# MATH 341 Study Guide
# Midterm 1

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## Definitions and Laws

1. $A$ and $B$ are **mutually exclusive** or **disjoint** if $A \cap B = \varnothing$.
2. $A_1, A_2, \dots, A_k$ are **exhaustive events** if $A_1 \cup A_2 \cup \cdots \cup A_k = S$.
3. Commutative laws
   - $A \cup B = B \cup A$
   - $A \cap B = B \cap A$
4. Associative laws
   - $A \cup (B \cup C) = (A \cup B) \cup C$
   - $A \cap (B \cap C) = (A \cap B) \cap C$
5. Distributive laws
   - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
6. De Morgan's laws
   - $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
   - $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

## Axioms

1. For any event $A$ in $S$, $0 \leq P(A) \leq 1$.
2. P(S) = 1.
3. For any sequence of mutually exclusive (disjoint) events $A_1, A_2, A_3, \dots \mathrm{in} S$(i.e., $A_i \cap A_j = \varnothing$ whenever $i \neq j$), then
$$P(A_1 \cup A_2 \cup A_3 \dots) = \sum_{i=1}^{=} P(A_i).$$

These axioms imply that

- $P(\varnothing) = 0$.
- $P(A \cup B) = P(A) + P(B)$ when $A$ and $B$ are mutually exclusive.

## Permutations and Combinations

| Select $r$ from $n$ | Order matters | Order does not matter |
|---|---|---|
| Without replacement | ${}_nP_r = \frac{n!}{(n-r)!}$ | ${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ |
| With replacement | $n^r$ | $\binom{n+r-1}{r}$ |

# Conditional Probability

The **conditional probability** of $A$ given $B$ has occured is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad\qquad P(A \cap B) = P(A \mid B)P(B).$$

# Bayes' Theorem

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}.$$