14. Ion trap physics

14.1. The Paul trap

Assume ion with positive unit charge e

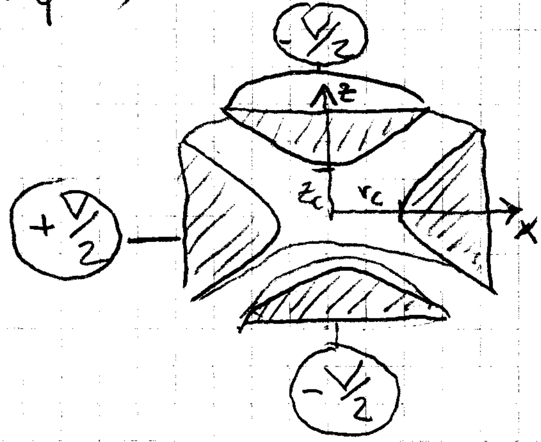
Trap ion with e.m. forces exerted by electrostatic potential $U(\vec{r})$? $\rightsquigarrow$ equilibrium position $\Leftrightarrow$ min. of potential.

* Problem: Gauss theorem $\Rightarrow \nexists$ min. of U in free space.
* The only stationary points of U in free space are saddle points.
* Simplest example of saddle point: origin of quadrupolar potential:

$$U_q(\vec{r}) = \frac{V}{4z_c^2}(x^2+y^2-2z^2) \quad (14.1)$$

* (14.1) is solution of Laplace equation $\Delta U_q(\vec{r}) = 0$.

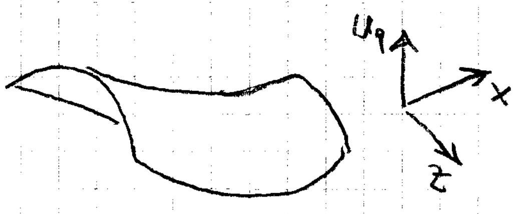
$U_q(\vec{r})$ can be created by cyl. symm. electrode config.:



$r_c = \sqrt{2}\, z_c$

Shape: hyperboloids

For $V>0$: $U_q(\vec{r})$ = min along x & max along z



Situation reversed for $V<0$

$\Rightarrow$ unstable equilibrium at origin.

Two routes to reach stable trapping:

* Add strong B-field along z ⇒ Penning trap (not discussed here)

* Make V a rapidly oscillating function of time:

$$V(t) = V_0 \cos(\omega_{rf} t) \quad (14.2)$$

* Equation of motion $\hat{=}$ Mathieu equation

e.g., along z: $\frac{d^2 z}{d\tau^2} - 2 q_z \cos(2\tau) z = 0 \quad (14.3)$

with dimensionless time $\tau = \omega_{rf} t / 2 \quad (14.4)$

and dimensionless param. $q_z = \frac{2eV_0}{m z_c^2 \omega_{rf}^2} = \frac{4eV_0}{m r_c^2 \omega_{rf}^2} \quad (14.5)$

ion's mass

* Eq. (14.3) can be solved analytically and yields stable motion under certain conditions, e.g., for

$$0 \leq q_z \leq 0{,}908$$
$$\text{and } \omega_{rf} > \frac{\sqrt{2eV_0}}{z_c\sqrt{0{,}908\, m}} \quad (14.6)$$

⇝ Show Fig. 8.2 of Haroche & Raimond

* Resulting motion: Fast harm. "micro-motion" with small amplitude ξ_0 which cancels at trap center + much larger and slower "macro-motion" corresponding to evolution in effective anisotropic harm. potential with $\omega_z = 2\omega_x = 2\omega_y$

More quantitatively: Express ion's position as $z = Z + \xi$ with ξ: small, rapidly oscillating micro-motion component with zero average value and

Z: ion's pos. averaged over a period of micro-motion (evolves on much larger time scale).

Under these conditions: Separation of fast and slow motions

$\Rightarrow$ eq. of motion for ξ: $m\dfrac{d^2\xi}{dt^2} \approx \dfrac{eV_0}{z_c^2}\cos(\omega_{rf}t)Z$ (14.7)

with solution $\xi = -\dfrac{q_z}{2}\cos(\omega_{rf}t)Z$ (14.8)

i.e., amplitude $\sim Z$

Time-averaged kin. energy of micro-motion:

$$\bar{E}_c(Z) = \frac{1}{16}mq_z^2\omega_{rf}^2 Z^2 = \frac{1}{2}m\omega_z^2 Z^2 \quad (14.9)$$

with $\omega_z = \dfrac{1}{2\sqrt{2}}q_z\omega_{rf}$ (14.10)

<u>Note</u>: * $\bar{E}_c(Z)$ has to be borrowed from kin. energy of macro-motion.

