ADA: Demographic Methods

Monica Alexander

Week 3: Mortality II

Overview

- Decomposition
- Interlude: oldest old mortality
- Mortality models

Decomposition

Decomposition

- Decomposition is a technique that allows for quantifying the impact of age structure (and other characteristics)
- Asking the question: how much is a difference between death rates (or life expectancy, etc) due to age structure versus differences in mortality risk?

Kitagawa decomposition

Write the difference between two CDR rates as

$$\Delta = CDR^B - CDR^A = \sum_i C_i^B \cdot M_i^B - \sum_i C_i^A \cdot M_i^A$$

- C_i is age structure (proportion of population in age group i)
- M_i is age-specific mortality rate

Kitagawa decomposition

Can rewrite this as

$$\Delta = \sum_i \left(C_i^B - C_i^A\right) \cdot \left[\frac{M_i^B + M_i^A}{2}\right] + \sum_i \left(M_i^B - M_i^A\right) \cdot \left[\frac{C_i^A + C_i^B}{2}\right]$$

$$= \begin{array}{l}\text{difference in age}\\ \text{composition}\end{array} \cdot \left[\begin{array}{l}\text{weighted by average}\\ \text{age-specific mortality}\end{array}\right]$$

$$+ \begin{array}{l}\text{difference in rate}\\ \text{schedules}\end{array} \cdot \left[\begin{array}{l}\text{weighted by}\\ \text{average age}\\ \text{composition}\end{array}\right]$$

$$= \begin{array}{c}\text{contribution of age compositional}\\ \text{differences to } \Delta\end{array} + \begin{array}{c}\text{contribution of rate schedule}\\ \text{differences to } \Delta\end{array}$$

Kitagawa decomposition

$$\Delta = \sum_i \left(C_i^B - C_i^A\right) \cdot \left[\frac{M_i^B + M_i^A}{2}\right] + \sum_i \left(M_i^B - M_i^A\right) \cdot \left[\frac{C_i^A + C_i^B}{2}\right]$$

- Decompose into age differences and mortality differences
- Evelyn Kitagawa (1955)

Decomposition of Canada versus Kenya

- CDR in Canada: 7.9 per 1,000 people
- CDR in Kenya: 7.2 per 1,000 people

$$\Delta = CDR_K - CDR_C = -0.000724$$

- Age contribution: -0.0107
- Rate contribution: 0.0099

Decomposition in the wild

Home | JAMA Network Open | Vol. 6, No. 5

Original Investigation | Infectious Diseases

COVID-19 Mortality by Race and Ethnicity in US Metropolitan and Nonmetropolitan Areas, March 2020 to February 2022

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Key Points

Question Why did racial and ethnic disparities in COVID-19 mortality in the US decrease in the Omicron wave compared with the initial wave of the pandemic?

Findings In this cross-sectional study of 977 018 adults who died from COVID-19, 60.3% of the national decrease in disparities in COVID-19 mortality for non-Hispanic Black compared with non-Hispanic White adults between the initial and Omicron waves could be explained by increases in mortality among non-Hispanic White adults and shifts in mortality to nonmetropolitan areas, where more non-Hispanic White adults reside.

Meaning This study found that racial and ethnic disparities in COVID-19 mortality decreased nationally for some groups during the first 2 years of the pandemic, but this decrease was mostly explained by increases in mortality for non-Hispanic White adults and changes in pandemic geography.

Statistical Analysis

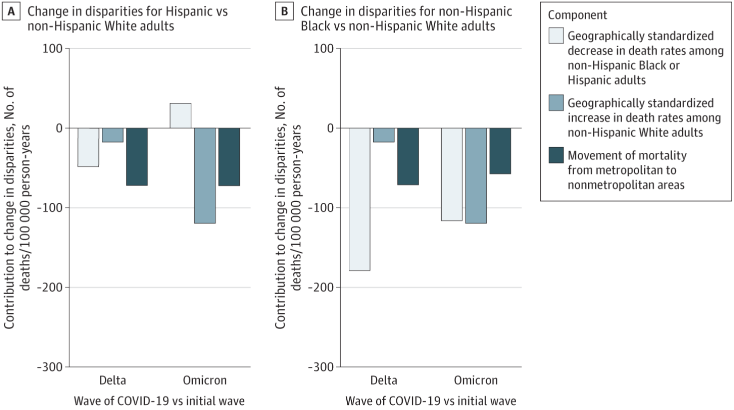
We decomposed the national change between the initial wave of the pandemic and the Omicron wave in the absolute disparity in age-standardized COVID-19 death rates among non-Hispanic Black compared with non-Hispanic White adults. We also decomposed the national change in disparities for Hispanic compared with non-Hispanic White adults and compared the initial wave with the second, Alpha, and Delta waves. We sought to understand the contribution of the following 4 components to national changes in disparities:

1. The geographically standardized decrease in death rates among non-Hispanic Black or Hispanic adults in a hypothetical population in which the non-Hispanic Black or Hispanic population had the geographic distribution of a standard population.

2. The geographically standardized increase in death rates among non-Hispanic White adults in a hypothetical population in which the non-Hispanic White population had the geographic distribution of a standard population.

