多元统计分析 (3)

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3-6 (均值向量的各分量间结构关系的检验)设总体

$$X \sim N_p(\mu, \Sigma) \quad (\Sigma > 0),$$

$X_{(\alpha)}$ $(\alpha=1,\cdots,n)$ $(n>p)$为来自 p 元正态总体 X 的样本,记 $\mu=(\mu_1, \cdots, \mu_p)'$. C 为 $k\times p$ 常数矩阵($k<p$), rank$(C)=k$, r 为已知 k 维向量. 试给出检验 H_0: $C\mu=r$ 的检验统计量及分布.

解. *(1) 当 Σ 已知时:*

因为 μ 的估计量为 $\bar{X}$, 故 $C\mu$ 的估计量为 $C\bar{X}$. 由于

$$\bar{X} \sim N_p(\mu, \frac{1}{n}\Sigma)$$

可推得

$$C\bar{X} \overset{H_0}{\sim} N_k(C\mu = r, \frac{1}{n}C\Sigma C')$$

而

$$\sqrt{n}\Sigma^{-\frac{1}{2}}(\bar{X} - \mu) \sim N_p(0, I_p)$$

故令

$$Y = (Y_1, ..., Y_k)' = \sqrt{n}(C\Sigma C')^{-\frac{1}{2}}(C\bar{X} - r) \sim N_k(0, I_k)$$

有

$$\sum_{i=1}^{k} Y_i^2 = Y'Y = n(C\bar{X} - r)'(C\Sigma C')^{-1}(C\bar{X} - r) \overset{H_0}{\sim} \chi^2(k)$$

(2) 当 Σ 未知时:

$$T^2 = n(C\bar{X} - r)'(CSC')^{-1}(C\bar{X} - r) \overset{H_0}{\sim} T^2(k, n-1)$$

或者

$$F = \frac{n-k}{(n-1)k}T^2 = \frac{n(n-k)}{k}(C\bar{X} - r)'(CAC')^{-1}(C\bar{X} - r) \overset{H_0}{\sim} F(k, n-k)$$

3-7 设总体 $X \sim N_p(\mu, \Sigma)$ $(\Sigma > 0)$，$X_{(\alpha)}(\alpha=1,\cdots,n)$ $(n>p)$ 为来自 p 元正态总体 X 的样本，样本均值为 $\overline{X}$，样本离差阵为 A. 记 $\mu=(\mu_1,\cdots,\mu_p)'$. 为检验 H_0: $\mu_1=\mu_2=\cdots=\mu_p$, H_1: $\mu_1,\mu_2,\cdots,\mu_p$ 至少有一对不相等，令

$$C=\begin{bmatrix}1 & -1 & 0 & \cdots & 0\\ 1 & 0 & -1 & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ 1 & 0 & 0 & \cdots & -1\end{bmatrix}_{(p-1)\times p},$$

则上面的假设等价于

$$H_0: C\mu = 0_{p-1}, \quad H_1: C\mu \neq 0_{p-1},$$

其中 0_{p-1} 为 $p-1$ 维零向量. 试求检验 H_0 的似然比统计量和分布.

解. 似然比统计量

$$\lambda = \frac{\max_{\theta\in\Theta_0} L(C\mu, C\Sigma C')}{\max_{\theta\in\Theta} L(C\mu, C\Sigma C')} = \frac{L(0, \frac{1}{n}CAC')}{L(C\bar{X}, \frac{1}{n}CAC')} \overset{H_0}{\sim} \chi^2(\frac{p(p+1)}{2})$$

3-8 假定人体尺寸有这样的一般规律：身高(X_1)，胸围(X_2)和上半臂围(X_3)的平均尺寸比例是 6 : 4 : 1. 假设 $X_{(\alpha)}(\alpha=1,\cdots,n)$ 为来自总体 $X=(X_1,X_2,X_3)'$ 的随机样本，并设 $X \sim N_3(\mu,\Sigma)$. 试利用表 3.4 中男婴这一组数据来检验其身高、胸围和上半臂围这三个尺寸(变量)是否符合这一规律(写出假设 H_0，并导出检验统计量).

表 3.4 某地区农村两周岁婴儿的体格测量数据

性别	身高(X_1)	胸围(X_2)	上半臂围(X_3)
男	78	60.6	16.5
男	76	58.1	12.5
男	92	63.2	14.5
男	81	59.0	14.0
男	81	60.8	15.5
男	84	59.5	14.0
女	80	58.4	14.0
女	75	59.2	15.0
女	78	60.3	15.0
女	75	57.4	13.0
女	79	59.5	14.0
女	78	58.1	14.5
女	75	58.0	12.5
女	64	55.5	11.0
女	80	59.2	12.5

解. 令 $C=\begin{pmatrix}4 & -6 & 0\\ 1 & 0 & -6\end{pmatrix}$, 则

$$H_0: C\mu = \boldsymbol{0} \leftrightarrow H_1: C\mu \neq \boldsymbol{0}$$

其中 $\mu=(\mu_1,\mu_2,\mu_3)'$. 由于 Σ 未知, 故用 *Hotelling* T^2 检验.

