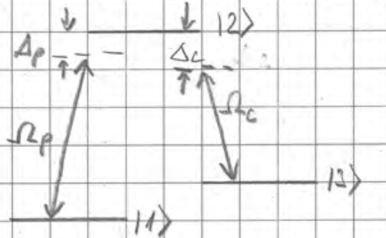
EIT Static case - recap

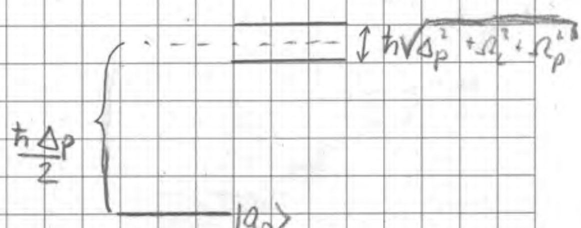


$$H^{RWA} = \begin{pmatrix} 0 & \Omega_p & 0 \\ \Omega_p & -2\Delta_p & \Omega_c \\ 0 & \Omega_c & -2(\Delta_p - \Delta_c) \end{pmatrix}$$

2-Photon resonance: $\Delta_p - \Delta_c = 0$

$$\hbar\omega_0 = 0$$

$$\hbar\omega_\pm = \frac{\hbar}{2}\left[\Delta_p \pm \sqrt{\Delta_p^2 + \Omega_p^2 + \Omega_c^2}\right]$$



$$|a_0\rangle = \cos\theta|1\rangle - \sin\theta|2\rangle$$

$$|a_+\rangle = \sin\theta\sin\varphi|1\rangle + \cos\varphi|2\rangle + \cos\theta\sin\varphi|3\rangle$$

$$|a_-\rangle = \sin\theta\cos\varphi|1\rangle - \sin\varphi|2\rangle + \cos\theta\cos\varphi|3\rangle$$

$$\tan\theta = \frac{\Omega_p}{\Omega_c} \qquad \tan 2\varphi = \frac{\sqrt{\Omega_c^2 + \Omega_p^2}}{\Delta_p}$$

Typical EIT regime: $\Delta_p = 0$; $\Omega_p \ll \Omega_c \Rightarrow |a_0\rangle \simeq |1\rangle$

Optical response (modification of probe field)

1. Master equation $\frac{d\rho}{dt} = \frac{1}{i\hbar}[H, \rho] + \mathcal{L}\rho$

   $\mathcal{L}$: Lindblad superoperator for decay and dephasing

2. Perturbative regime: Ω_p small $\Rightarrow \dot{\rho}_{11} \simeq 0$ and $\rho_{11} \simeq 1$

3. Induced atomic polarization (1d uniform situation)

$$P_{12} = \frac{N_{at}}{V} \mu_{12} \rho_{12} = \varepsilon_0 \chi(\omega_p) E$$

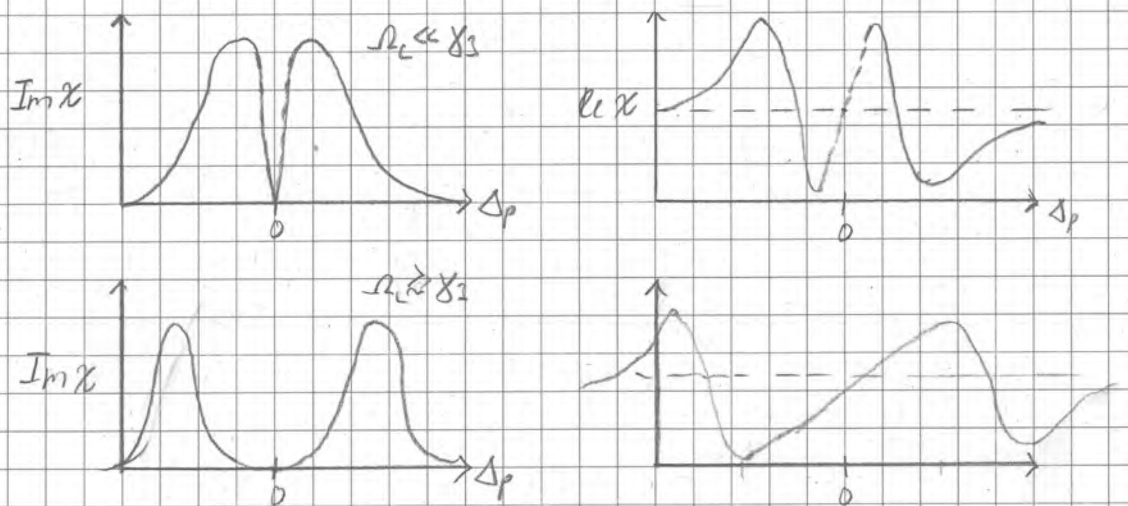
$$\Rightarrow \chi(\omega_p) = \frac{N_{at}}{V} \frac{|\mu_{12}|^2}{\varepsilon_0 \hbar} \times \left\{ \frac{4\delta(|\Omega_c|^2 - 4\delta\Delta_p) - 4\Delta_p \gamma_3^2}{\left| |\Omega_c|^2 + (\gamma_2 + 2i\Delta_p)(\gamma_3 + 2i\delta) \right|^2} + i \frac{8\delta^2 \gamma_2 + 2\gamma_3(|\Omega_c|^2 + \gamma_3\gamma_2)}{\left| |\Omega_c|^2 + (\gamma_2 + 2i\Delta_p)(\gamma_3 + 2i\delta) \right|^2} \right.$$