* Ion is trapped due to "ponderomotive" force corresponding to gradient of $\bar{E}_c(Z)$ $\Rightarrow$ harm. oscillation along z with freq. $\omega_z \ll \omega_{rf}$

* Smaller ponderomotive force along x, y such that $\omega_x = \omega_y = \omega_z/2$.

* Example: $^{40}Ca^+$-ion in trap with $z_c = 1\,mm$, $V_0 = 100\,V$ and $\omega_{rf} = 2\pi \times 10\,MHz$

$\Rightarrow q_z = 0{,}15$, $\omega_z/2\pi = 530\,kHz$ ($\approx \omega_{rf}/20$)

& $\bar{E}_c(z_c) = \frac{1}{8}q_z e V_0$ ($\approx 2\%$ of static pot. depth).

$\Rightarrow \bar{E}_c(z_c)/k_B \approx$ several thousand Kelvin 8

* Effective trap is conservative (like dipole trap for neutral atoms) ⇝ use dissipative process (e.g., via background gas coll.) to load trap or produce ions near trap centre (e.g., via e^- impact or photo-ionisation).

* Paul traps can be realized with dust particles at ambient conditions with moderate voltages

⇝ Show Bärlauch-Sporen-Fallen-Foto & Fig 8.2 (b) of Haroche & Raimond

14.2 The Linear Paul trap

Often used in quantum information experiments.

<u>Idea</u>: Realize strong radial confinement using lin. RF quadrupol-field (in analogy to mass filter) and close potential in axial direction with DC field (using endcaps)

⇝ Show slide lin. Paul trap

* Typical trap frequencies: $\omega_x = \omega_y \approx 2\pi \times 2\text{-}4\,\mathrm{MHz}$
  $\omega_z \approx 2\pi \times 500\,\mathrm{kHz}$ (harm. pot. along x, y & z)

* Typical trap depth $\sim 10^4\,\mathrm{K}$

* High temp. of ions ⇝ unordered motion of ions in trap potential
* Very low temp. ⇝ ions form ordered structure

14.2.1 Ion crystals

- Consider two ions in lin. Paul trap:
Equilibrium pos.: $\pm z_0/2$, where z_0 corresponds to min. of total pot. energy (trap + Coulomb interaction)

$$E_T(z) = \frac{1}{4} m \omega_z^2 z^2 + \frac{e^2}{4\pi\varepsilon_0 z} \quad (14.11)$$

$$\Rightarrow \text{min. for } z_0 = \left(\frac{e^2}{2\pi\varepsilon_0 m \omega_z^2}\right)^{1/3} \quad (14.12)$$

Example: $^{40}Ca^+$ ⇝ typ. $z_0 \approx 9\,\mu m$ (same order of magn. for $N_{ion} > 2$)

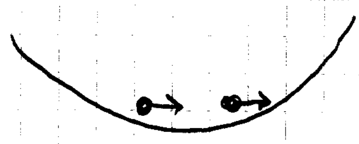
⇝ show ion crystals

14.2.2 Vibrational modes

Consider oscillation of two ions along z-axis

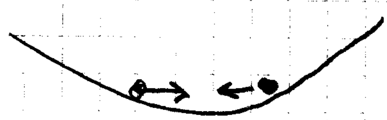
⇝ Class. eq. of motions yields two normal modes:

Center of mass mode (CM):



⇝ freq. ω_z

Stretch mode:



⇝ freq. $\sqrt{3}\,\omega_z$

For three ions there are three modes:

Center of mass mode: •→ •→ •→ freq. ω_z
Stretch mode: ←• • •→ freq. $\sqrt{3}\,\omega_z$
Scissor mode: •→ ←• •→ freq. $\sqrt{29/5}\,\omega_z$

⇒ Three indep. harm. osc.

N ions: CM and stretch mode remain modes with lowest freq. ω_z & $\sqrt{3}\omega_z$ indep. of N.

Note: The frequencies of the higher modes are more and more closely spaced for increasing N.

⇝ Show CM & stretch mode for 7 ions.