3. The change in mortality outcomes associated with shifts in where deaths occurred from metropolitan to nonmetropolitan areas, where more non-Hispanic White adults reside relative to the national geographic distribution (ie, the differential outcomes associated with changes in racial or ethnic–specific and geography-specific mortality rates because of the actual geographic distribution of the US population, with non-Hispanic White adults overrepresented in nonmetropolitan areas).

4. The change in mortality outcomes associated with shifts in the racial and ethnic population composition in metropolitan and nonmetropolitan areas, which we expected to be minor.


A
Change in disparities for Hispanic vs non-Hispanic White adults
B
Change in disparities for non-Hispanic Black vs non-Hispanic White adults
Contribution to change in disparities, No. of deaths/100 000 person-years
100
0
-100
-200
-300
Delta
Omicron
Wave of COVID-19 vs initial wave
Component
Geographically standardized decrease in death rates among non-Hispanic Black or Hispanic adults
Geographically standardized increase in death rates among non-Hispanic White adults
Movement of mortality from metropolitan to nonmetropolitan areas

The change between periods in racial and ethnic disparities in mortality is equivalent to the racial and ethnic disparity in the change in mortality:

$$(\overline{m}_{b,2} - \overline{m}_{w,2}) - (\overline{m}_{b,1} - \overline{m}_{w,1}) = (\overline{m}_{b,2} - \overline{m}_{b,1}) - (\overline{m}_{w,1} - \overline{m}_{w,1}) \tag{1}$$

Due to that equivalence, we proceed by geographically decomposing the changes in race and ethnicity-specific aggregate mortality $(\overline{m}_{i,2} - \overline{m}_{i,1})$ before plugging those components into Eq. (1) to produce our final decomposition equation.

The change in race and ethnicity-specific aggregate mortality can be geographically decomposed as:

$$\begin{aligned}
\overline{m}_{i,2} - \overline{m}_{i,1} &= \sum_k \left[c_{i,1,k}(m_{i,2,k} - m_{i,1,k}) + (c_{i,2,k} - c_{i,1,k})m_{i,2,k}\right] \\
&= \sum_k \left[(c_k^{st} + \tilde{c}_{i,1,k})(m_{i,2,k} - m_{i,1,k}) + \left(c_k^{st} + \tilde{c}_{i,2,k} - (c_k^{st} + \tilde{c}_{i,1,k})\right) m_{i,2,k}\right] \\
&= \sum_k \left[c_k^{st}(m_{i,2,k} - m_{i,1,k}) + \tilde{c}_{i,1,k}(m_{i,2,k} - m_{i,1,k}) + (\tilde{c}_{i,2,k} - \tilde{c}_{i,1,k})m_{i,2,k}\right]
\end{aligned}$$

Mortality models

Continuous mortality

The continuous version of mortality rates are called **hazard rates** $h(x)$ (or $\lambda(x)$) or, if you're vintage demography, the **force of mortality** $\mu(x)$.

Let's take the limit of ${}_nM_x$ as $n \rightarrow 0$

$$\begin{aligned} h(x) &= \lim_{n\rightarrow 0} \frac{{}_nd_x}{{}_nLx} \\ &= \lim_{n\rightarrow 0} \frac{l_x - l_{x+n}}{n \cdot l_x} \\ &= -\frac{dln(l_x)}{dx} \end{aligned}$$

Hazard rates

$$h(x) = -\frac{dln(l_x)}{dx}$$

Implies

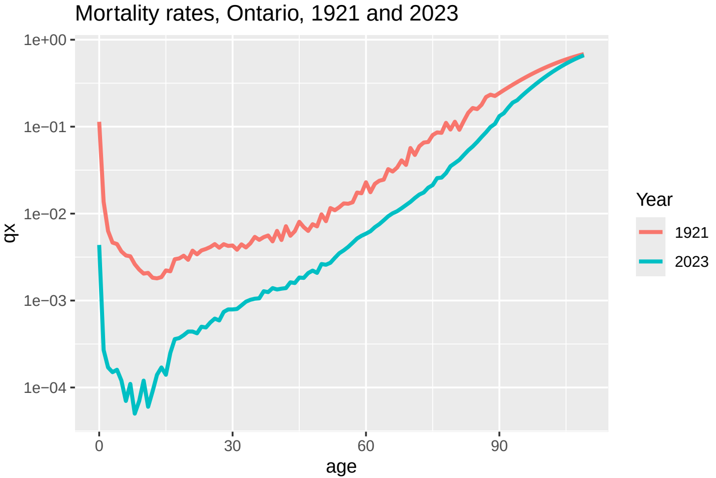
$$l(x+n) = l(x)e^{-\int_x^{x+n} h(x)dx}$$

Note the similarities between this and exponential growth from the first week.

