Math 341 Project 1

Isaac Boaz

February 19, 2023

Probability Questions

1. asdf
2. As binomial only checks two outcomes (success or failure), we can assign a safety car leading as a success and otherwise as a failure. One thing to note is that these laps aren't necessarily independent, as each lap may have an impact on the safety car's deployments further down the line.
3. The Poisson distribution measures the # of occurrences of an event within a fixed time/space. A requirement of this distribution is that $n \to \infty,\ p \to 0,\ \lambda = np$. As n represents the number of laps, we can assume that it should be relatively high (given a timespan of a few seasons for example). Similarly, we can assume the safety car's deployment rate should be relatively low.
4. The # of safety car deployments can be modeled as a poisson distribution, and thus, the interval between each deployment can be represented by an exponential distribution.
5. We can assume that the two time periods are independent of each other as they are disjoint.
6. $$\begin{aligned} P(X \geq t_1 + t_2 \mid X \geq t_1) &= P(X \geq t_2), t_1 \geq 0, t_2 \geq 0 \\ t_1 &= 3 \\ t_2 &= 5 \\ P(X \geq 5 + 3 \mid X \geq 3) &= P(X \geq 8 \mid X \geq 3) \\ &= P(X \geq 5) \end{aligned}$$

   As the memoryless' property name implies, the probability of an event occurring at a time t is independent of the time that has passed since the event occurred. Thus, the probability of an event occurring at time $t_1 + t_2$ is independent of the probability of the event occurring at time t_1. Therefore, the probability of an event occurring at time $t_1 + t_2$ is equal to the probability of the event occurring at time t_2.

Statistics Questions

1. Judging by the best poisson line fit, the distributions seem to follow a poisson distribution.

2. Since the poisson distribution's equation is $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$, we can see that as λ increases, the probability of an event occurring increases. Since poisson takes in the average amount of times an event occurs within a given timespan, it makes sense to use the mean # of accidents for λ.

3. In general it seems that the interval between safety car deployments follows an exponential distribution.

4. Since the exponential distribution's equation is $P(X \leq t) = 1 - e^{-\lambda t}$, we can see that as λ increases, the probability of an event occurring increases. Since exponential takes in the average amount of time between events, it makes sense to use the mean interval between safety car deployments (ie the inverse of # of deployments within a set timespan) for λ.

5. Means

first_half	0.7291667
interval1	1.328827
second_half	0.6960784
interval2	1.421543

Going by the given means, we can see that each interval is approximately the inverse of the # of safety car deployments per half (i.e: interval $\approx \frac{1}{\#ofdeployments}$). This makes sense as the exponential distribution (time between deployments) is the distribution of the time between events, and the poisson distribution (number of deployments within a timespan), are inversely related.