Math 341 Homework 6

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Problem 2

Let X be a random variable representing the number of shooting stars per hour. Assume X is Poisson distributed with $E[x] = \lambda$ (i.e: $X \sim Poisson(\lambda)$).

a) Instead of a 1-hour interval, consider an interval of t hours for some $t > 0$. If Y denotes the number of stars in t hours, what is its distribution and parameter value?

$$Y \sim Poisson(t\lambda)$$

b) Calculate the probability that no shooting stars are observed in t hours (i.e $P(Y = 0)$).

$$P(Y = y) = \frac{e^{-t\lambda} \cdot (t\lambda)^y}{y!}$$

$$\begin{aligned} P(Y = 0) &= \frac{e^{-t\lambda} \cdot (t\lambda)^0}{0!} \\ &= \frac{e^{-t\lambda}}{1} \\ &= e^{-t\lambda} \end{aligned}$$

c) Suppose we measure the time until the first shooting star. Let T denote this time. Explain why the event $A = \{T > t\}$ is equivalent to the event $B = \{Y = 0\}$

Event B represents the event that no shooting stars are observed in t hours.

Event A represents the event that the first shooting star is observed after t hours.

Since event A is defined as $T > t$, we know that at least t hours have passed before the first shooting star is observed.

d) Using (c), compute the cdf of T. Then, state the distribution of T and its parameter value.

$$\begin{aligned} A = B &\implies P(A) = P(B) \\ P(T > t) &= P(Y = 0) \\ &= e^{-t\lambda} \end{aligned}$$

$$\begin{aligned} \text{CDF of T: } F(t) &= P(T \leq t) \\ &= 1 - e^{-t\lambda} \end{aligned}$$

Exponential distribution with parameter λ.