MATH 341 HW 3

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January 23, 2023

Problem 4

Measuring the effectiveness of treatment.

- RR: Relative Risk
- ARR: Absolute Risk Reduction
- T: Treatment Group $\implies$ $\overline{T}$: Control Group
- B: 'Bad Outcome'

a) $RR = P(B|T)/P(B|\overline{T})$
RR should be > 1 if the treatment increases the chances of harm, and < 1 if it reduces the chances of harm. In this instance, the numerator represents the number of people harmed in total, with the treatment group

b) $ARR = P(B|\overline{T}) - P(B|T)$
As the paper explains how $P(B|\overline{T})$

c) Is it possible to have a sutation where $RR \approx 0$, and $ARR \approx 0$?
Consider a "Rare disease", where

$$P(B|\overline{T}) = 0.001$$
$$P(B|T) = \text{(smaller than above)}$$
$$ARR = P(B|\overline{T}) - P(B|T)$$

d) Show that $P(B|\overline{T}) = \frac{ARR}{1-RR}$

$$\frac{ARR}{1 - RR} = \frac{P(B|\overline{T}) - P(B|T)}{1 - [\frac{P(B|T)}{P(B|\overline{T})}]}$$
$$= \frac{P(B|\overline{T})}{P(B|\overline{T})} \cdot \frac{P(B|\overline{T}) - P(B|\overline{T})}{1 - [\frac{P(B|T)}{P(B|\overline{T})}]}$$

Problem 8

The following website compares different algorithms for predicting Australian credit approval results for 517 individuals. We focus on the confusion matrix (where each number is divided by the total) for the logistic regression model.

	Predicted Denied	Predicted Approved	Total
Actually Denied	249/517	38/517	287/517
Actually Approved	18/517	149/517	230/517
Total	330/517	187/517	1

a) Let A denote 'Actually Approved' and B denote 'Predicted Approved'. Show mathematically whether or not A and B are independent

We know A and B are independent if $P(A \cap B) = P(A) \cdot P(B)$.

$$P(A) = \frac{230}{517}$$
$$P(B) = \frac{187}{517}$$
$$P(A) \cdot P(B) = \frac{230}{517} \cdot \frac{187}{517}$$
$$= \frac{43010}{267289} = \frac{3910}{24299}$$
$$P(A \cap B) = \frac{149}{517}$$
$$\frac{149}{517} \neq \frac{3910}{24299}$$

Since this is not the case, we can conclude that A and B are not independent.

b) Does the result from (a) seem to indicate that the logistic regression model has some ability to predict the outcome correctly? In other words, discuss what must happen if the logistic regression model simply generates random predictions.

This model would not be able to accurately predict the outcome correctly, as the events are not independent.

c) Let C denote 'Actually Denied' and D denote 'Predicted Denied'. Without doing any calculation, discuss whether or not C and D are independent by referring to the result from (a) and one of the results mentioned in the lecture notes.

We can rewrite these as

$$C = \text{'Actually Denied'} = \overline{A}$$
$$D = \text{'Predicted Denied'} = \overline{B}$$

From the result in (a), C and D are not independent. We know this because the probability of C given D is not equal to the unconditional probability of C. A reference to the lecture notes states that if two events are independent then their conditional probability will be equal to their unconditional probability.

Problem 11

$$P(D) = 0.025$$
$$P(C|\overline{N}) = 0.80$$
$$P(\overline{C}|N) = 0.97$$