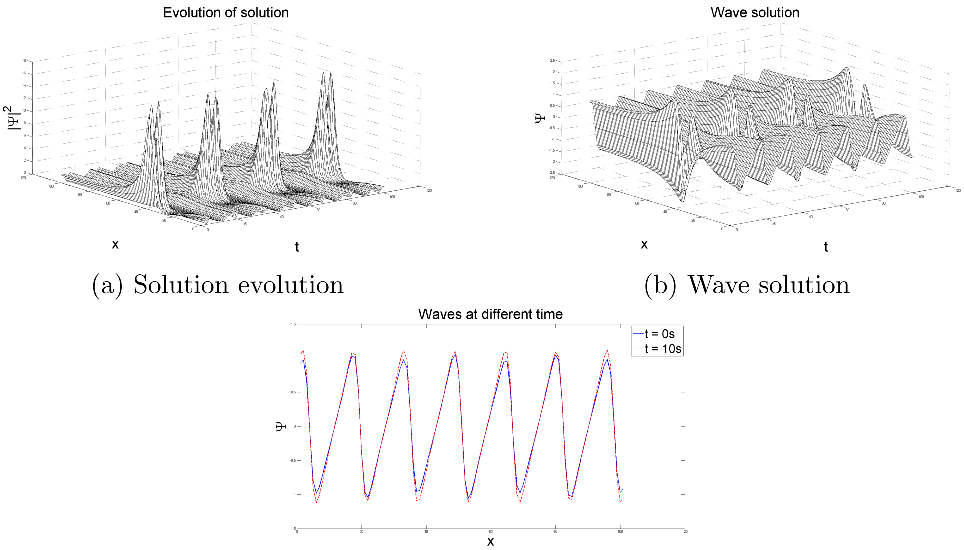
Figure 5 indicates this model ion-acoustic wave will oscillates boundlessly since figure (c) obviously show this property. Besides that a close look at wave solution also unveils the fluctuation of wave in specified time range which cannot be ignored. This model just like the model discussed above all of them can be studied through observation without construction of Lyapunov function to prove its in/stability especially for model 5 the jumping property is sumptuous.
The case 2 of family solution 3 compared with above five models is not that easy to observe the results since its wave stability diagram fluctuates in a certain range. Without computation, it is incorrect for anyone to say family 3 solution case 2 is stable.

3 Lyapunov stability for Family 3 case 2

Analytical solution for family 3 case 2 :

$$\Psi = B_1(z)\left[-\frac{\delta\beta}{2M(1-\delta^2)^{1/2}} + \frac{M\beta(\delta cosh(\xi)+cos(\eta))}{(1-\delta^2)^{1/2}(cosh(\xi)+\delta\cos(\eta))} + \frac{i\beta sin(\eta)}{cosh(\xi)+\delta cos(\eta)}\right]e^{i\Delta}$$
$$\xi = G_5 t + \frac{1}{2}G_5 G_4 ln(k_2(z)) + G_3$$
$$\eta = \frac{G_5^2}{4(1-\delta^2)^{1/2}Mk_2(z)} + G_1$$
$$\Delta = k_2(z)t^2 + G_4 k_2(z)t + \frac{2G_4^2 k_2(z)(\delta^2-1) - \delta^2 G_5^2 + 8G_2 k_2(z)(\delta^2-1)}{8k_2(z)(\delta^2-1)} \quad (16)$$

With specified condition: $G_1 = G_4 = 2, G_2 = G_3 = 1, G_5 = 0.4, \delta = 0.1, \beta = 1.3, M = 1$ and $k_2(z) = 1.5cos(z), B_1(z) = \dfrac{-0.4sin(2z)}{1-0.8cos^2(z)}$



(a) Solution evolution

(b) Wave solution

(c) Wave Stability

Figure 6: Family 3 solution(case 2)

Focusing on figure (c) from Figure 6, no one can directly to claim that this model is stable since the latter wave is pretty close to the initial wave. Thus in order to study its stability well, Lyapunov function will introduced here to serve as an efficient tool.