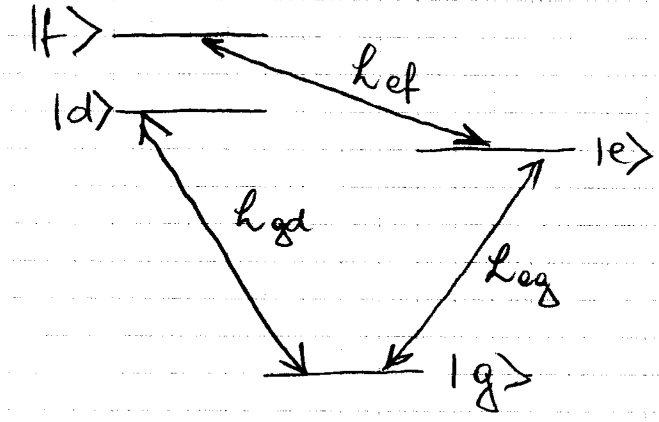
14.3. Ground state cooling

Quantum state manipulation or precision spectroscopy of trapped ions ⇝ prepare motional ground state

Problem: Initial kin. energy after capture is on the order of the trap depth ⇝ huge number of phonons ($\sim 10^9$) must be suppressed

Solution: Combination of Doppler & sideband cooling

Possible, e.g., in four level configuration:



$|e\rangle$ & $|g\rangle$: long-lived states, can be used to encode qubit states $|0\rangle$ & $|1\rangle$

$|f\rangle$ & $|d\rangle$: auxiliary short-lived upper levels

14.3.1 Doppler cooling

Idea: Couple $|e\rangle$ & $|f\rangle$ with resonant laser h_{ef}.
⇝ Excitation, combined with spont. emission from $|f\rangle$ to $|g\rangle$, optically pumps ions to $|g\rangle$.
Simultaneously shine in laser h_{gd}, slightly red-detuned w.r.t. g-d-transition ⇝ Doppler cooling

Friction force (cf. eqs. (13.4) & (13.5)) leads to exponentially damped oscillation. Average energy decreases with rate

$$\frac{1}{\tau_D} = \frac{\hbar k_e^2}{4m} \quad (14.13)$$

for optimal values of Rabi freq. and detuning of h_{gd} ($\Omega = \Gamma_{dg}$ & $\delta = -\Gamma_{dg}/2$ with $\Gamma_{dg}/2\pi$: spont. emission rate of level $|d\rangle$).

Heating due to random direction of photon emission

⇝ Equilibrium at finite energy

$$E \approx \hbar\Gamma_{dg} \quad (14.14)$$

corresponding, on average, to $n_{th} \approx \Gamma_{dg}/\omega_z$ phonons left in the ionic motional state.

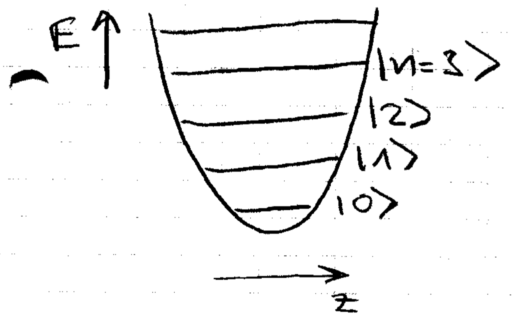
In the experiment, this typically corresponds to a few or a few tens of phonons.

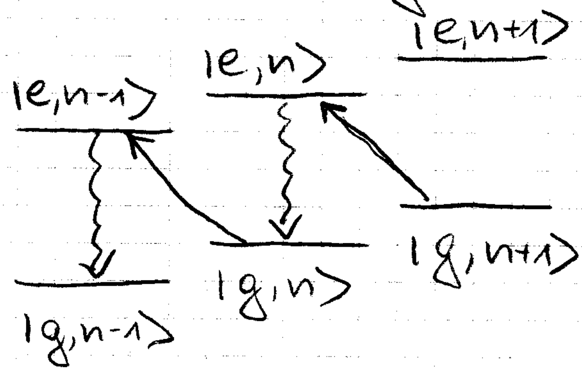
14.3.2 Sideband cooling

Problem: For many experiment, Doppler cooling is not sufficient

Idea: Irradiate ion with a laser that induces the first red sideband transition between the two long-lived states $|e\rangle$ & $|g\rangle$:

q.m. motion:





laser frequency: $\omega_\ell = \omega_{eg} - \omega_z$

* Photon absorption on red sideband ⇝ ion undergoes transition from $|g\rangle$ to $|e\rangle$ and loses one phonon.

* Recycle ion to state $|g\rangle$ by coupling $|e\rangle$ with short-lived state $|f\rangle$ using laser L_{ef} ⇝ $|f\rangle$ decays spontaneously to $|g\rangle$.