where γ_i: decoherence rate, δ: $\Delta_p - \Delta_c$

Remember: $\mathrm{Im}\,\chi \rightsquigarrow$ absorption (see later)

$\mathrm{Re}\,\chi \rightsquigarrow$ refractive index ($n = \sqrt{1 + \mathrm{Re}\,\chi}$)

Dependence for $\Delta_c = 0$ ($\gamma_2 = 0$)



Remarks:

1. Kramers-Kronig relation: $\mathrm{Re}\,\chi(\Delta_p) \sim \frac{d\,\mathrm{Im}\,\chi(\Delta_p)}{d\Delta_p}$

2. For $\gamma_2 = 0$: Steep variation of n without absorption

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3. Shape of $\mathrm{Im}\,\chi$:

a) $\Omega_c \ll \gamma_3$: Broad Lorentzian ($\sim\Gamma_3$) − Narrow Lorentzian ($\sim\Omega_c$)
$\Rightarrow$ EIT based on interference

b) $\Omega_c \gg \gamma_3$: Sum of two Lorentzians with width $\sim\Gamma_3$
$\Rightarrow$ EIT based on AC-Stark effect (Autler-Townes)

4. General case for $\delta = \Delta = 0$:

$$\mathrm{Im}\,\chi(0) = \frac{N_{at}}{V}\frac{|\mu_a|^2}{\varepsilon_0\hbar} \times \frac{2\gamma_3(|\Omega|_c^2 + \gamma_2\gamma_3)}{||\Omega_c|^2+\gamma_2\gamma_3|^2} \propto \frac{1}{1+\frac{|\Omega_c|^2}{\gamma_2\gamma_3}}$$

$\Rightarrow$ Negligible absorption only for $|\Omega_c|^2 \gg \gamma_2\gamma_3$.
Ground-state decoherence γ_3 "destroys" dark state!
$\Rightarrow$ Also less refractive-index variation!

Pulse propagation

If coupling field constant while probe pulse propagates through medium:

Frequency domain $E(\omega, z) = \underbrace{e^{ikz\chi(\omega)}}_{\text{transfer function}} E(\omega, 0)$

gives attenuation, delay, distortion ...

In particular: $I_{out}(\omega) \propto |E(\omega, L)|^2 = I_{in}(\omega)\, e^{-kL\,\mathrm{Im}\,\chi(\omega)}$

$\Rightarrow$ Optical depth $d = kL\,\mathrm{Im}\,\chi(\omega)$

Resonant optical depth for $\Omega_c = 0$:

$$\mathrm{Im}\,\chi(0) = \frac{N_{at}}{V}\frac{|\mu_{12}|^2}{\varepsilon_0\hbar}\frac{2\gamma_3^2\gamma_2}{\gamma_2^2\gamma_3^2} = \frac{N_{at}}{V}\frac{2|\mu_{12}|^2}{\varepsilon_0\hbar\gamma_2} \Rightarrow d_0 = \frac{N_{at}}{V}\frac{2|\mu_{12}|^2}{\varepsilon_0\hbar}\frac{kL}{\gamma_2}$$

