Math 341 Homework 8

Isaac Boaz

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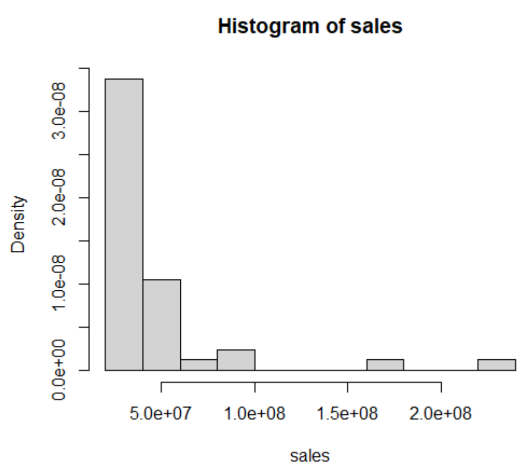
Problem 7

(a)

$$\alpha = 1 - 0.9 = 0.1$$
$$\alpha/2 = 0.05$$

$$\begin{aligned} t_{\alpha,n-1} &= t_{0.05,42} \\ &= \text{qt}(0.05,\ 42,\ \text{FALSE}) \\ &= 1.681952 \end{aligned}$$

$$\begin{aligned} &\bar{x} \pm t_{\alpha,n-1}\frac{s}{\sqrt{n}} \\ &= 43698770 \pm 1.681952\frac{40268914}{\sqrt{43}} \\ &= 43698770 \pm 10328788.03 \\ &= [33369981.97, 54027558.03] \end{aligned}$$



(b) A rough distribution for this data would be an exponential distribution, as the sales seem to go from high at the beginning and fall off exponentially as time passes. Since the exponential distribution only has one parameter (λ), we know it can be reasonably estimated by its mean ($\lambda \approx \mu = 43698770$).

(c) Our estimation of μ is most reliable when the random variables are normally distributed and independent. Since our dataset is clearly not normally distributed, it would require significantly more data samples ($n \to 100+$) to be able to estimate μ with a high degree of confidence.

(d)

$$\bar{x_1} - \bar{x_2} \pm t_{\alpha/2,n_1+n_2-2}s_p\sqrt{1/n_1 + 1/n_2}$$
$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

(e) Requires part d.

(f) It seems like the spreads are similar, so we can presumably trust the common variance assumption.

Problem 12

Problem 13

(a) Binomial Distrbution with $n = 100$ and $p = 0.05$ (success being a CI does not contain the mean μ).

(b)

$$\begin{aligned}
&P(3 \leq X < 8) \\
&\quad = P(X \leq 7) - P(X < 3) \\
&\quad = P(X \leq 7) - P(X \leq 2) \\
&\quad \approx 0.872039 - 0.1182629 \\
&\quad \approx 0.7537765
\end{aligned}$$

(c)

$$\begin{aligned}
Poisson &\sim Bionmial \\
Y &\sim Poisson(np) \\
&\sim Poisson(5)
\end{aligned}$$

$$\begin{aligned}
P(3 \leq Y < 8) &= P(Y \leq 7) - P(Y < 3) \\
&= P(Y \leq 7) - P(Y \leq 2) \\
&\approx 0.866628 - 0.124652 \\
&\approx 0.7419763
\end{aligned}$$

(d) The Poisson approximation is a good approximation for the binomial distribution when n is large and p is small, and the two probabilities reflect this.

Problem 14

(a) To achieve a 95% confidence interval with a margin of error being at most 0.025:

$$n = \frac{z^2_{a/2}\sigma^2}{E^2}$$
$$n = \frac{z^2_{a/2}p(1-p)}{E^2}$$

$$\begin{aligned} E &= 0.025 \\ \alpha &= 1 - 0.98 = 0.02 \\ Z_{\alpha/2} &= Z_{0.01} \\ \text{invNorm}(0.01,\ 0,\ 1,\ \text{RIGHT}) &= 2.326 \\ P &\to 0.5 \end{aligned}$$

$$\begin{aligned} n &\geq \frac{2.326^2 0.5(1-0.5)}{0.025^2} \\ &\geq \lceil 2164.1104 \rceil = 2165 \end{aligned}$$

(b) For the life of a light bulb that is normally with $\sigma = 25$. To find a CI with 95% and error ≤ 2.5:

$$\begin{aligned} E &= 2.5 \\ \alpha &= 1 - 0.95 = 0.05 \\ Z_{\alpha/2} &= Z_{0.025} \\ &= \text{invNorm}(0.025,\ 0,\ 1,\ \text{RIGHT}) = 1.959 \\ n &\geq \frac{1.959^2 \cdot 25^2}{2.5^2} \\ &\geq \lceil 383.76 \rceil \to 384 \end{aligned}$$

(c) Creating a 95% CI with an error at most 0.05 assuming $0.1 \leq p \leq 0.3$:

$$\begin{aligned} E &= 0.05 \\ \alpha &= 1 - 0.95 = 0.05 \\ Z_{\alpha/2} &= Z_{0.025} \\ &= \text{invNorm}(0.025,\ 0,\ 1,\ \text{RIGHT}) = 1.959 \\ P &\to 0.3 \\ n &\geq \frac{1.959^2 \cdot 0.3(1-0.3)}{0.05^2} \\ &\geq \lceil 322.365204 \rceil = 323 \end{aligned}$$