Note: * Broadening of $|e\rangle$ due to coupling to $|f\rangle$ must be as large as possible while remaining small enough (typically 100 kHz) to spectrally resolve the sidebands (typically at 1 MHz).

* Spontaneous photon on recycling transition predominantly emitted on carrier. Red and blue sideband transition amplitude suppressed by factor

$$\eta = \sqrt{\frac{\hbar k_\ell^2}{2m\omega_z}} = \sqrt{\frac{E_r}{\hbar\omega_z}} \qquad (14.15)$$

where η : Lamb-Dicke-parameter
k_ℓ : wavenumber of laser light
$E_r = \frac{\hbar^2 k_\ell^2}{2m}$: recoil energy of ion under photon absorption

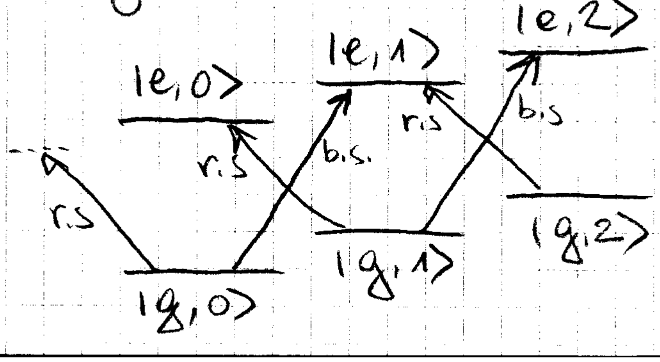
Typical exp. values : $\eta = 0.05 - 0.3$ ⇝ ion most likely emits photon without changing phonon number (momentum conservation ensured by recoil of trap as a whole ⇒ cf. Mössbauer effect).

* After n cycles, the ion ends up in $|g,0\rangle$ ⇝ "dark state" for sideband excitation ⇝ ion has been cooled to ground state of motion!

* Final stage of cooling process can be monitored by measuring the sideband spectrum: After cooling, send probe laser pulse on red or blue sideband and measure prob. for transfer of ion to state $|e\rangle$.

<u>red sideband</u> : transfer to $|e\rangle$ only possible if initially in state $|g, n \geq 1\rangle$

<u>blue sideband</u> : transfer to $|e\rangle$ possible for all n ⇝ signal can be taken as a reference



⇝ show Fig. 8.12 aus Haroche & Raimond

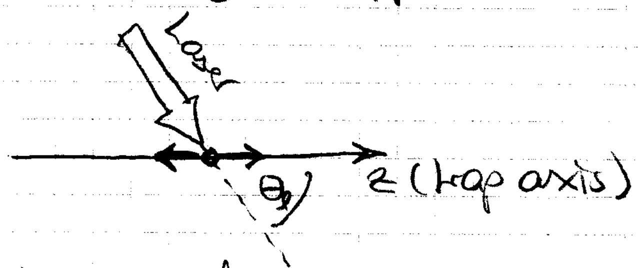
* Ground state population >99,9% has been achieved.

* Cooling of all three vibrational mode in anisotropic traps ($\omega_x \neq \omega_y \neq \omega_z$) has been achieved using the appropriate number of laser fields

* Sideband cooling method can also be aplied to ion crystals ⇒ cool each vibrational mode by addressing one of the ions of the string on the corresponding red sideband frequency.

14.4. Coherent manipulation of internal & external states

Consider a single trapped ion and a single vib. mode:



Hamiltonian for ion-laser-interaction:

$$\hat{H}_\ell(t) = -\frac{\hbar\Omega_\ell}{2}\hat{\sigma}_+ \exp(ik_\ell \cos\theta_\ell \hat{z})\exp(-i(\omega_\ell t+\varphi)) + \text{c.c.} \quad (14.16)$$

Rabi freq. ~ laser amplitude & dipole matrix element for e-g-transition ($\Omega_\ell > 0$ chosen)

$|e\rangle\langle g|$

CM-operator describes spatial dependence of field and ensures momentum conservation along z upon absorption & emission of photons

oscillatory term depending on frequency and phase of laser field.

Expansion:

$$\exp(ik_\ell\cos\theta_\ell\hat{z}) = 1 + ik\hat{z}\cos\theta_\ell + \ldots = 1 + i\eta\cos\theta_L(\hat{a}+\hat{a}^\dagger) + \ldots \quad (14.17)$$

where $\hat{a}^\dagger$ & $\hat{a}$: raising & lowering ops. for phonons in vib. mode.