$C\mu$ 的估计量 $C\bar{X}\sim N_3(C\mu,C\Sigma C')$,$C\bar{X}\overset{H_0}{\sim}N_3(0,C\Sigma C')$. 构造检验统计量:

$$T^2=n(C\bar{X}-0)'(CSC')^{-1}(C\bar{X}-0)=n(n-1)(C\bar{X})'(CAC')^{-1}(C\bar{X})\sim T^2(p,n-1)$$

或

$$F=\frac{n-p}{(n-1)p}T^2\overset{H_0}{\sim}F(p,n-p)$$

其中 $p=3,n=6$.

```
> tige<-read.csv(”E:/4.多元统计分析/zuoye/3/1.csv”,stringsAsFactors = TRUE)
> print(tige)
   gender X1   X2   X3
1        B 78 60.6 16.5
2        B 76 58.1 12.5
3        B 92 63.2 14.5
......
13       G 75 58.0 12.5
14       G 64 55.5 11.0
15       G 80 59.2 12.5
> X<-data.frame(tige[c(1:6),])
> X<-data.frame(X[,-1])
> a<-colMeans(X)   #样本均值
> a<-as.matrix(a,byrow=TRUE)
> c<-matrix(data=c(4,-6,0,1,0,-6),byrow=FALSE,ncol=3)    #C矩阵
> A<-5*cov(X)    #样本离差阵
> ca<-c%*%a
> ca2<-t(ca)
> c2<-t(c)   #C的转置
> CAC<-c%*%A%*%c2
> CAC3<-solve(CAC)
> F<-6*ca2%*%CAC3%*%ca    #F检验统计量
> F
         [,1]
[1,] 290.0087
> 1-pf(F,3,3)   #p值
             [,1]
[1,] 0.0003416187   #小于0.0005,故否定原假设,认为三个尺寸不符合这一规律.
```

3-11　表 3.4 给出 15 名两周岁婴儿的身高(X_1)，胸围(X_2)和上半臂围(X_3)的测量数据. 假设男婴的测量数据 $X_{(\alpha)}(\alpha=1,\cdots,6)$为来自总体 $N_3(\mu^{(1)},\Sigma)$的随机样本；女婴的测量数据 $Y_{(\alpha)}(\alpha=1,\cdots,9)$为来自总体 $N_3(\mu^{(2)},\Sigma)$的随机样本. 试利用表 3.4 中的数据检验 H_0: $\mu^{(1)}=\mu^{(2)}(\alpha=0.05)$.

解. $n_1=6, n_2=9$, 同方差的两正态总体均值向量检验的检验统计量为

$$\frac{n_1 n_2}{n_1+n_2}(\bar{X}-\bar{Y})'(\frac{A_1+A_2}{n_1+n_2-2})^{-1}(\bar{X}-\bar{Y}) \overset{H_0}{\sim} T^2(p, n_1+n_2-2)$$

```
> Y<-data.frame(tige[c(7:15),-1])
> a1<-colMeans(X)
> a2<-colMeans(Y)
> a1<-as.matrix(a1,byrow=TRUE)  #X的均值向量
> a2<-as.matrix(a2,byrow=TRUE)  #Y的均值向量
> A1<-5*cov(X)    #X的样本离差阵
> A2<-8*cov(Y)    #Y的样本离差阵
> a1.a2<-a1-a2
> a1.a2_t<-t(a1.a2)
> A1.A2<-solve((A1+A2)/13)  #混合样本协方差阵的逆
> T2<-(6*9/15)*a1.a2_t%*%A1.A2%*%a1.a2
> T2
         [,1]
[1,] 5.311726
> F<-11/13/3*T2     #转换为F分布
> F
         [,1]
[1,] 1.498179
> 1-pf(F,3,11)
          [,1]
[1,] 0.2692616    #p值大于0.05,故接受H_0
```

3-13　对表 3.3 给出的三组观测数据分别检验是否来自 4 元正态分布.

(1) 对每个分量检验是否是一元正态？

(2) 利用 χ^2 图检验法对三组观测数据分别检验是否来自 4 元正态分布.