3.1 Lyapunov function

From chapter 4 of *Nonlinear System*[12] and Khail research paper[13] intricate that Lyapunov stability give sufficient conditions for stability, asymptotically stability and so on. Lyapunov stability analysis can be used to show boundedness of the solution even when the system has no equilibrium points. Lyapunov also stipulates that if the functioned created based on system variables is positive semi-definite and it derivative negative semi-definite then this system is stable and if this function is positive definite and its derivative negative definite we can say this system is asymptotically stable. This function is called Lyapunov function.
Construction Lyapunov function is not a easy work, one can define any function to be Lyapunov function and if the Lyapunov function selected does not satisfy Lyapunov core principle, it is unwise to say this model is unstable since Lyapunov has already pointed out that there are countless functions of function if one satisfied this system is stable.
At the beginning, try Lyapunov function be:

$$f(\Psi) = \frac{1}{2}\Psi^2 \tag{17}$$

Since Ψ is a complex wave which means it contained complex number so the square of wave should be Ψ dot product conjugate of Ψ So the fist thing is to calculate Ψ with given condition. In order to save, Program will run in Mathematica to obtain the algebra solution of Ψ:

$$-\frac{0.4\sin(x)}{-0.8\cos^2(x)+1}\left(-0.0653274579918488+\frac{1.3i\sin\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)}{0.1\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+\cos(0.4t+0.4\log(1.5\cos(x))+1.0)}\right.$$
$$\left.+\frac{1.3\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+0.13\cosh(0.4t+0.4\log(1.5\cos(x))+1.0)}{0.099498743710662\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+0.99498743710662\cosh(0.4t+0.4\log(1.5\cos(x))+1.0)}\right)$$
$$e^{1.0i\left(1.5t^2\cos(x)+3.0t\cos(x)-\frac{-1.0\cos(x)-0.666801346801347}{\cos(x)}\right)}$$

After calculation $f(\Psi)$ is positive definite. Now considering its derivative:

$$\dot{f}(\Psi) = \Psi\Psi_t \tag{18}$$

Through Mathematica Ψ_t obtained:

$$- \frac{0.4i\sin(x)}{-0.8\cos^2(x)+1}(3.0t\cos(x)+3.0\cos(x))\Bigg(-0.0653274579918488 + \frac{1.3i\sin\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)}{0.1\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+\cos(0.4t+0.4\log(1.5\cos(x))+1.0)}$$

$$+ \frac{1.3\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+0.13\cosh(0.4t+0.4\log(1.5\cos(x))+1.0)}{0.099498743710662\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+0.99498743710662\cosh(0.4t+0.4\log(1.5\cos(x))+1.0)}\Bigg) e^{1.0i\left(1.5t^2\cos(x)+3.0t\cos(x)-\frac{-1.0\cos(x)-}{}\right.}$$

$$- \frac{0.4\sin(x)}{-0.8\cos^2(x)+1}\Bigg(\frac{0.52i\sin\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)\sin(0.4t+0.4\log(1.5\cos(x))+1.0)}{\left(0.1\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+\cos(0.4t+0.4\log(1.5\cos(x))+1.0)\right)^2}$$

$$+ \frac{0.052\sinh(0.4t+0.4\log(1.5\cos(x))+1.0)}{0.099498743710662\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+0.99498743710662\cosh(0.4t+0.4\log(1.5\cos(x))+1.0)}$$

$$- \frac{0.397994974842648\left(1.3\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+0.13\cosh(0.4t+0.4\log(1.5\cos(x))+1.0)\right)\sinh(0.4t+0.4\log(1.5\cos(x))+1}{\left(0.099498743710662\cos\left(2.0+\frac{0.0268010084069123}{\cos(x)}\right)+0.99498743710662\cosh(0.4t+0.4\log(1.5\cos(x))+1.0)\right)^2}$$

$$\Bigg) e^{1.0i\left(1.5t^2\cos(x)+3.0t\cos(x)-\frac{-1.0\cos(x)-0.666801346801347}{\cos(x)}\right)}$$

After the calculation, the derivative of function $f(\Psi)$ is negative definite this function prove that model six (family solution 3 case 2) is asymptotically stable.

