Math 341 Homework 4

Isaac Boaz

February 1, 2023

Problem 6

Let X be a random variable representing the number of times a randomly selected student goes to a gym in a week. The amount of time usually spent is $50X - 20$, find the expected number of times they go to the gym ($E[X]$), expected time (in minutes $E[50X - 20]$), and variability in the amount of time at the gym ($Var[50X - 20]$). Assuming X has the following pmf.

a) $f(x) = \frac{-3x+15}{24},\ x = 1, 2, 4$
Let's make a table to better understand and represent the pmf.

x	1	2	4
$f(x)$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$
$xf(x)$	$\frac{1}{2}$	$\frac{6}{8}$	$\frac{4}{8}$

Summing up all $xf(x)$ tells us the expected number of times a random person goes to the gym $E[X] = 1.75$ times. Applying the same table technique to $50X - 20$ for time spent:

x	1	2	4
$f(x)$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$
$(50x - 20)f(x)$	15	30	22.5

Gives us an estimated time spent of $E[50X - 20] = 67.5$ minutes. Finally, to calculate the $Var[50X - 20]$ we need to calculate the expected value of the squared deviation from the mean.

$$\begin{aligned} Var(50X - 20) &= Var(50X) = 50^2 Var(X) = 2500 Var(X) \\ &= 2500(E[x^2] - \mu^2) \\ \mu &= 1.75 \\ E[x^2] &= \frac{1^2}{2} + \frac{2^2 \cdot 3}{8} + \frac{4^2 \cdot 1}{8} \\ &= \frac{1}{2} + \frac{3}{2} + 2 = 4 \\ Var(50X - 20) &= 2500(4 - 1.75^2) \\ &= 2500(0.9375) = 2343.75 \end{aligned}$$

b) $f(x) = \begin{cases} 0.65, & x = 1 \\ 0.35, & x = 5 \end{cases}$
We can apply the same steps as before:

x	1	5
$f(x)$	0.65	0.35
$xf(x)$	0.65	1.75

$E[X] = 2.4$ times.

x	1	5
$f(x)$	0.65	0.35
$(50x - 20)f(x)$	19.5	80.5

E[50X - 20] = 100 minutes. Finally:

$$\begin{aligned} Var(50X - 20) &= Var(50X) = 50^2 Var(X) = 2500 Var(X) \\ &= 2500(E[x^2] - \mu^2) \\ \mu &= 2.4 \\ E[x^2] &= 19.5 + 430.5 \\ &= 450 \\ Var(50X - 20) &= 2500(450 - 2.4^2) \\ &= 2500(444.24) = 1110600 \end{aligned}$$

Problem 11

1. A sports team has a win/loss record of 23-24. Let p denote the probability that they win the next game. Let X denote a random variable for the # of wins in the next 10 games.

   a) Based on the win/loss record, suggest a value for p and a distribution for X.

$$\begin{aligned} p &= \frac{23}{24+23} \approx 0.4894 \\ X &= 10 \cdot 0.4894 = 4.894 \\ Var(x) &= p(1-p) \\ &= 0.4894(1-0.4894) = 0.247 \end{aligned}$$

   b) Discuss the reasonableness of the distribution in (a) in terms of 'independent and identical trials'
   Games are not necessarily independent, depending on the lineup, the better a team may progress, they are more likely to face more difficult teams.

   c) Calculate the probability that they will have 8 wins in the next 10.

$$\begin{aligned} P(X=8) &= \binom{10}{8} 0.4894^8 (1-0.4894)^2 \\ &\approx 0.0386088 \end{aligned}$$

   d) Calculate the expected number of wins and its standard deviation in the next 10 games.

$$\begin{aligned} E(X) &= 10 \cdot 0.4894 = 4.894 \\ Var(X) &= 10 \cdot 0.4894(1-0.4894) = 2.47 \end{aligned}$$