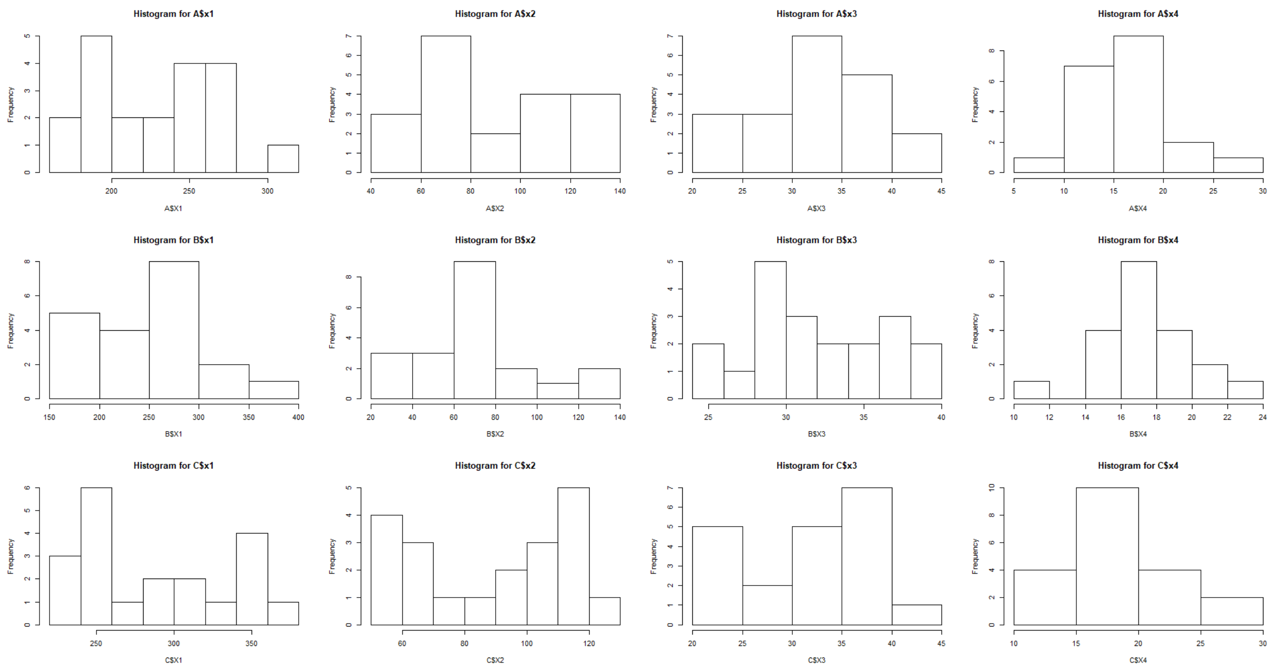
例 3.3.2　为了研究某种疾病，对一批人同时测量了 4 个指标：β 脂蛋白(X_1)，甘油三酯(X_2)，α 脂蛋白(X_3)，前 β 脂蛋白(X_4). 按不同年龄、不同性别分为三组(20 至 35 岁的女性、20 至 25 岁的男性和 35 至 50 岁的男性)，数据见表 3.3. 试问这三个组的 4 项指标间有无显著性差异($\alpha=0.01$)？

表 3.3　身体指标化验数据

X_1	X_2	X_3	X_4	组	X_1	X_2	X_3	X_4	组	X_1	X_2	X_3	X_4	组
260	75	40	18	1	310	122	30	21	2	320	64	39	17	3
200	72	34	17	1	310	60	35	18	2	260	59	37	11	3
240	87	45	18	1	190	40	27	15	2	360	88	28	26	3
170	65	39	17	1	225	65	34	16	2	295	100	36	12	3
270	110	39	24	1	170	65	37	16	2	270	65	32	21	3
205	130	34	23	1	210	82	31	17	2	380	114	36	21	3
190	69	27	15	1	280	67	37	18	2	240	55	42	10	3
200	46	45	15	1	210	38	36	17	2	260	55	34	20	3
250	117	21	20	1	280	65	30	23	2	260	110	29	20	3
200	107	28	20	1	200	76	40	17	2	295	73	33	21	3
225	130	36	11	1	200	76	39	20	2	240	114	38	18	3
210	125	26	17	1	280	94	26	11	2	310	103	32	18	3
170	64	31	14	1	190	60	33	17	2	330	112	21	11	3
270	76	33	13	1	295	55	30	16	2	345	127	24	20	3
190	60	34	16	1	270	125	24	21	2	250	62	22	16	3
280	81	20	18	1	280	120	32	18	2	260	59	21	19	3
310	119	25	15	1	240	62	32	20	2	225	100	34	30	3
270	57	31	8	1	280	69	29	20	2	345	120	36	18	3
250	67	31	14	1	370	70	30	20	2	360	107	25	23	3
260	135	39	29	1	280	40	37	17	2	250	117	36	16	3