More notes:

- if h is constant in the interval then $l(x+n) = l(x)e^{-hn}$
- can now express all the life table functions in continuous form, e.g. ${}_nd_x = \int_x^{x+n} l(a)h(a)da$

Modeling mortality rates over age



Gompertz model

Gompertz (1825). Hazards are log linear:

$$h(x) = \alpha e^{\beta x}$$

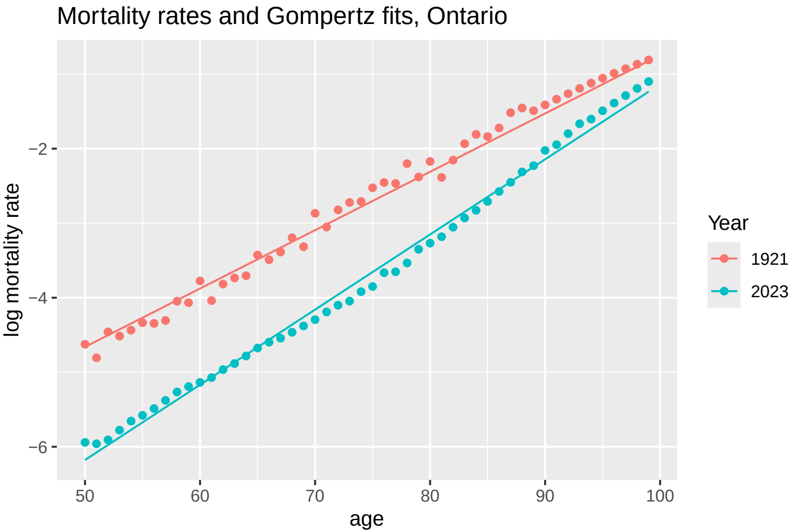
or

$$\log h(x) = \log(\alpha) + \beta x$$

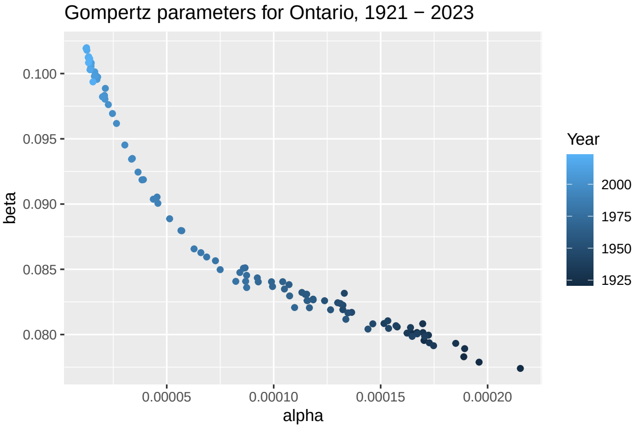
So we can take the log of mortality rates over age and fit a simple regression to get estimates of α and β.

Only fit to adult ages!

Gompertz fits



Gompertz parameters



Think about the implications for forecasting here!

Gompertz distribution

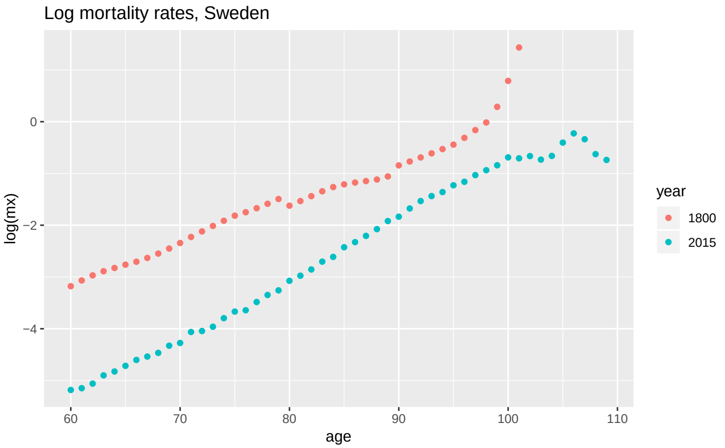
Note that because we know the form of Gompertz hazards $h(x)$, we can get closed form expressions for survivorship l_x etc. For example, the Gompertz death distribution is

$$d(x) = \alpha \exp\left(\beta x - \frac{\alpha}{\beta}\left(e^{\beta x - 1}\right)\right)$$

This is a PDF because $\int d(x) = 1$ i.e. everyone dies.

Other parametric models

Gompertz is unrealistic at young ages, but may also be misleading at old ages



Other parametric models

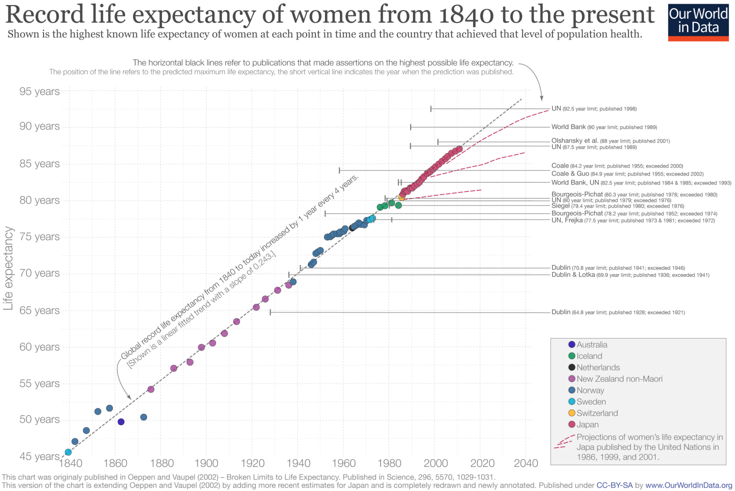
e.g., the log quadratic model

$$h(x) = e^{\alpha+\beta x+\gamma x^2}$$

allows for deceleration at older ages.

