MATH 341 HW 3

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Problem 4

Measuring the effectiveness of treatment.

- RR: Relative Risk
- ARR: Absolute Risk Reduction
- T: Treatment Group $\implies \overline{T}$: Control Group
- B: 'Bad Outcome'

a) $RR = P(B|T)/P(B|\overline{T})$
RR should be > 1 if the treatment increases the chances of harm, and < 1 if it reduces the chances of harm. In this instance, the numerator represents the number of people harmed in total, with the treatment group

b) $ARR = P(B|\overline{T}) - P(B|T)$
As the paper explains how $P(B|\overline{T})$

c) Is it possible to have a sutation where $RR \approx 0$, and$ARR \approx 0$?
Consider a "Rare disease", where

$$\begin{aligned} P(B|\overline{T}) &= 0.001 \\ P(B|T) &= \text{(smaller than above)} \\ ARR &= P(B|\overline{T}) - P(B|T) \end{aligned}$$

d) Show that $P(B|\overline{T}) = \frac{ARR}{1-RR}$

$$\begin{aligned} \frac{ARR}{1-RR} &= \frac{P(B|\overline{T}) - P(B|T)}{1 - [\frac{P(B|T)}{P(B|\overline{T})}]} \\ &= \frac{P(B|\overline{T})}{P(B|\overline{T})} \cdot \frac{P(B|\overline{T}) - P(B|\overline{T})}{1 - [\frac{P(B|T)}{P(B|\overline{T})}]} \end{aligned}$$

Problem 8

The following website compares different algorithms for predicting Australian credit approval results for 517 individuals. We focus on the confusion matrix (where each number is divided by the total) for the logistic regression model.

	Predicted Denied	Predicted Approved	Total
Actually Denied	249/517	38/517	287/517
Actually Approved	18/517	149/517	230/517
Total	330/517	187/517	1

a) Let A denote 'Actually Approved' and B denote 'Predicted Approved'. Show mathematically whether or not A and B are independent
We know A and B are independent if $P(A \cap B) = P(A) \cdot P(B)$.

$$\begin{aligned}
P(A) &= \frac{230}{517} \\
P(B) &= \frac{187}{517} \\
P(A) \cdot P(B) &= \frac{230}{517} \cdot \frac{187}{517} \\
&= \frac{43010}{267289} = \frac{3910}{24299} \\
P(A \cap B) &= \frac{149}{517} \\
\frac{149}{517} &\neq \frac{3910}{24299}
\end{aligned}$$

Since this is not the case, we can conclude that A and B are not independent.

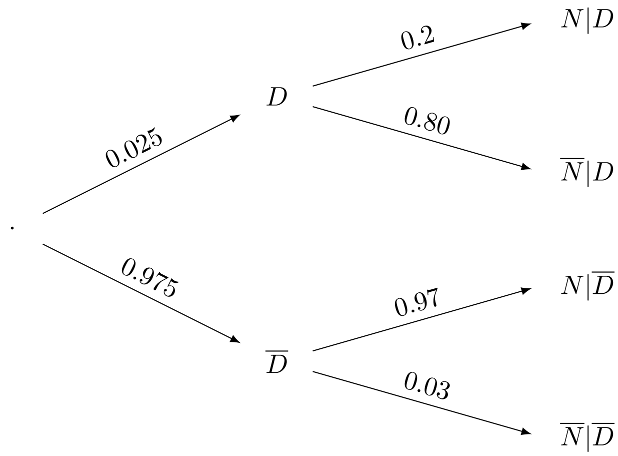
b) Does the result from (a) seem to indicate that the logistic regression model has some ability to predict the outcome correctly? In other words, discuss what must happen if the logistic regression model simply generates random predictions.
This model would not be able to accurately predict the outcome correctly, as the events are not independent.

c) Let C denote 'Actually Denied' and D denote 'Predicted Denied'. Without doing any calculation, discuss whether or not C and D are independent by referring to the result from (a) and one of the results mentioned in the lecture notes.
We can rewrite these as

$$\begin{aligned}
C &= \text{'Actually Denied'} = \overline{A} \\
D &= \text{'Predicted Denied'} = \overline{B}
\end{aligned}$$

Since we know that A and B are not independent, we can conclude that C and D are also not independent.

Problem 11



a) What is the probability that an individual tests positive?

$$
\begin{aligned}
P(\overline{N}) &= P(\overline{N}|D) + P(\overline{N}|\overline{D}) \\
&= 0.8 \cdot 0.025 + 0.03 \cdot 0.975 \\
&= 0.04905
\end{aligned}
$$

b) Given an individual tests positive, what is the probability that they have the disease?

$$
\begin{aligned}
P(D \mid \overline{N}) &= \frac{P(D \cap \overline{N})}{P(\overline{N})} \\
&= \frac{P(\overline{N} \cap D)}{P(\overline{N})} \\
&= \frac{P(\overline{N} \mid D)P(D)}{P(\overline{N})} \\
&= \frac{0.8 \cdot 0.025}{0.04905} = 0.40774719673
\end{aligned}
$$

c) Given an individual tests negative, what is the probability that they don't have the disease?

$$
\begin{aligned}
P(\overline{D}|N) &= \frac{P(\overline{D} \cap N)}{P(N)} \\
&= \frac{P(N \cap \overline{D})}{P(N)} \\
&= \frac{P(N|\overline{D})P(\overline{D})}{P(N)} \\
&= \frac{0.97 \cdot 0.975}{1 - P(\overline{N})} \\
&= \frac{0.97 \cdot 0.975}{1 - 0.04905} \approx 0.9945317
\end{aligned}
$$

d) Comment on (b) and (c).
I found these results rather surprising, as it shows that WHO cares (statistically) more about false positives than false negatives. Considering the probability of having the disease given a positive test is significantly lower than the probability of not having the disease given a negative test, it goes to show how the multiplicative aspect probabilities can heavily impact raw statistics.