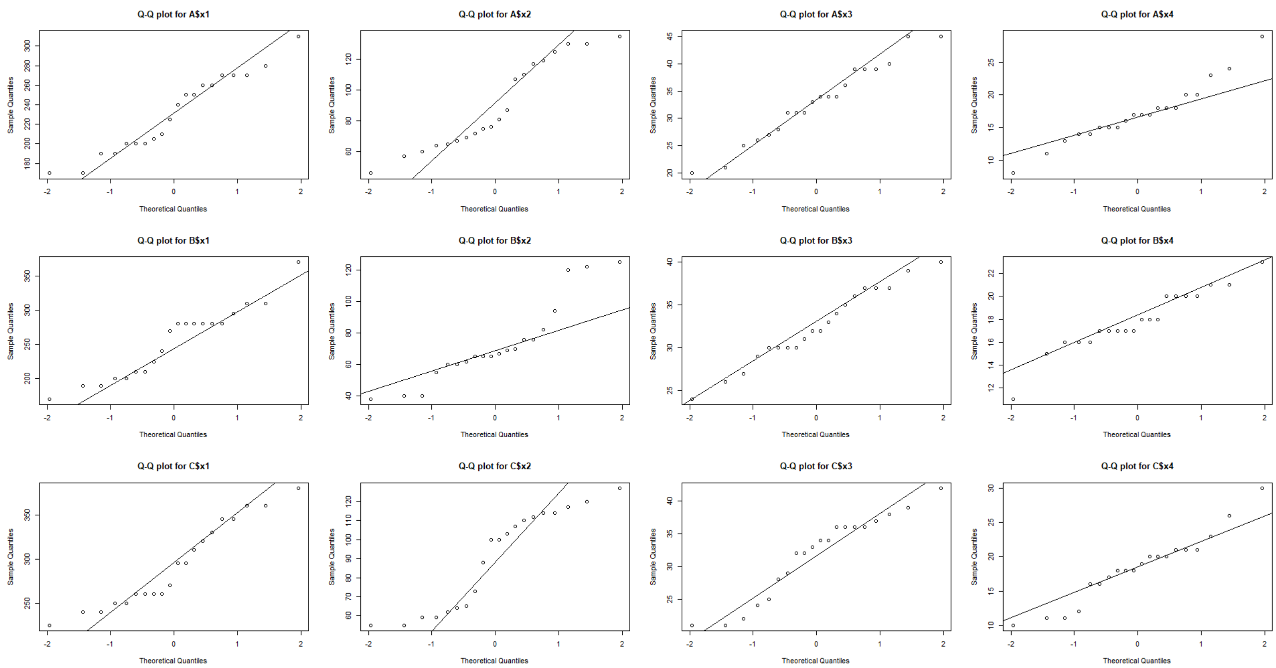
解. (第一问):

1.1 直方图



由图观察, 组 1 的第 4 特征和组 2 的第 4 特征接近一元正态.

1.2 QQ 图



由图观察, 组 1 的第 3 特征和组 2 的第 4 特征接近一元正态.

1.3 偏度和峰度

由结果 (结果见附录), 组 1 的第 4 特征, 组 2 的第 4 特征, 组 3 的第 4 特征接近一元正态.

1.4 统计检验

(1)Shapiro-Wilks test

由检验结果 (见附录), 组 1 的第 3 特征, 组 1 的第 4 特征, 组 2 的第 3 特征, 组 3 的第 4 特征接近一元正态.

(2)Kolmogorov-Smirnov test

由检验结果 (见附录), 组 1 的第 3 特征, 组 2 的第 4 特征接近一元正态.

(3)Cramer-von Mises test

由检验结果 (见附录), 组 1 的第 3 特征, 组 2 的第 3 特征接近一元正态.

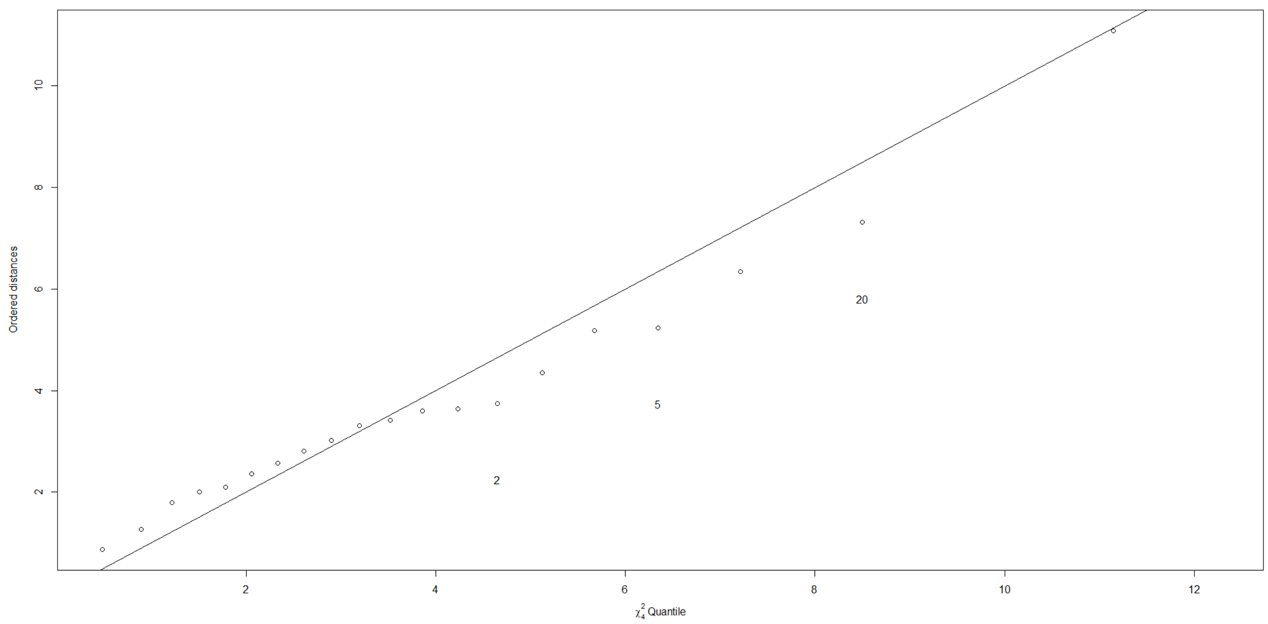
(4)Anderson-Darling test

由检验结果 (见附录), 组 1 的第 3 特征, 组 2 的第 3 特征接近一元正态.