There's a good discussion of parametric mortality models in Feehan (2018).

The remarkable graph from Oeppen and Vaupel (2002)



The ongoing debate

Is there a human mortality plateau at older ages?

The plateau of human mortality: Demography of longevity pioneers

Elisabetta Barbi[1,*], **Francesco Lagona**[2], **Marco Marsili**[3], **James W. Vaupel**[4,5,6,7], **Kenneth W. Wachter**[8]

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Science 29 Jun 2018:
Vol. 360, Issue 6396, pp. 1459-1461
DOI: 10.1126/science.aat3119

Human mortality plateau

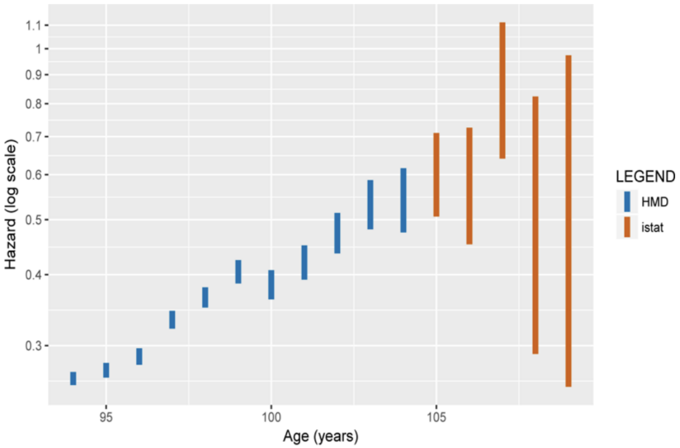
Abstract

Theories about biological limits to lifespan and evolutionary shaping of human longevity depend on facts about mortality at extreme ages. The facts have remained in dispute. Do hazard curves typically ultimately level out into high plateaus, as seen in other species, or do exponential increases go on and on? Here we estimate hazard rates from data on all Italian inhabitants aged over 105 between 2009 and 2015 (born 1896–1910), 3836 carefully documented cases. We find level hazard curves, essentially constant beyond age 105. The estimates are free from artifacts of aggregation limiting earlier studies and provide the best evidence so far for the existence of extreme-age mortality plateaus in humans.

Summary

Above age 105 human mortality appears constant over age at levels that are slowly declining across cohorts.

Data



Hazard (log scale)
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
95
100
105
Age (years)
LEGEND
HMD
istat

Method

Fit a Gompertz model, with proportional effects for cohort and gender

$$h(x) = \alpha e^{\beta x} e^{\beta_1 C + \beta_2 M}$$

Summary

Parameters estimated by standard maximum likelihood methods for truncated and censored survival data (14) are shown in Table 2. A likelihood ratio test fails to reject the constant-hazard null model at a level as generous as 0.44. Under the alternative hypothesis, the Gompertz slope parameter estimate $b = 0.013$ with standard error 0.017, is not statistically significant at the 5% level and is practically indistinguishable from zero. This near-negligible slope stands in striking contrast to the slope as large as 0.103 at younger ages (65–80) in Fig. 1b, which is paired with a log hazard at 65 of $log(0.015)$. For variant models and power calculations, see Tables S1 and S2.

Are you convinced?

A lot of people aren't

This conclusion is not without its controversy. A competing perspective is that a mortality plateau could come about from age misreporting, which is common among centenarians (Newman 2018; Gavrilov and Gavrilova 2019). Using 'extinct generations' methods applied to monthly US Social Security data, Gavrilov and Gavrilova (2011) contended that mortality even up to the oldest ages is best characterized by a Gompertz hazard. Based on the bold claims made in the Barbi et al. (2018) study, Camarda et al. (2019) argued that the sample size of survivors was too small to confidently reject other possibilities for the shape of ageing at oldest ages, including a Gompertz hazard. Wrigley-Field (2014) warned that the link between mortality selection and mortality deceleration is often more complex than theorized, with multiple mortality decelerations or even accelerations being possible results of the changing composition of frailty within cohorts.

(From Van Raalte (2021))

- There's also more recent work (https://www.jstor.org/stable/48728206) showing the pattern doesn't replicate in France.

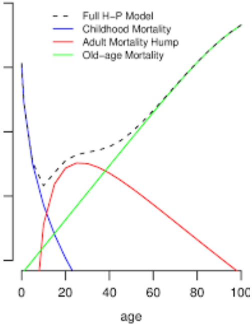
Models for all ages

Parametric models get tricky across all ages

Most well-known parametric model for the whole mortality curve is Heligman-Pollard (1980):

$$_1q_x = A^{(x+B)^C} + De^{-E(ln(x)-ln(F))^2} + GH^x$$

Eight parameters! Hard to fit, even with good data.