Refractive index close to $\delta = \Delta_p = 0$

$$\mathrm{Re}\,\chi \simeq \frac{d_0}{2}\frac{\gamma_2}{kL}\times\frac{4\Delta_p(|\Omega_c|^2 - \gamma_3^2)}{\left||\Omega_c|^2 + \gamma_2\gamma_3\right|^2} = \frac{2d_0}{kL}\frac{\Delta_p}{\gamma_2}\frac{\frac{|\Omega_c|^2}{\gamma_3^2} - 1}{\left|1 + \frac{|\Omega_c|^2}{\gamma_2\gamma_3}\right|^2}$$

(sanity check: slope for $|\Omega_c|^2 = 0$ or large)

$$n = \sqrt{1 + \mathrm{Re}\,\chi} \simeq 1 + \frac{1}{2}\mathrm{Re}\,\chi$$

$$v_{gr} = \frac{c}{n_{gr}} \qquad n_{gr}(\omega) = n(\omega) + \omega\frac{dn(\omega)}{d\omega}$$

$$= 1 + ck\frac{d_0}{kL}\frac{1}{\gamma_2}\frac{\frac{|\Omega_c|^2}{\gamma_3^2} - 1}{\left|1 + \frac{|\Omega_c|^2}{\gamma_2\gamma_3}\right|^2}$$

Group delay:

$$\tau_d = L\left(\frac{1}{v_{gr}} - \frac{1}{c}\right) = L\left(\frac{n_{gr} - 1}{c}\right) = d_0\frac{1}{\gamma_2}\frac{\frac{|\Omega_c|^2}{\gamma_3^2} - 1}{\left|1 + \frac{|\Omega_c|^2}{\gamma_2\gamma_3}\right|^2}$$

Special case: $\gamma_3 \simeq 0 \Rightarrow \tau_d = d_0\left[\frac{\gamma_2}{|\Omega_c|^2} \underbrace{- \frac{\gamma_2^2}{|\Omega_c|^4}\gamma_3}_{\text{finite } \gamma_3 \text{ reduces } \tau_d!} + O(\gamma_3^2)\right]$

Fractional delay:

- Ratio of pulse delay to pulse duration
- Pulse should fit inside transparency window

Estimation of width of transparency window:

Transmission $T(\Delta_p) = e^{-\frac{d_0}{2}\gamma_2 \operatorname{Im}\chi(\Delta_p)}$

$$= \operatorname{Exp}\left[\frac{-d_0}{2}\gamma_2 \frac{8\Delta_p^2\gamma_2 + \overbrace{2\gamma_3(|\Omega_c|^2+\gamma_2\gamma_3)}^{a)}}{|\,|\Omega_c|^2 + (\gamma_2+2i\Delta_p)(\gamma_3+2i\Delta_p)|^2}\right]$$

a) constant factor

b) not relevant for width estimation

$$\sim \operatorname{Exp}\left[\frac{-\Delta^2}{2\sigma^2}\right] \text{ where } \sigma \sim \frac{1}{\sqrt{d_0}}\left(\gamma_3 + \frac{|\Omega|^2}{\gamma_2}\right)$$

order of magn.

Then $$\frac{\tau_d}{\tau_p} \sim \tau_d\,\sigma \simeq \sqrt{d_0}\left(\frac{-\gamma_2}{|\Omega_c|^2} - \frac{\gamma_2^2}{|\Omega_c|^4}\gamma_3\right)\left(\frac{|\Omega_c|^2}{\gamma_2} + \gamma_3\right)$$

$$= \sqrt{d_0}\left(1 - \frac{\gamma_2^2\gamma_3^2}{|\Omega_c|^4}\right)$$

Remarks:

- Achievable fractional delay determined by d_0 only!
- Ground state decoherence reduces fractional delay.
- This was for static case (Ω_c constant in time)

Solution:

