MATH 341 HW 2

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Problem 2

Prove the result (3) in "MATH341 07 Chapter 1 #7 Some Important Results Related to Probability". That is,

$$P(A \cup B \cup C)$$

= $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Proof. Additive Rule.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$B \cap \overline{A} = C$$

$$A \cap C = \emptyset$$

$$P(A \cup C) = P(A) + P(C)$$

$$= P(A) + P(B \cap \overline{A})$$

$$B = (B \cap \overline{A}) \cup (A \cap B)$$

$$P(B) = P(B \cap \overline{A}) + P(A \cap B)$$

$$P(B) - P(A \cap B) = P(B \cap \overline{A})$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Revisiting the original question:

Proof. Let $D = B \cup C$

$$\begin{split} P(A \cup B \cup C) &= P(A \cup D) \\ P(A \cup D) &= P(A) + P(D) - P(A \cap D) \\ &= P(A) + P(B \cup C) - P(A \cap (B \cup C)) \\ P(B \cup C) &= P(B) + P(C) - P(B \cap C) \\ P(A \cup B \cup C) &= P(A) + (P(B) + P(C) - P(B \cap C)) - P(A \cap D) \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap D) \\ P(A \cap D) &= P(A \cap (B \cup C)) \\ &= P((A \cap B) \cup (A \cap C)) \end{split}$$

Let
$$X = (A \cap B), Y = (A \cap C)$$

$$P((A \cap B) \cup (A \cap C)) = P(X) + P(Y) - P(X \cap Y)$$

$$= P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C))$$

$$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$

$$P(A \cup B \cup C)$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Problem 3

According to the survey conducted by The List, 60% of respondents are dog lovers and 23% of them are cat lovers. Also, 11% of them are not fans of either pet.

a) Assuming P(D) = 0.6, P(C) = 0.23

	С	\overline{C}	Total
D	-0.06	0.66	0.6
\overline{D}	0.29	0.11	0.4
Total	0.23	0.77	1

This table does not make sense, as $C \cap D$ is < 0, which is impossible under $0 \le P(A) \le 1$.

b) We assume that "dog lovers" and "cat lovers" are exclusive (i.e. $D \cap C = \emptyset$). However, this assumption is incorrect.

c) Assuming P(D) = 0.8, P(C) = 0.3:

	$^{\mathrm{C}}$	\overline{C}	Total
D	0.14	0.66	0.8
\overline{D}	0.16	0.11	0.2
Total	0.3	0.7	1

Allows us to calculate $P(\overline{C} \cup D) = 0.66$.

Finally, finding respondents who love cats (and not dogs) is $P(C \cap \overline{D}) = 0.16$

Problem 13

- a) Compute the probability of getting a "full house".
 - 1. One way of calculating this is first assigning the first pair's rank. (13 ranks, picking 1)
 - 2. Assign the suit of the pair. (4 suits, picking 2)
 - 3. Assign the rank for the last 3 cards. (12 ranks, picking 1)
 - 4. Lastly, these 3 cards can be of any suit (4 suits, picking 3)

$$\begin{pmatrix} 13\\1 \end{pmatrix} \begin{pmatrix} 4\\2 \end{pmatrix} \cdot \begin{pmatrix} 12\\1 \end{pmatrix} \begin{pmatrix} 4\\3 \end{pmatrix}$$

- 1. Alternatively, you could assign the three-card's rank first (13 ranks, picking 1)
- 2. Assign the suits (4 suits, picking 3)
- 3. Assign the remaining pair's rank (12 ranks, picking 1)
- 4. Pick remaining suits (4 suits, picking 2)

$$\begin{pmatrix} 13\\1 \end{pmatrix} \begin{pmatrix} 4\\3 \end{pmatrix} \cdot \begin{pmatrix} 12\\1 \end{pmatrix} \begin{pmatrix} 4\\2 \end{pmatrix}$$

Both methods predictably provide the same "ways of getting a full house" at 3744. Finally, plugging in the total ways one can draw any 5 cards, we get:

$$\frac{3744}{\binom{52}{5}} = \frac{3744}{2598960} = 0.00144$$

b) Compute the number of ways of having a pair of aces and a triple of cards of another rank.

The math for this is the same as the previous problem, with the removal of picking any rank (1 rank, picking 1)

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Giving us 288 ways of getting a pair of aces and a triple of cards of another rank. Doing the math:

$$\frac{288}{\binom{52}{5}} = \frac{288}{2598960} = 0.0001108136$$

Shows us the probability is < 0.02%.