Relational models

- Mortality across age is very non-linear
- But the general ‘shape’ is quite regular across populations
- Use information from one population as the basis for a model for another population
- Add in parameters to shift and twist shape of mortality curve

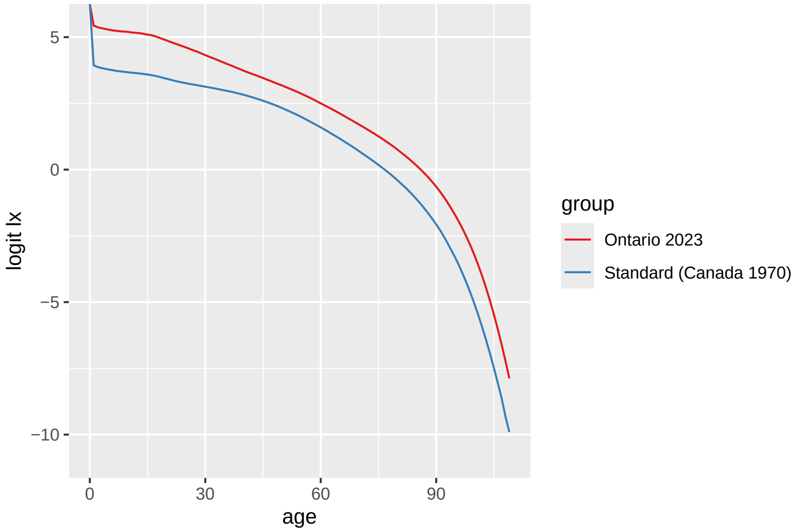
Brass relational logit

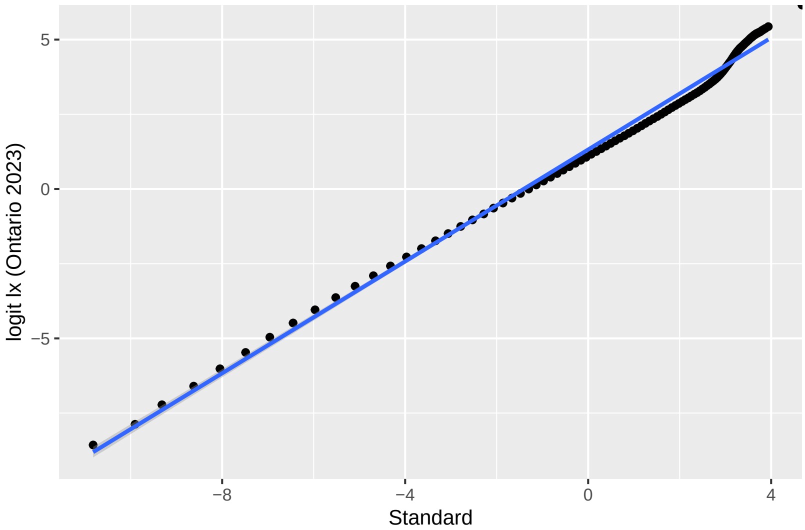
Define a mortality standard as Y_x. Then the model is

$$\text{logit } lx = \alpha + \beta Y_x$$

- α is a level parameter, β shifts the balance between young and old age mortality.
- Why is this set-up useful? Can get sensible mortality curves for populations where we have limited data.
- DIY standard: choose your favorite l_x, take the logit.

Example with Canada





logit lx (Ontario 2023)
5
0
−5
−8
−4
0
4
Standard

Lee Carter

Journal of the American Statistical Association ›

Volume 87, 1992 - Issue 419

Applications and Case Studies

Modeling and Forecasting U.S. Mortality

Ronald D. Lee & Lawrence R. Carter

Pages 659-671 | Received 01 Sep 1990, Published online: 27 Feb 2012

Cite this article https://doi.org/10.1080/01621459.1992.10475265

Lee-Carter mortality forecasting model

- forecasting mortality rates is important (insurance, social security)
- life expectancy is a non-linear function of age-specific mortality rates; trends are driven by trends at different ages
- want to take into consideration age patterns but reduce the dimensionality of the model

Lee-Carter model

$$\log m_{x,t} = a_x + b_x \cdot k_t + \varepsilon_{x,t}$$

- $m_{x,t}$ are age-specific mortality rates for time t
- a_x is some baseline mortality schedule
- b_x is the contribution of age group x to mortality change over time
- k_t is a time index, telling us how much mortality is changing

So if you have a_x and b_x, then you can forecast k_t to obtain forecasts of mortality.

Obtaining values of the parameters

Do Singular Value Decomposition (SVD) on matrix of demeaned, logged age-specific rates over time. Call this matrix X. This matrix has dimensions T (number of years) by A (number of age groups). Then the SVD is

$$X = UDV'$$

Or let's write it as

$$X_{t,a} = u_{t,1} * d_{1,1} * (v_{a,1})' + u_{t,2} * d_{2,2} * (v_{a,2})' + u_{t,3} * d_{3,3} * (v_{a,3})' + \dots$$

Why is this useful?