(第二问):

1. 第一组

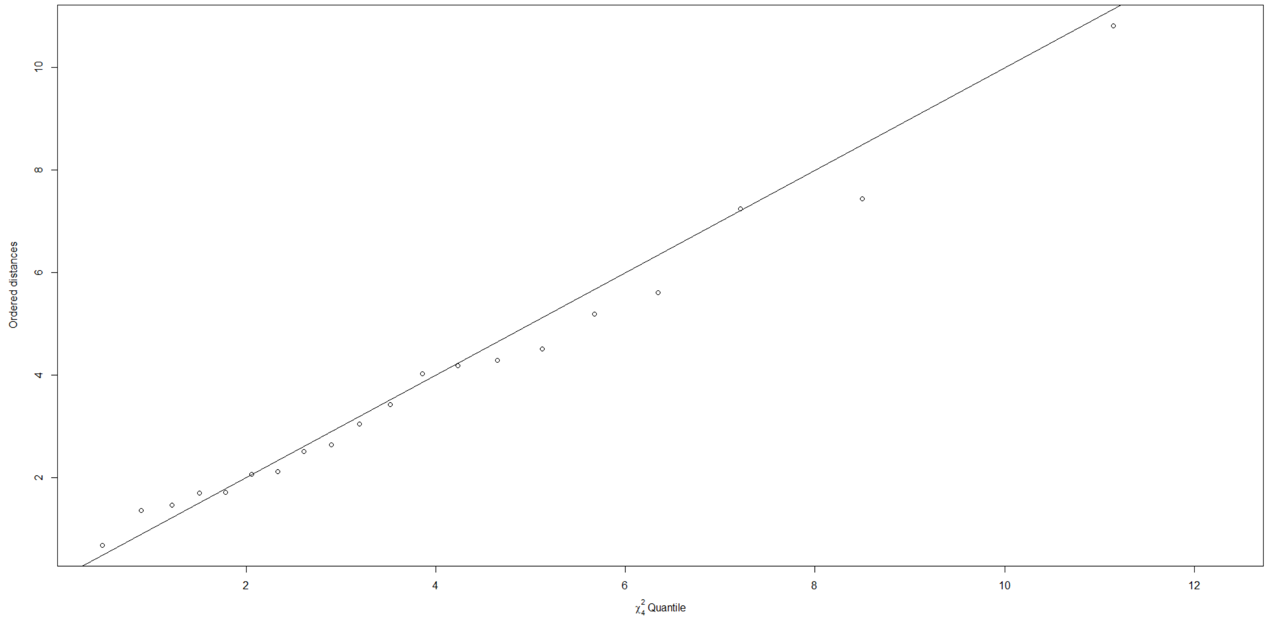
```
> y <- A
> cm <- colMeans(y)
> S <- cov(y)
> d <- apply(y, 1, function(y) t(y - cm) %*% solve(S) %*% (y - cm))
> plot(qc <- qchisq((1:nrow(y) - 1/2) / nrow(y), df = 4), sd <- sort(d),
+       xlab = expression(paste(chi[4]^2, ” Quantile”)),
+       ylab = ”Ordered distances”, xlim = range(qc) * c(1, 1.1))
> abline(a = 0, b = 1)
```



拒绝正态性假设.