- Control $\Omega_c(t)$ dynamically

$\Rightarrow$ Dark-state polaritons

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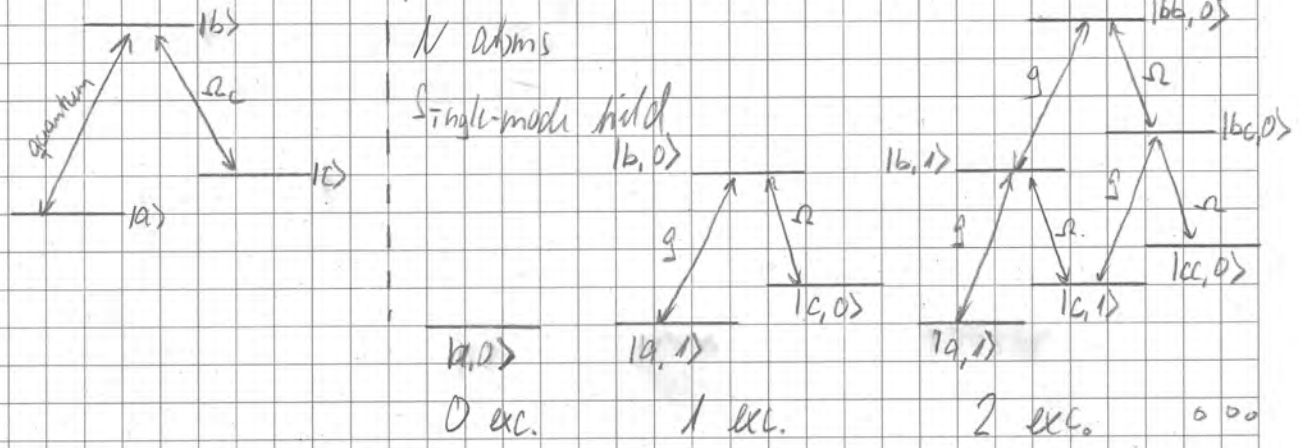
Dark-State Polaritons

Consider now a quantum field for the probe



Interaction Hamiltonian

$$H = \hbar g \sum_{i=1}^{N} \hat{a}\,\sigma_{ba}^{i} - \hbar\Omega(t)\, e^{-i\omega t} \sum_{i=1}^{N} \sigma_{bc}^{i} + H.c. \qquad \sigma_{ij} = |i\rangle\langle j|$$

State space:

$$|a\rangle = |a_1, a_2 \dots a_N\rangle$$

$$|b\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |a_1, a_2 \dots b_j \dots a_N\rangle \qquad |bb\rangle = \frac{1}{\sqrt{2N(N-1)}} \sum_{i \neq j} |a_1 \dots b_i \dots b_j \dots a_N\rangle$$

$$|c\rangle = \frac{1}{\sqrt{N}} \sum_{j=0} |a_1, a_2 \dots c_j \dots a_N\rangle \qquad |cc\rangle = \dots$$

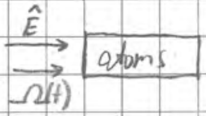
Dark states: $|D,1\rangle = \cos\theta\,|a,1\rangle - \sin\theta\,|c,0\rangle \quad \tan\theta = \frac{g\sqrt{N}}{\Omega(t)}$

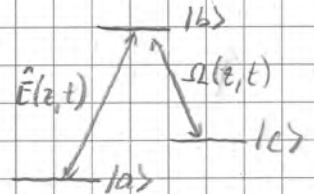
$\Rightarrow$ Adiabatic rotation $\theta = 0 \to \frac{\pi}{2} \quad \left(\Omega(t) \gg g\sqrt{N} \to \Omega(t) = 0\right)$

gives perfect conversion from photons to collective excitations.

- Works for n photons iff $n \leq N$
- Question: is the energy stored?

Pulse propagation (non-stationary field)





(assume no detunings)

$$\hat{H} = -\mu_{ab}\sum_j\left(\sigma^j_{ba}\hat{E}^{(+)}(z_j,t) + H.c.\right) - \hbar\sum_j\left(\sigma^j_{bc}\Omega(z_j,t)e^{i(k_c z_j - \omega_c t)} + H.c.\right)$$

Slowly varying variables

$$\hat{E}^{(+)}(z,t) = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\,\hat{\mathcal{E}}(z,t)e^{i\frac{\omega}{c}(z-ct)}$$

$$\hat{\sigma}^j_{\mu\nu}(t) = \tilde{\sigma}^j_{\mu\nu}(t)e^{-i\frac{\omega_{\mu\nu}}{c}(z-ct)}$$