Obtaining values of the parameters

Do Singular Value Decomposition (SVD) on matrix of demeaned, logged age-specific rates over time. Call this matrix X. Then the SVD is

$$X = UDV'$$

- a_x is just the mean age schedule
- b_x is the first right singular vector, v_1, normalized to sum to 1.
- k_t is the first left single vector, multiplied by the first singular value $u_1 \cdot d_{1,1}$

Once you have these, you can forecast k_t using ARIMA etc

Matrices (Mx)

	0	1	2	3	4	5	6	7	8	9	10
1	0.11376	0.01370	0.00631	0.00464	0.00447	0.00367	0.00330	0.00323	0.00262	0.00227	0.00204
2	0.10436	0.01383	0.00634	0.00404	0.00360	0.00284	0.00233	0.00240	0.00228	0.00172	0.00180
3	0.10480	0.01434	0.00596	0.00447	0.00314	0.00258	0.00272	0.00224	0.00211	0.00181	0.00156
4	0.09535	0.01131	0.00525	0.00424	0.00315	0.00229	0.00182	0.00208	0.00176	0.00176	0.00185
5	0.09701	0.01150	0.00514	0.00397	0.00298	0.00217	0.00224	0.00200	0.00161	0.00192	0.00169
6	0.09293	0.01280	0.00552	0.00324	0.00282	0.00255	0.00215	0.00196	0.00163	0.00167	0.00154
7	0.08456	0.01025	0.00453	0.00359	0.00308	0.00235	0.00210	0.00228	0.00184	0.00165	0.00172
8	0.08617	0.00987	0.00515	0.00373	0.00253	0.00236	0.00208	0.00201	0.00200	0.00141	0.00174
9	0.09228	0.01253	0.00529	0.00385	0.00313	0.00248	0.00247	0.00190	0.00192	0.00175	0.00139
10	0.09353	0.01050	0.00447	0.00333	0.00273	0.00240	0.00230	0.00190	0.00202	0.00181	0.00159
11	0.08547	0.00864	0.00371	0.00274	0.00207	0.00153	0.00196	0.00141	0.00128	0.00137	0.00134
12	0.07152	0.00848	0.00407	0.00227	0.00150	0.00165	0.00162	0.00147	0.00139	0.00119	0.00107
13	0.06448	0.00794	0.00323	0.00267	0.00176	0.00133	0.00132	0.00111	0.00135	0.00095	0.00104
14	0.05863	0.00724	0.00283	0.00251	0.00206	0.00135	0.00152	0.00133	0.00103	0.00118	0.00094
15	0.05838	0.00676	0.00335	0.00255	0.00185	0.00169	0.00155	0.00137	0.00114	0.00120	0.00116

Matrices (log Mx)

	0	1	2	3	4	5	6	7	8	9	10
1	-2.173664	-4.290359	-5.065620	-5.373041	-5.410367	-5.607564	-5.713833	-5.735273	-5.944581	-6.087975	-6.194805
2	-2.259909	-4.280915	-5.060877	-5.511511	-5.626821	-5.863951	-6.061887	-6.032287	-6.083580	-6.365431	-6.319969
3	-2.255702	-4.244702	-5.122685	-5.410367	-5.763532	-5.959966	-5.907123	-6.101279	-6.161067	-6.314428	-6.463069
4	-2.350201	-4.482068	-5.249527	-5.463192	-5.760353	-6.079203	-6.308919	-6.175387	-6.342441	-6.342441	-6.292570
5	-2.332941	-4.465408	-5.270702	-5.528989	-5.815832	-6.133028	-6.101279	-6.214608	-6.431521	-6.255430	-6.383027
6	-2.375909	-4.358310	-5.199377	-5.732182	-5.871018	-5.971662	-6.142287	-6.234811	-6.419175	-6.394932	-6.475973
7	-2.470294	-4.580478	-5.397033	-5.629603	-5.782826	-6.053340	-6.165818	-6.083580	-6.297990	-6.406980	-6.365431
8	-2.451433	-4.618255	-5.268759	-5.591347	-5.979536	-6.049094	-6.175387	-6.209621	-6.214608	-6.564166	-6.353870
9	-2.382928	-4.379630	-5.241937	-5.559682	-5.766722	-5.999497	-6.003537	-6.265901	-6.255430	-6.348139	-6.578452
10	-2.369473	-4.556380	-5.410367	-5.704783	-5.903454	-6.032287	-6.074846	-6.265901	-6.204658	-6.314428	-6.444021
11	-2.459590	-4.751353	-5.596723	-5.899797	-6.180207	-6.482488	-6.234811	-6.564166	-6.660895	-6.592945	-6.615086
12	-2.637778	-4.770045	-5.504112	-6.087975	-6.502290	-6.406980	-6.425329	-6.522493	-6.578452	-6.733802	-6.840097
13	-2.741400	-4.835842	-5.735273	-5.925677	-6.342441	-6.622576	-6.630124	-6.803395	-6.607651	-6.959049	-6.868535
14	-2.836509	-4.928134	-5.867479	-5.987473	-6.185049	-6.607651	-6.489045	-6.622576	-6.878196	-6.742241	-6.969631
15	-2.840782	-4.996732	-5.698795	-5.971662	-6.292570	-6.383027	-6.469500	-6.592945	-6.776727	-6.725434	-6.759335