4 Conclusion

In this paper, five soliton solutions obtained through solving 1D IHNLSE with Bäcklund transformation and ANKS system. All properties of these solution has been presented via auxiliary computer simulation.

For stability each solution varied with different given condition. Among these three family solution, family 2 solution seems to be the most reliable model. The wave will become asymptotically stable as time approaches infinity.

For family 1 solution, coefficient function $B_1(z)$ dominates the stability of wave while for family 2 solution changing of specified conditions seldom affected the stability of wave but this does not mean family 2 will always be stable no what given condition.

Family 3 is more complicated than family 1 and family 2 since one cannot observe stability of wave. In order to ensure its stability property, Lyapunov stability method is introduced to work it out.

Stabilization of a certain type of wave can be achieved through study the self-designing conditions and the further study of plasma will concentrate on selection of appropriate conditions.

Appendix A Code for Family 1 solution ($\delta = 0$)

```
function INNLSE_c1_dt
%------------------------------------
%constants:
i = sqrt(-1);
beta = 13;
C_1 = 0;
C_2 = 0;
C_3 = 0;
C_4 = 2;
C_5 = 2;
M = 1;
%------------------------------------
%linspace for time and space:
t = linspace(-10,10,101);
x = linspace(-10,10,101);
%------------------------------------
%coefficients:
B_1 = 2*sech(0.01*x);
a_1 = cos(x);
a_2 = 2*C_5*a_1./(B_1.^2*beta^2);
a_3 = sin(x);
a_4 = 6*C_5.*a_3./(B_1.^2*beta^2);
a_5 = -a_4;
%------------------------------------
%variable:
%why not calculate in for-loop due to efficiency
    xi_s = -2*(sin(x)-cos(x));
    eta_s = sin(x) - 3*cos(x);
    delta_s = sin(x)- cos(x);
for k = 1:101
    for j = 1:101
        xi(k,j) = t(k) + xi_s(j);
        eta(k,:) = eta_s;
        delta(k,j) = t(k) - delta_s(j);
%calculate the wave psi:
        psi(k,j) = M*beta*sech(xi(k,j))...
        *exp(i*(delta(k,j) + M*eta(k,j)));
        psi_r(k,j) = real(psi(k,j));
        psi_i(k,j) = imag(psi(k,j));
        psi_se(k,j) = psi(k,j)*conj(psi(k,j));
```

```
    end
end
figure;
surf(psi_se,'FaceColor',[1 1 1]);
title('Evolution of solution','FontSize',40)
xlabel('t','FontSize',40);
ylabel('x','FontSize',40);
zlabel('{|\Psi|}^2','FontSize',40);
figure;
surf(psi_r,'FaceColor',[1 1 1]);
title('Wave solution','Fontsize',40);
xlabel('t','FontSize',40);
ylabel('x','FontSize',40);
zlabel('\Psi','FontSize',40);

figure;
plot(psi_r(1,:),'Linewidth',1.5);
hold on
plot(psi_r(101,:),'r--','Linewidth',1.5);
xlabel('x','FontSize',40);
ylabel('\Psi','FontSize',40);
title('Waves at different time','FontSize',40);
h_l = legend('t = 0s','t = 10s');
set(h_l,'FontSize',35);

save('INNLSE_f1_dt');
evalin('base','load INNLSE_f1_dt');
end
```