2. 第二组

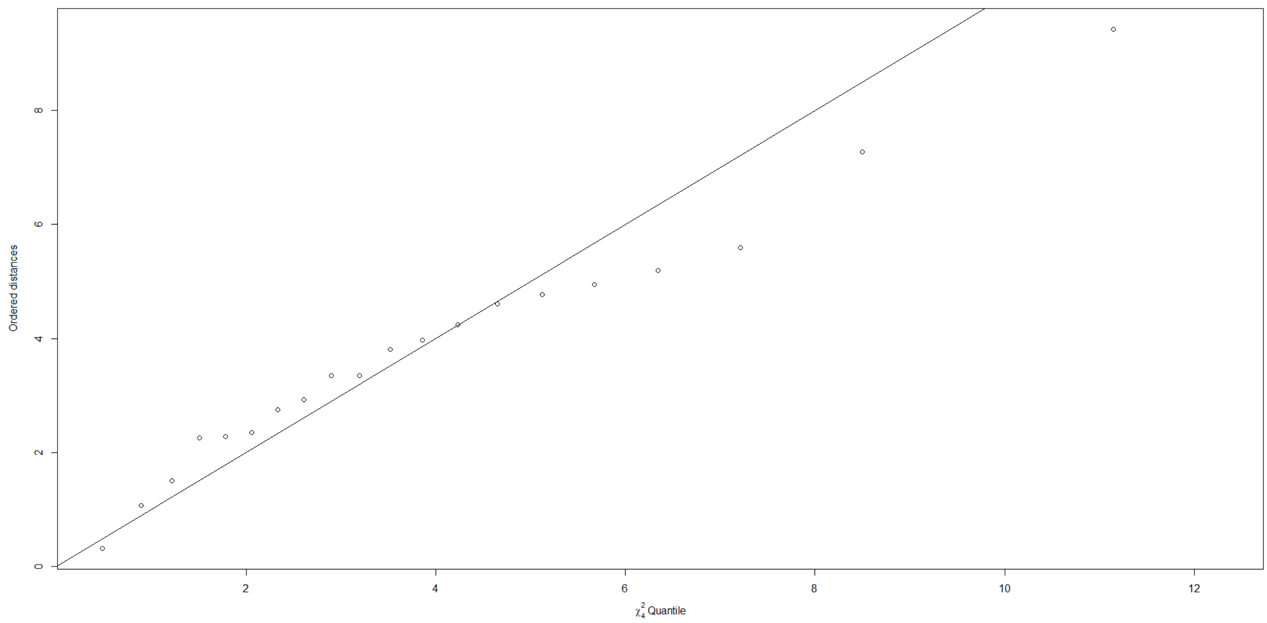
```
> y <- B
> cm <- colMeans(y)
> S <- cov(y)
> d <- apply(y, 1, function(y) t(y - cm) %*% solve(S) %*% (y - cm))
> plot(qc <- qchisq((1:nrow(y) - 1/2) / nrow(y), df = 4), sd <- sort(d),
+       xlab = expression(paste(chi[4]^2, " Quantile")),
+       ylab = "Ordered distances", xlim = range(qc) * c(1, 1.1))
> abline(a = 0, b = 1)
```



勉强接受正态性假设.

3. 第三组

```
> y <- C
> cm <- colMeans(y)
> S <- cov(y)
> d <- apply(y, 1, function(y) t(y - cm) %*% solve(S) %*% (y - cm))
> plot(qc <- qchisq((1:nrow(y) - 1/2) / nrow(y), df = 4), sd <- sort(d),
+       xlab = expression(paste(chi[4]^2, " Quantile")),
+       ylab = "Ordered distances", xlim = range(qc) * c(1, 1.1))
> abline(a = 0, b = 1)
```



拒绝正态性假设.

附录

A 3-13 1.1 直方图

```r
> data<-read.csv("E:/4.多元统计分析/zuoye/3/2.csv",stringsAsFactors = TRUE)
> A<-data.frame(data[c(1:20),-1])      #将三个组分出来
> B<-data.frame(data[c(21:40),-1])
> C<-data.frame(data[c(41:60),-1])
> par(mfrow=c(3,4))
> hist(A$X1,main="Histogram for A$x1")
> hist(A$X2,main="Histogram for A$x2")
> hist(A$X3,main="Histogram for A$x3")
> hist(A$X4,main="Histogram for A$x4")
> hist(B$X1,main="Histogram for B$x1")
> hist(B$X2,main="Histogram for B$x2")
> hist(B$X3,main="Histogram for B$x3")
> hist(B$X4,main="Histogram for B$x4")
> hist(C$X1,main="Histogram for C$x1")
> hist(C$X2,main="Histogram for C$x2")
> hist(C$X3,main="Histogram for C$x3")
> hist(C$X4,main="Histogram for C$x4")
```

B 3-13 1.2 QQ 图

```r
par(mfrow=c(3,4))
qqnorm(A$X1,main="Q-Q plot for A$x1"); qqline(A$X1)
qqnorm(A$X2,main="Q-Q plot for A$x2"); qqline(A$X2)
qqnorm(A$X3,main="Q-Q plot for A$x3"); qqline(A$X3)
qqnorm(A$X4,main="Q-Q plot for A$x4"); qqline(A$X4)
qqnorm(B$X1,main="Q-Q plot for B$x1"); qqline(B$X1)
qqnorm(B$X2,main="Q-Q plot for B$x2"); qqline(B$X2)
qqnorm(B$X3,main="Q-Q plot for B$x3"); qqline(B$X3)
qqnorm(B$X4,main="Q-Q plot for B$x4"); qqline(B$X4)
qqnorm(C$X1,main="Q-Q plot for C$x1"); qqline(C$X1)
qqnorm(C$X2,main="Q-Q plot for C$x2"); qqline(C$X2)
qqnorm(C$X3,main="Q-Q plot for C$x3"); qqline(C$X3)
qqnorm(C$X4,main="Q-Q plot for C$x4"); qqline(C$X4)
```