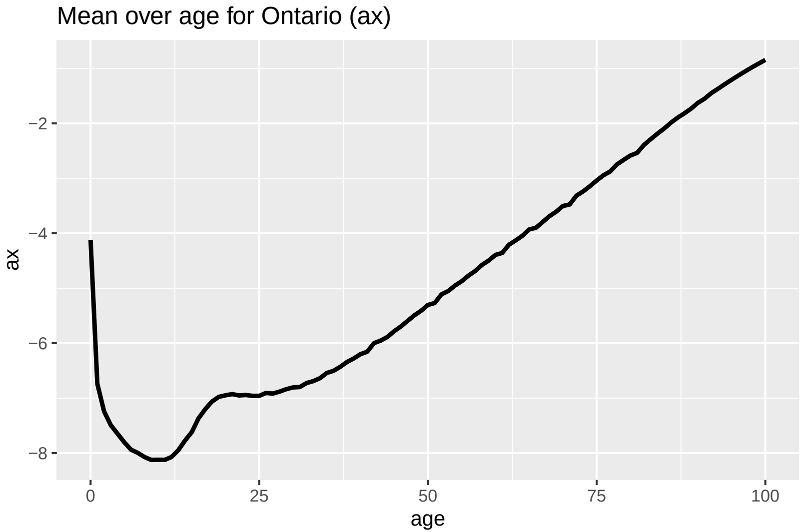
Matrices (demeaned log Mx)

	0	1	2	3	4	5	6	7	8	9
1	1.94648702	2.44841075	2.17619947	2.120204424	2.23916996	2.197087318	2.22587703	2.26210378	2.12839672	2.03737716
2	1.86024251	2.45785506	2.18094256	1.981734750	2.02271540	1.940699708	1.87782283	1.96509038	1.98939784	1.75992162
3	1.86444982	2.49406775	2.11913427	2.082878467	1.88600435	1.844685055	2.03258644	1.89609751	1.91191035	1.81092418
4	1.76995038	2.25670221	1.99229187	2.030053327	1.88918400	1.725447474	1.63079106	1.82198954	1.73053621	1.78291114
5	1.78721012	2.27336195	1.97111687	1.964256153	1.83370485	1.671622824	1.83843043	1.78276882	1.64145658	1.86992252
6	1.74424257	2.38046009	2.04244165	1.761063388	1.77851844	1.832989015	1.79742241	1.76256611	1.65380241	1.73042096
7	1.64985739	2.15829262	1.84478573	1.863642261	1.86671115	1.751310984	1.77389191	1.91379708	1.77498797	1.71837262
8	1.66871814	2.12051477	1.97306050	1.901898292	1.67000085	1.755557275	1.76432246	1.78775636	1.85836958	1.56118704
9	1.73722348	2.35914069	1.99988204	1.933563206	1.88281456	1.805154216	1.93617271	1.73147553	1.81754759	1.77721312
10	1.75067829	2.18239017	1.83145220	1.788462362	1.74608316	1.772364393	1.86486369	1.73147553	1.86831991	1.81092418
11	1.66056149	1.98741750	1.64509567	1.593447979	1.46933016	1.322163391	1.70489904	1.43321135	1.41208248	1.53240807
12	1.48237318	1.96872537	1.73770679	1.405269890	1.14724666	1.397670944	1.51438071	1.47488404	1.49452615	1.39155064
13	1.37875115	1.90292819	1.50654593	1.567568531	1.30709536	1.182074598	1.30958630	1.19398166	1.46532699	1.16630404
14	1.28364256	1.81063612	1.37434050	1.505772811	1.46448753	1.197000248	1.45066490	1.37480058	1.19478120	1.38311177
15	1.27936942	1.74203781	1.54302414	1.521583417	1.35696719	1.421624185	1.47020949	1.40443238	1.29625066	1.39991889