Appendix B Code for Family 2 solution case 1

```
function INNLSE_f2_c1
%----------------------------------
%constants:
dt = 2;
beta = 1.5;
C_1 = 1;
C_2 = 1;
C_3 = 1;
C_4 = 3;
C_5 = 2;
C_6 = 5;
M = 1;
im = sqrt(-1);
%----------------------------------
%linspace for time and space:
t = linspace(-10,10,101);
x = linspace(-10,10,101);
%----------------------------------
%coefficients:
A1 = 0.5*tanh(0.8*x);
B1 = 0.5*sech(9*x);
%calclulating coefficient:
xi = C_6*t+C_3;
eta = C_4*t+C_1;
%i for time, j for space:

for i  = 1:101
    for j = 1:101
        psi(i,j) = -A1(j)*dt+M*beta*B1(j)/(dt^2-1)...
            *(dt*cosh(xi(i))+cos(eta(i)))...
            /(cosh(xi(i)) + dt*cos(eta(i))) + ...
            im*B1(j)*beta*sin(eta(i))/...
            (cosh(xi(i))+dt*cos(eta(i)));
%----------------------------------
%real and imaginary parts:
    psi_r = real(psi);
    psi_i = imag(psi);
%solution evolution:
    psi_se(i,j) = psi(i,j)*conj(psi(i,j));
    end
```

```
end
figure;
surf(psi_se,'FaceColor',[1 1 1]);
title('Evolution of solution','FontSize',40)
xlabel('t','FontSize',40);
ylabel('x','FontSize',40);
zlabel('{|\Psi|}^2','FontSize',40);
figure;
surf(psi_r,'FaceColor',[1 1 1]);
title('Wave solution','Fontsize',40);
xlabel('t','FontSize',40);
ylabel('x','FontSize',40);
zlabel('\Psi','FontSize',40);

figure;
plot(psi_r(1,:),'Linewidth',1.5);
hold on
plot(psi_r(101,:),'r--','Linewidth',1.5);
xlabel('x','FontSize',40);
ylabel('\Psi','FontSize',40);
title('Waves at different time','FontSize',40);
h_l = legend('t = 0s','t = 10s');
set(h_l,'FontSize',35);

save('INNLSE_f2_c1');
evalin('base','load INNLSE_f2_c1');
end
```

Appendix C Code for Family 3 solution case 1

```
function INNLSE_f3_c1
% when a3 = a4 = a5 = 0
%——————————————————
%constants:
C1 = 0.2;
C2 = 1;
C3 = 1;
C4 = 0.3;
C5 = 8;
M = 1;
dt = 0.8;
alpha = 0.4;
beta = 0.3;
im = sqrt(-1);
%————————————————————————
%linspace for time and space:
t = linspace(-10,10,101);
x = linspace(-10,10,101);
%————————————————————————
%coefficients:
B1 = -0.4*tanh(2*x)./(1-0.8*sin(x).^2);
k2 = 0.7*sech(0.1*x);

  xi_s = dt*C4*k2/alpha + dt*C4*k2...
           *( C4*(beta^2*dt^2-2*beta^2-alpha^2)...
           + 2*M*C5*beta*sqrt(alpha^2+beta^2)...
           *sqrt(dt^2-1))...
           /(4*alpha*M*sqrt(alpha^2+beta^2)...
            *sqrt(dt^2-1));
for i = 1:101
    for j = 1:101
%complicated coefficients:
        xi(i,:) = xi_s;

        eta(i,j) = k2(j)*C4*t(i) + C4*k2(j)...
            *( C4*(beta^2*dt^2+2*beta*dt^2*alpha^2 ...
             -alpha^2*beta) +2*M*C5*sqrt(alpha^2+beta^2)...
             *sqrt(dt^2-1))/( 4*alpha^2*M...
             *sqrt(alpha^2+beta^2)*sqrt(dt^2-1));
```

```
        delta(i,j) = k2(j)*t(i)^2+C5*k2(j)*t(i)...
            +k2(j)*( C4*(alpha^2+beta^2*dt^2)^2 ...
            + 2*alpha^2*C5^2*(alpha^2+beta^2)...
            *(dt^2-1))/( 8*alpha^2*(alpha^2+beta^2)...
            *(dt^2-1) );

%wave:

        psi(i,j) = B1(j)*( (alpha^2+beta^2*dt^2)...
            /(2*dt*M*(alpha^2+beta^2)*sqrt(dt^2 -1))...
            + M*sqrt(alpha^2+beta^2)...
            /sqrt(dt^2-1)*(dt*cosh(xi(i,j))...
            +cos(eta(i,j)))...
            /(cosh(xi(i,j))+dt*cos(eta(i,j)))...
            + im*(alpha*sinh(xi(i,j))+beta*sin(eta(i,j)))...
            /(cosh(xi(i,j))+dt...
            *cos(eta(i,j))))*exp(im*delta(i,j));
%------------------------------
%real and imaginary parts:
       psi_r(i,j) = real(psi(i,j));
       psi_i(i,j) = imag(psi(i,j));
%------------------------------
%solution evolution:
    psi_se(i,j) = psi(i,j)*conj(psi(i,j));

    end
end
figure;
surf(psi_se,'FaceColor',[1 1 1]);
title('Evolution of solution','FontSize',40)
xlabel('t','FontSize',40);
ylabel('x','FontSize',40);
zlabel('{|\Psi|}^2','FontSize',40);
figure;
surf(psi_r,'FaceColor',[1 1 1]);
title('Wave solution','Fontsize',40);
xlabel('t','FontSize',40);
ylabel('x','FontSize',40);
zlabel('\Psi','FontSize',40);

figure;
plot(psi_r(1,:),'Linewidth',1.5);
```

```
hold on
plot(psi_r(101,:),'r--','Linewidth',1.5);
xlabel('x','FontSize',40);
ylabel('\Psi','FontSize',40);
title('Waves at different time','FontSize',40);
h_l = legend('t = 0s','t = 10s');
set(h_l,'FontSize',35);

save('INNLSE_f3_c1');
evalin('base', 'load INNLSE_f3_c1');
end
```