C 3-13 1.3 偏度和峰度

```
> library(moments)
> skewness(A$X1); kurtosis(A$X1)
[1] 0.1204204
[1] 1.933383
> skewness(A$X2); kurtosis(A$X2)
[1] 0.2773125
[1] 1.587005
> skewness(A$X3); kurtosis(A$X3)
[1] -0.05769829
[1] 2.307432
> skewness(A$X4); kurtosis(A$X4)
[1] 0.601332
[1] 3.789564
> skewness(B$X1); kurtosis(B$X1)
[1] 0.2049252
[1] 2.363692
> skewness(B$X2); kurtosis(B$X2)
[1] 0.8553859
[1] 3.002884
> skewness(B$X3); kurtosis(B$X3)
[1] -0.04971583
[1] 2.19002
> skewness(B$X4); kurtosis(B$X4)
[1] -0.4271917
[1] 3.707689
> skewness(C$X1); kurtosis(C$X1)
[1] 0.3598219
[1] 1.764333
> skewness(C$X2); kurtosis(C$X2)
[1] -0.1937781
[1] 1.401273
> skewness(C$X3); kurtosis(C$X3)
[1] -0.4423124
[1] 2.038037
> skewness(C$X4); kurtosis(C$X4)
[1] 0.2004094
[1] 3.058989
```

D 3-13 1.4.1 Shapiro-Wilks test

```
> shapiro.test(A$X1)

Shapiro-Wilk normality test

data:  A$X1
W = 0.94438, p-value = 0.2898

> shapiro.test(A$X2)

Shapiro-Wilk normality test

data:  A$X2
W = 0.90348, p-value = 0.04795

> shapiro.test(A$X3)

Shapiro-Wilk normality test

data:  A$X3
W = 0.97035, p-value = 0.7622

> shapiro.test(A$X4)

Shapiro-Wilk normality test

data:  A$X4
W = 0.9573, p-value = 0.4914

> shapiro.test(B$X1)

Shapiro-Wilk normality test

data:  B$X1
W = 0.93141, p-value = 0.1644

> shapiro.test(B$X2)

Shapiro-Wilk normality test

data:  B$X2
W = 0.87954, p-value = 0.01736
```

```
> shapiro.test(B$X3)

Shapiro-Wilk normality test

data:  B$X3
W = 0.97246, p-value = 0.8057

> shapiro.test(B$X4)

Shapiro-Wilk normality test

data:  B$X4
W = 0.94251, p-value = 0.2674

> shapiro.test(C$X1)

Shapiro-Wilk normality test

data:  C$X1
W = 0.92007, p-value = 0.09939

> shapiro.test(C$X2)

Shapiro-Wilk normality test

data:  C$X2
W = 0.87082, p-value = 0.01215

> shapiro.test(C$X3)

Shapiro-Wilk normality test

data:  C$X3
W = 0.92527, p-value = 0.1252

> shapiro.test(C$X4)

Shapiro-Wilk normality test

data:  C$X4
```

```
W = 0.9509, p-value = 0.381
```

E 3-13 1.4.2 One-sample Kolmogorov-Smirnov test

```
> library(nortest)  # package for normality tests
> ks.test(A$X1,"pnorm",mean(A$X1),sqrt(var(A$X1)))

One-sample Kolmogorov-Smirnov test

data:  A$X1
D = 0.14982, p-value = 0.7604
alternative hypothesis: two-sided

Warning message:
In ks.test(A$X1, "pnorm", mean(A$X1), sqrt(var(A$X1))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(A$X2,"pnorm",mean(A$X2),sqrt(var(A$X2)))

One-sample Kolmogorov-Smirnov test

data:  A$X2
D = 0.18174, p-value = 0.5235
alternative hypothesis: two-sided

Warning message:
In ks.test(A$X2, "pnorm", mean(A$X2), sqrt(var(A$X2))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(A$X3,"pnorm",mean(A$X3),sqrt(var(A$X3)))

One-sample Kolmogorov-Smirnov test

data:  A$X3
D = 0.10512, p-value = 0.9799
alternative hypothesis: two-sided

Warning message:
In ks.test(A$X3, "pnorm", mean(A$X3), sqrt(var(A$X3))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(A$X4,"pnorm",mean(A$X4),sqrt(var(A$X4)))

One-sample Kolmogorov-Smirnov test
```

```

data:  A$X4
D = 0.17354, p-value = 0.5835
alternative hypothesis: two-sided

Warning message:
In ks.test(A$X4, ”pnorm”, mean(A$X4), sqrt(var(A$X4))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(B$X1,”pnorm”,mean(B$X1),sqrt(var(B$X1)))

One-sample Kolmogorov-Smirnov test

data:  B$X1
D = 0.19427, p-value = 0.4372
alternative hypothesis: two-sided

Warning message:
In ks.test(B$X1, ”pnorm”, mean(B$X1), sqrt(var(B$X1))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(B$X2,”pnorm”,mean(B$X2),sqrt(var(B$X2)))

One-sample Kolmogorov-Smirnov test

data:  B$X2
D = 0.19605, p-value = 0.4256
alternative hypothesis: two-sided

Warning message:
In ks.test(B$X2, ”pnorm”, mean(B$X2), sqrt(var(B$X2))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(B$X3,”pnorm”,mean(B$X3),sqrt(var(B$X3)))

One-sample Kolmogorov-Smirnov test

data:  B$X3
D = 0.11193, p-value = 0.9636
alternative hypothesis: two-sided

Warning message:
In ks.test(B$X3, ”pnorm”, mean(B$X3), sqrt(var(B$X3))) :
Kolmogorov - Smirnov检验里不应该有连结
```

```
> ks.test(B$X4,"pnorm",mean(B$X4),sqrt(var(B$X4)))

One-sample Kolmogorov-Smirnov test

data:  B$X4
D = 0.137, p-value = 0.8471
alternative hypothesis: two-sided

Warning message:
In ks.test(B$X4, "pnorm", mean(B$X4), sqrt(var(B$X4))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(C$X1,"pnorm",mean(C$X1),sqrt(var(C$X1)))

One-sample Kolmogorov-Smirnov test

data:  C$X1
D = 0.20397, p-value = 0.3761
alternative hypothesis: two-sided

Warning message:
In ks.test(C$X1, "pnorm", mean(C$X1), sqrt(var(C$X1))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(C$X2,"pnorm",mean(C$X2),sqrt(var(C$X2)))

One-sample Kolmogorov-Smirnov test

data:  C$X2
D = 0.19913, p-value = 0.4059
alternative hypothesis: two-sided

Warning message:
In ks.test(C$X2, "pnorm", mean(C$X2), sqrt(var(C$X2))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(C$X3,"pnorm",mean(C$X3),sqrt(var(C$X3)))

One-sample Kolmogorov-Smirnov test

data:  C$X3
D = 0.16575, p-value = 0.6419
alternative hypothesis: two-sided