Make slices such that
- slowly varying amplitudes do not change
- many atoms per slice

$\Rightarrow \tilde{\sigma}_{\mu\nu}(z,t) = \frac{1}{N_z}\sum_{z_j\in N_z}\tilde{\sigma}^j_{\mu\nu}(t)$ and exchange sum with integral

$$\Rightarrow \hat{H} = -\int\frac{dz}{L}\left[\hbar g N\tilde{\sigma}_{ba}(z,t)\hat{\mathcal{E}}(z,t) + \hbar\Omega(z,t)N\tilde{\sigma}_{bc}(z,t) + H.c.\right]$$

Time evolution of quantum field: (Heisenberg eqs. of motion)

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}}(z,t) = igN\tilde{\sigma}_{ab}(z,t) \qquad \text{(slowly varying ampl.)}$$

Atomic operators similar to Master-equation approach used to derive $\chi(\omega)$.

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Low intensity approximation:

- Rabi frequency of quantum field $\ll \Omega$
- Photon density $\ll$ atom density

$$\Rightarrow \tilde{\sigma}_{ab} = -\frac{i}{\Omega^*}\frac{\partial}{\partial t}\tilde{\sigma}_{ac} \quad \rightsquigarrow \text{collective ground-state spin}$$

$$\tilde{\sigma}_{bc} = -g\frac{\hat{\mathcal{E}}}{\Omega} + \text{terms containing } \frac{\partial}{\partial t}$$

Adiabatic limit: neglect time-derivative

$$\Rightarrow \left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}}(z,t) = -\frac{g^2 N}{\Omega^*(z,t)}\frac{\partial}{\partial t}\frac{\hat{\mathcal{E}}(z,t)}{\Omega(z,t)}$$

Tricky to solve in general, but easy for

- $\Omega(z,t) = \Omega(z) \rightsquigarrow$ spatially varying group velocity
- $\Omega(z,t) = \Omega(t) \rightsquigarrow$ temporally — " —

Temporal control of group velocity:

Introduce quantum fields

$$\hat{\Psi}(z,t) = \cos\theta(t)\,\hat{\mathcal{E}}(z,t) - \sin\theta(t)\sqrt{N}\,\tilde{\sigma}_{ac}(z,t) \quad \text{„dark"}$$

$$\hat{\Phi}(z,t) = \sin\theta(t)\,\hat{\mathcal{E}}(z,t) + \cos\theta(t)\sqrt{N}\,\tilde{\sigma}_{ac}(z,t) \quad \text{„bright"}$$

with $\tan^2\theta(t) = \frac{g^2 N}{|\Omega(t)|^2} = n_{gr}(t)$

$\rightsquigarrow$ quasiparticles with field and atomic spin contributions

$\rightsquigarrow$ "polaritons" are bosonic in low-excitation limit

$\rightsquigarrow$ admixture controlled through $\theta(t)$

With this

$$\left(\frac{\partial}{\partial t} + c\cos^2\theta \frac{\partial}{\partial z}\right)\hat{\Psi} = -\dot{\theta}\,\hat{\Phi} - \sin\theta\cos\theta\, c\,\frac{\partial}{\partial z}\hat{\Phi}$$

Adiabatic limit (again): $\hat{\Phi} = 0$

$$\Rightarrow \hat{\mathcal{E}}(z,t) = \cos\theta(t)\,\hat{\Psi}(z,t)$$
$$\sqrt{N}\,\hat{\sigma}_{ac} = -\sin\theta(t)\,\hat{\Psi}(z,t) \qquad v_g(t) = c\cos^2\theta(t)$$

i.e. $|\Omega(t)|^2 \gg g^2 N$ polariton are photonic and $v_g \simeq c$

$|\Omega(t)|^2 \ll g^2 N$ polaritons are atomic and $v_g \simeq 0$

Show examples:

- Fleischhauer & Lukin, PRA 65, 022314 (2002)
- Liu et al., Nature 409, 490 (2001)
- Phillips et al., PRL 86, 783 (2001)
- Phillips et al., PRA 78, 023801 (2008)
- Chen et al., PRL 110, 083601 (2013)