Math 341 Project 1

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Probability Questions

- 1. Both roles rely heavily on data analysis, with the ability to crunch vast amounts of information to make informed decisions being a vital aspect of each. In the fast-paced world of Formula One racing, Computer Science majors can utilize their skills in programming and data analysis to process the mountains of data generated by a race.
 - Another similarity between these roles is the need for algorithmic thinking. In Formula One racing, strategists must break down complex variables like tire wear, fuel consumption, and track conditions into manageable components to develop effective race strategies. Similarly, Computer Science majors use algorithmic thinking to break down complex problems into smaller, more manageable pieces.
- 2. As binomial only checks two outcomes (success or failure), we can assign a safety car leading as a success and otherwise as a failure. One thing to note is that these laps aren't necessarily independent, as each lap may have an impact on the safety car's deployments further down the line.
- 3. The Poisson distribution measures the # of occurrences of an event within a fixed time/space. A requirement of this distribution is that $n \to \infty$, $p \to 0$, $\lambda = np$. As n represents the number of laps, we can assume that it should be relatively high (given a timespan of a few seasons for example). Similarly, we can assume the safety car's deployment rate should be relatively low.
- 4. The # of safety car deployments can be modeled as a Poisson distribution, and thus, the interval between each deployment can be represented by an exponential distribution.
- 5. We can assume that the two time periods are independent of each other as they are disjoint.

6.

$$P(X \ge t_1 + t_2 \mid X \ge t_1) = P(X \ge t_2), t_1 \ge 0, t_2 \ge 0$$

$$t_1 = 3$$

$$t_2 = 5$$

$$P(X \ge 3 + 5 \mid X \ge 3) = P(X \ge 8 \mid X \ge 3)$$

$$= P(X \ge 5)$$

As the memoryless' property name implies, the probability of an event occurring at a time t is independent of the time that has passed since the event occurred. Thus, the probability of an event occurring at time $t_1 + t_2$ is independent of the probability of the event occurring at time t_1 . Therefore, the probability of an event occurring at time $t_1 + t_2$ (given t_1 has passed) is equal to the probability of the event occurring at time t_2 .

Statistics Questions

- 1. Judging by the best Poisson line fit, the distributions seem to follow a Poisson distribution.
- 2. Since the Poisson distribution's equation is $P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$, we can see that as λ increases, the probability of an event occurring increases. Since Poisson takes in the average amount of times an event occurs within a given timespan, it makes sense to use the mean # of accidents for λ .
- 3. In general it seems that the interval between safety car deployments follows an exponential distribution.
- 4. Since the exponential distribution's equation is $P(X \le t) = 1 e^{-\lambda t}$, we can see that as λ increases, the probability of an event occurring increases. Since exponential takes in the average amount of time between events, it makes sense to use the mean interval between safety car deployments (ie the inverse of # of deployments within a set timespan) for λ .

	Means		proximately the inverse of the # of safety car deployments per
	$first_half$	0.7291667	half (i.e. interval $\approx \frac{1}{\#ofdeployments}$). This makes sense as the
5.	interval1	1.328827	exponential distribution (time between deployments) is the dis-
	second_half	0 6960784	tribution of the time between events, and the Poisson distribu-
			tion (number of deployments within a timespan), are inversely
	interval2		related.
	Going by the	given means	, we can see that each interval is ap-