```

```
Warning message:
In ks.test(C$X3, "pnorm", mean(C$X3), sqrt(var(C$X3))) :
Kolmogorov - Smirnov检验里不应该有连结
> ks.test(C$X4,"pnorm",mean(C$X4),sqrt(var(C$X4)))

One-sample Kolmogorov-Smirnov test

data:  C$X4
D = 0.15187, p-value = 0.7455
alternative hypothesis: two-sided

Warning message:
In ks.test(C$X4, "pnorm", mean(C$X4), sqrt(var(C$X4))) :
Kolmogorov - Smirnov检验里不应该有连结
```

F 3-13 1.4.3 Cramer-von Mises normality test

```
> cvm.test(A$X1)

Cramer-von Mises normality test

data:  A$X1
W = 0.086853, p-value = 0.1584

> cvm.test(A$X2)

Cramer-von Mises normality test

data:  A$X2
W = 0.14504, p-value = 0.02482

> cvm.test(A$X3)

Cramer-von Mises normality test

data:  A$X3
W = 0.029414, p-value = 0.8454

> cvm.test(A$X4)

Cramer-von Mises normality test
```

```

data:  A$X4
W = 0.074724, p-value = 0.2295

> cvm.test(B$X1)

Cramer-von Mises normality test

data:  B$X1
W = 0.12397, p-value = 0.04817

> cvm.test(B$X2)

Cramer-von Mises normality test

data:  B$X2
W = 0.16261, p-value = 0.01441

> cvm.test(B$X3)

Cramer-von Mises normality test

data:  B$X3
W = 0.041269, p-value = 0.6392

> cvm.test(B$X4)

Cramer-von Mises normality test

data:  B$X4
W = 0.085822, p-value = 0.1635

> cvm.test(C$X1)

Cramer-von Mises normality test

data:  C$X1
W = 0.11306, p-value = 0.06823

> cvm.test(C$X2)

```

```
Cramer-von Mises normality test

data:  C$X2
W = 0.17689, p-value = 0.00932

> cvm.test(C$X3)

Cramer-von Mises normality test

data:  C$X3
W = 0.10375, p-value = 0.09207

> cvm.test(C$X4)

Cramer-von Mises normality test

data:  C$X4
W = 0.0695, p-value = 0.2695
```

G 3-13 1.4.4 Anderson-Darling normality test

```
> ad.test(A$X1)

Anderson-Darling normality test

data:  A$X1
A = 0.48879, p-value = 0.1974

> ad.test(A$X2)

Anderson-Darling normality test

data:  A$X2
A = 0.80834, p-value = 0.02999

> ad.test(A$X3)

Anderson-Darling normality test

data:  A$X3
A = 0.20767, p-value = 0.8443
```

```

> ad.test(A$X4)

Anderson-Darling normality test

data:  A$X4
A = 0.42246, p-value = 0.2906

> ad.test(B$X1)

Anderson-Darling normality test

data:  B$X1
A = 0.67108, p-value = 0.06764

> ad.test(B$X2)

Anderson-Darling normality test

data:  B$X2
A = 0.97028, p-value = 0.0115

> ad.test(B$X3)

Anderson-Darling normality test

data:  B$X3
A = 0.2485, p-value = 0.7141

> ad.test(B$X4)

Anderson-Darling normality test

data:  B$X4
A = 0.52176, p-value = 0.162

> ad.test(C$X1)

Anderson-Darling normality test

data:  C$X1
```

```
A = 0.65859, p-value = 0.07283

> ad.test(C$X2)

Anderson-Darling normality test

data:  C$X2
A = 1.0481, p-value = 0.007258

> ad.test(C$X3)

Anderson-Darling normality test

data:  C$X3
A = 0.61015, p-value = 0.09707

> ad.test(C$X4)

Anderson-Darling normality test

data:  C$X4
A = 0.43689, p-value = 0.2674
```