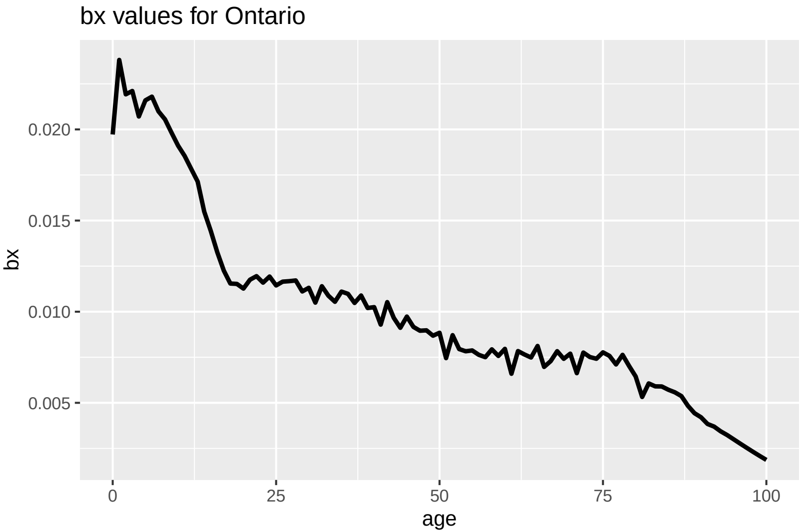
One result of SVD

	V1	V2	V3	V4	V5	V6	V7
1	-0.17644803	-0.043671918	2.911831e-01	-0.051626000	-0.0791457445	0.096408354	-0.1126018880
2	-0.21303696	-0.133457617	2.082918e-01	0.072049258	0.0486585968	0.091221296	0.2155764310
3	-0.19614433	-0.045402578	6.408482e-02	-0.049022944	-0.0174101312	0.016059620	0.1640398433
4	-0.19779899	0.009540088	1.622828e-01	0.049387065	-0.0746159969	0.013750214	0.0545144946
5	-0.18526720	-0.005993825	1.090192e-01	0.039956930	-0.1462442440	0.228581865	0.0738058391
6	-0.19320127	0.086930641	4.152262e-02	0.079263109	-0.0694527961	-0.003556025	-0.1396332362
7	-0.19500306	0.043602135	4.425317e-02	0.153084293	0.0168602357	0.154165294	-0.3108429431
8	-0.18779893	0.028451255	5.430883e-02	0.240634961	-0.4044162204	-0.582911268	-0.1306710516
9	-0.18386417	0.026742076	6.799394e-02	-0.054761866	0.3274144046	0.196924797	0.2554777672
10	-0.17731655	0.014137392	1.325315e-01	0.096814492	-0.5330217116	0.362324411	0.0531516421
11	-0.17097382	-0.013762020	1.660930e-01	0.045297034	0.0117451792	-0.192486242	0.2253864554
12	-0.16593855	-0.022366962	8.742433e-02	0.070571906	0.0759460518	-0.280440246	-0.2572398291
13	-0.15959397	-0.035732041	2.475833e-01	0.447352173	0.4705077159	0.084231168	-0.2864042120
14	-0.15330006	-0.018801949	1.237092e-01	0.136522199	-0.1415086018	0.161984925	0.0172767472
15	-0.13859267	-0.038934148	-6.719656e-02	0.111899249	0.0290906117	-0.157763150	-0.0006484033

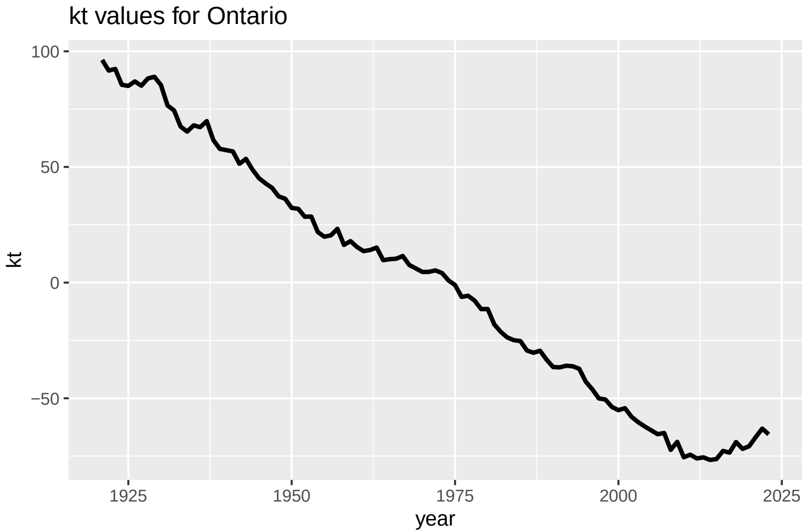
Lee-Carter for Ontario



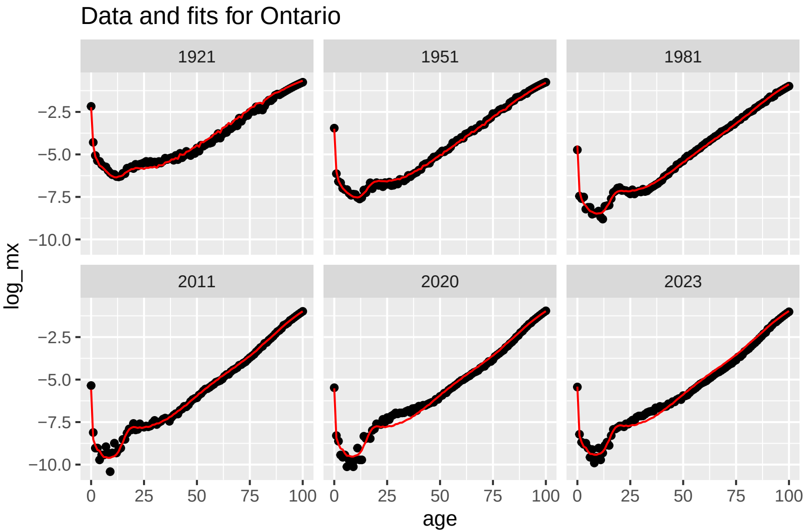
Lee-Carter for Ontario



Lee-Carter for Ontario



Lee-Carter for Ontario



Mortality as a more general regression framework

- So far, have considered mortality rates or probabilities with no exposure adjustment
- But often want to incorporate information about the size of the population (especially when estimating in smaller populations)
- Can consider observed deaths as an outcome of a Poisson process (or Binomial if dealing with smaller populations)

e.g. We could use Brass in a hierarchical model!

Consider deaths / mortality by population group g

$$
\begin{aligned}
D_{xgt} &\sim \text{Poisson}(P_{xgt} \cdot h_{xgt}) \\
l_{xgt} &= e^{-\sum h_{xgt}} \\
\text{logit} ł_{xgt} &= \alpha_{gt} + \beta_{gt} Y_x \\
\alpha_{gt} &\sim N(\mu_{sex[g]} + \mu_{race[g]}, \sigma_\alpha^2) \\
\beta_{gt} &\sim N(\beta_{g,t-1}, \sigma_\beta^2)
\end{aligned}
$$

MORE ON THIS LATER!!!

Lab now