To first order in η, we have:

$$\hat{H}_\ell(t) \approx \hat{H}_\ell^{\text{carrier}}(t) + \hat{H}_\ell^{\text{red sb}}(t) + \hat{H}_\ell^{\text{blue sb}}(t) \qquad (14.18)$$

where

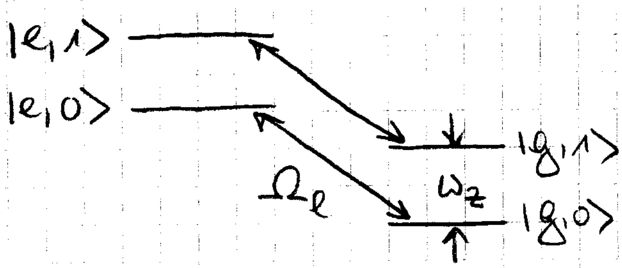
$$\hat{H}_\ell^{\text{carrier}}(t) = -\frac{\hbar\Omega_\ell}{2}\hat{\sigma}_+ e^{-i\varphi}\exp(i(\omega_{eg}-\omega_\ell)t) + \text{c.c.} \qquad (14.19)$$

$$\hat{H}_\ell^{\text{red sb}}(t) = -i\,\frac{\hbar\Omega_\ell\eta\cos\theta_\ell}{2}\hat{a}\hat{\sigma}_+ e^{-i\varphi}\exp(i(\omega_{eg}-\omega_z-\omega_\ell)t) + \text{c.c.} \qquad (14.20)$$

$$H_\ell^{\text{blue sb}}(t) = -i\,\frac{\hbar\Omega_\ell\eta\cos\theta_\ell}{2}\hat{a}^\dagger\hat{\sigma}_+ e^{-i\varphi}\exp(i(\omega_{eg}+\omega_z-\omega_\ell)t) + \text{c.c.} \qquad (14.21)$$

14.4.1 Carrier transitions

~> 1-Qubit-Gates that do not act on vib. mode

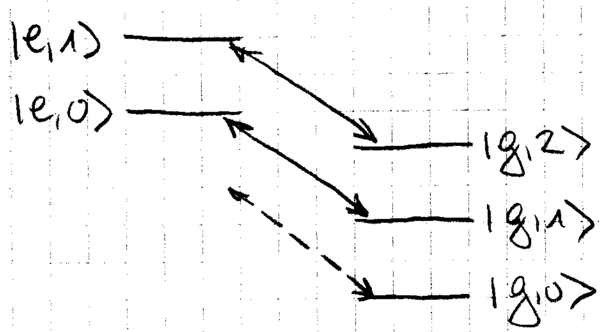


Rabi-Oscillation between $|e,n\rangle \leftrightarrow |g,n\rangle$ for $\omega_\ell = \omega_{eg}$

η small -> vib. modes are only "spectators"

14.4.2 Transitions on 1st red sideband

~> Ion excitation combined with phonon annihilation



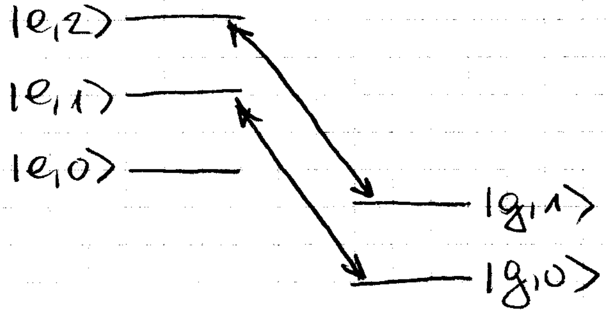
Rabi-osc. between $|e,n\rangle$ and $|g,n+1\rangle$ for $\omega_\ell = \omega_{eg} - \omega_z$ (Rabi-freq. $\Omega_\ell\eta\cos\theta_L\sqrt{n+1}$) while $|g,0\rangle$ remains uncoupled

~> used for sideband cooling and for entangling the qubit with the vib. mode

~> formally equivalent to Jaynes-Cummings-Hamiltonian

14.4.3 Transitions on 1st blue sideband

⇝ Ion excitation combined with phonon creation



Rabi-osc. between $|e,n+1\rangle$ and $|g,n\rangle$ for $\omega_\ell = \omega_{eg} + \omega_z$ (Rabi freq. $\Omega_\ell \eta \cos\theta_\ell \sqrt{n+1}$) while $|e,0\rangle$ remains uncoupled

⇝ used for preparing phonon Fock-states and for entangling the qubit with the vib. mode.

⇝ "anti-Jaynes-Cummings" dynamics.