Math 341 Homework 5

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Problem 2

a) A reasonable distribution model would be the binomial distribution using a time of 1:21.072 as a success and any other time a failure. This would be modeled as $x \sim b(8, 0.001)$ where $n = 8$, $p = 0.001$.

b) No, the cars differ, track conditions may be different, different drivers Additionally, one race being faster may motivate a driver and impact them mentally to do better/worse. In short, the trials are neither independent nor identical.

c) The poisson distribution requires n to be large, p to be small, and that np to be fixed. In this case, n is large (nowhere close to ∞).

d) Approximating X using a Poisson distribution of Y:

$$Poisson(X) \sim Y$$
$$\lambda = np$$
$$\binom{n}{x} p^x (1-p)^{n-x} \rightarrow e^{-\lambda} \frac{\lambda^x}{x!}$$
$$\binom{8}{x} 0.001^x (1-0.001)^{8-x} \rightarrow e^{-8 \cdot 0.001} \frac{(8 \cdot 0.001)^x}{x!}$$
$$Y = e^{-0.008} \frac{0.008^x}{x!}$$

Using this to find $E[Y]$ and $Var[Y]$

$$E[Y] = Var[Y] = \lambda = 8 \cdot 0.001 = 0.008$$

e) $P(Y = 0)$, $P(X = 0)$

$$P(Y = 0) = \frac{e^{-0.008} \cdot 0.008^0}{0!}$$
$$= \frac{e^{-0.008}}{1}$$
$$= e^{-0.008}$$
$$= 0.992032$$

$$P(X = 0) = \binom{8}{0} 0.001^0 (1-0.001)^{8-0}$$
$$= (1)1(0.999)^8$$
$$= 0.999^8 \approx 0.992028$$

f) Calculate the probability that no driver will get this specific time with two sessions.

$$\lambda = 2 \cdot 0.008 = 0.016$$
$$Poisson(Z) = e^{-0.016} \frac{0.016^0}{0!} = e^{-0.016}$$
$$= 0.984127$$

Problem 3

a) The two distributions look remarkably similar, the biggest distinction with binomial the skewness towards 0. Poisson seems to more smoothly lead towards zero compared to binomial which seems to go sinusodially.

b) Binomial takes $\approx 2\times$ longer at 2.34 for poisson vs 4.72 for binomial. Looking at the two equations, we can see the poisson distribution is much simpler than the binomial distribution due to the $_nC_r$ computation.

Problem 14

Suppose X is uniformly distributed in $[a, b]$.

a) Let $a \leq c \leq x \leq b$ with $a \leq c < b$. Compute $P(X \leq x \mid X \geq c)$.

$$X \sim U(a, b)$$

$$P(X \leq x \mid X \geq c) = \frac{P(X \leq x \cap X \geq c)}{P(X \geq c)}$$

$$= \frac{P(c \leq X \leq x)}{P(X \geq c)}$$

$$= \frac{\int_c^x f(t)dt}{\int_c^b f(t)dt}$$

$$= \frac{x - c}{b - c}$$

b) Explain
The lower bound is not a, we are bounding it to c. Since everything else is the same, we can conclude that the distribution is uniform in $[c, b]$.

c) Blah blah blah
Let $V \sim U(a, c)$, show $E[X] = wE[V] + (1 - w)E[Y]$ where $w = \frac{c-a}{b-a}$

$$E[Y] = \frac{b + c}{2}$$

$$E[V] = \frac{a + c}{2}$$