References

[1] Gogoi R, Devi N and Das G C, *Ind. J. of Pure and Appld.*, Phys. 46, 621-628, 2008.

[2] Salahuddin M, Saleem H and Saddiq, *Phys.Rev.*, E**66**, 036407, 2002.

[3] Gogoi R, Kalita L and Devi N, Thermo physical Properties-NCTP'07, Proc. 4th Nat. Kerala AIP Conf. Proc. **1004** 219, 2008.

[4] Kakutani T and Sugimoto N, *Phys.*, *Fluids*, **17**, 1617 1974.

[5] Gogoi, R., L. Kalita, and N. Devi, *"Stability analysis and investigation of higher order Schrödinger equation for strongly dispersive ion-acoustic wave in plasma."* , *Journal of Physics: Conference Series*, Vol. 208. No. 1. IOP Publishing 2010.

[6] Huan-Ping, Zhang, Li Biao, and Chen Yong, *"Some exact solutions to the inhomogeneous higher-order nonlinear Schrödinger equation by a direct method."* , *Chinese physics B* , 19.6 (2010): 060302.

[7] Li, Biao, Yong Chen, and Yu-Qi Li, *"A generalized sub-equation expansion method and some analytical solutions to the inhomogeneous higher-order nonlinear Schrödinger equation."* , Zeitschrift für Naturforschung, A 63.12 (2008): 763-777.

[8] Yang, Rongcao, et al, *"Dark soliton solution for higher-order nonlinear Schrödinger equation with variable coefficients."* , Optics communications 242.1 (2004): 285-293.

[9] Zedan, Hassan A., E. Aladrous,, and S. Shapll, *"Exact solutions for a perturbed nonlinear Schrödinger equation by using Bäcklund transformations."* , Nonlinear Dynamics 74.4 (2013): 1145-1151.

[10] Tian, Bo, and Yi-Tian Gao, *"Symbolic-computation study of the perturbed nonlinear Schrödinger model in inhomogeneous optical fibers."* , *Physics Letters* A 342.3 (2005): 228-236.

[11] Li, Lu, et al, *"Modulation instability and solitons on a cw background in inhomogeneous optical fiber media."* , *Optics Communications* 234.1 (2004): 169-176.

[12] Khalil, Hassan K *Nonlinear System* , Prentice Hall, 2002

[13] Khalil, Hassan K *"Robust servomechanism output feedback controllers for feedback linearizable systems."* , *Automatica* 30.10 (1994): 1587-1599.