Math 341 Homework 5

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Problem 2

a) Can be modeled as the Binomial Distribution $x \sim b(8, 0.001)$ where $n = 8,\ p = 0.001$

b) No, the cars differ, track conditions may be different, different drivers Additionally, one race being faster may motivate a driver and impact them mentally to do better/worse

c) No, the poisson distribution requires n to be large, p to be small, and that np to be fixed. In this case, n is very small.

d) $Y \approx X$ in distribution.

$$Y \sim Poisson(\lambda).$$
$$\frac{e^{-Y}Y^x}{x!}$$

e) $P(Y = 0),\ P(X = 0)$

f) Q1, Q2 Referencing d for one session, doubling these sessions keeps the poisson distribution. Let W be the # of drivers.

$$W \sim Poisson(2\lambda)$$
$$f$$

Problem 3

a) Assume $p = 0.1,\ n = 24$. Poisson with parameter $np = 2.4$

b) Binomial takes $\approx 2\times$ longer at 2.34 for poisson vs 4.72 for binomial.

Problem 14

Suppose X is uniformly distributed in $[a, b]$.

a) Let $a \leq c \leq x \leq b$ with $a \leq c < b$. Compute $P(X \leq x \mid X \geq c)$.

$$X \sim U(a, b)$$
$$\begin{aligned} P(X \leq x \mid X \geq c) &= \frac{P(X \leq x \cap X \geq c)}{P(X \geq c)} \\ &= \frac{P(c \leq X \leq x)}{P(X \geq c)} \\ &= \frac{\int_c^x f(t)dt}{\int_c^b f(t)dt} \\ &= \frac{x - c}{b - c} \end{aligned}$$

b) Explain
The lower bound is not a, we are bounding it to c. Since everything else is the same, we can conclude that the distribution is uniform in $[c, b]$.

c) Blah blah blah

Let $V \sim U(a, c)$, show $E[X] = wE[V] + (1 - w)E[Y]$ where $w = \frac{c-a}{b-a}$

$$E[Y] = \frac{b+c}{2}$$
$$E[V] = \frac{a+c}{2}$$