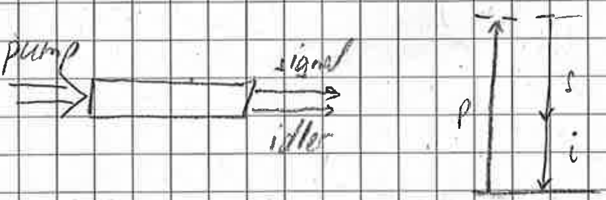
13.2 - Photon-Pair sources (continued)

PPS based on non-linear crystals:

Principle



Single-mode model: $H = \hbar\varkappa\left(a b^\dagger c^\dagger + a^\dagger b c\right)$

Non-depleted pump approximation: $a \to \alpha$ (c-number)

Output state (signal, idler): $|\psi_{out}\rangle = e^{-iH\tau/\hbar}|0,0\rangle$, τ: interaction time $= \frac{L}{v_p}$

2nd-order approximation: $\hbar^2\varkappa^2(\alpha b^\dagger c^\dagger + \alpha^* bc)(\alpha b^\dagger c^\dagger + \alpha^* bc)$

$\alpha^2 b^{\dagger 2} c^{\dagger 2} + |\alpha|^2(b^\dagger c^\dagger bc + bcb^\dagger c^\dagger) + \alpha^{*2} b^2 c^2$

$$e^{-iH\tau/\hbar} \simeq 1 - iH\tau/\hbar - H^2\tau^2/2\hbar^2$$

$$\Rightarrow |\psi_{out}\rangle \simeq |0,0\rangle - i\alpha\varkappa\tau|1,1\rangle - \frac{1}{2}\varkappa^2\tau^2\left(2\alpha^2|2,2\rangle + |\alpha|^2|0,0\rangle\right)$$

$$= \left(1 - \frac{1}{2}|\alpha|^2\varkappa^2\tau^2\right)|0,0\rangle - i\alpha\varkappa\tau|1,1\rangle - (\alpha\varkappa\tau)^2|2,2\rangle$$

Entangled

Let $p = -(\alpha\varkappa\tau)^2$ and assume $\alpha\varkappa\tau < 0$

$$\Rightarrow |\psi_{out}\rangle = \left(1 - \frac{1}{2}p\right)|0,0\rangle + \sqrt{p}|1,1\rangle + p|2,2\rangle$$

Normalisation: $\langle\psi_{out}|\psi_{out}\rangle = \left(1 - \frac{1}{2}p^2\right)^2 + p^2 + p^4$

$= 1 + \frac{5}{4}p^2 \simeq 1$

02.06.2016

Correlation functions:

$\langle b^+ b \rangle = p + 2p^2$ $\langle b^+ b^+ b b \rangle = 2p^2$ (same for $\langle c^+ c \rangle \dots$)

$\Rightarrow g^{(2)}_{b,b} = \frac{2p^2}{(p+2p^2)^2} \simeq \underline{2}$ like thermal state

$\langle b^+ c^+ c b \rangle = p + 4p^2$ $\Rightarrow g^{(2)}_{b,c} = \frac{p+4p^2}{(p+2p^2)^2} = \frac{p+4p^2}{p^2+4p^3+4p^4} \simeq \frac{1}{p}$

strong for $p \ll 1$

Note: $p \ll 1 \Rightarrow$ mostly vacuum. Single pair with prob. p.

Exact solution: Two-mode squeezed state

$$|\psi_{out}\rangle = \frac{1}{\cosh(\alpha \chi \tau)} \sum_{n=0}^{\infty} \tanh^n(\alpha \chi \tau) |n,n\rangle$$

$$= \sqrt{1-p} \sum_{n=0}^{\infty} p^{n/2} |n,n\rangle \quad \text{with } p = \tanh^2(\alpha \chi \tau)$$

Signal and idler separately are in thermal state

$$\rho_a = Tr_b(|\psi_{out}\rangle\langle\psi_{out}|) = \sum_{n=0}^{\infty} \frac{\mu^n}{(1+\mu)^{n+1}} |n\rangle\langle n|$$

where $\mu = \langle n \rangle = \frac{p}{1-p}$ $\left(= \left[e^{\hbar\omega/k_B T} - 1\right]^{-1}\right)$

$\Rightarrow g^{(2)}_{a,a} = g^{(2)}_{b,b} = 2$ $g^{(2)}_{a,b} = 1 + \frac{1}{p}$

Non-classical according to Cauchy-Schwartz !

$$g^{(2)}_{a,b} \leq \sqrt{g^{(2)}_{a,a}\, g^{(2)}_{b,b}}$$

Source of heralded single photons:

Conditional 3rd-order correlation $\sim \dfrac{P_{aa|b}}{P_{a|b}P_{a|b}} = \dfrac{P(a\,\&\,a\,\&\,b)}{P_b} \cdot \left(\dfrac{P_b}{P(a\,\&\,b)}\right)^2$

$$\Rightarrow g^{(2)}_{aa|b} = \frac{\langle a_a^\dagger a_a^\dagger a_b^\dagger a_b a_a a_a \rangle \langle a_b^\dagger a_b \rangle}{\langle a_a^\dagger a_b^\dagger a_b a_a \rangle^2} = 2p \frac{2+p}{(1+p)^2} \simeq 4p$$

Spectral properties of collinear SPDC

Consider now a realistic situation

Classical wave equation:

$$\vec{\nabla}^2 \vec{E}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}(\vec{r},t) = \mu_0 \frac{\partial^2}{\partial t^2}\vec{P}(\vec{r},t)$$

Taylor expansion of $\vec{P} = \varepsilon_0 \chi \vec{E} + \chi^{(2)} \vec{E}\vec{E} + \chi^{(3)} \vec{E}\vec{E}\vec{E} + \dots$

non-linear susceptibility tensors

Example: 3-wave mixing (classical)

Input fields: $E = A\{\cos(\omega_1 t) + \cos(\omega_2 t)\}$

Non-linear polarisation to lowest order

$$P_2 = \chi^{(2)} A^2 \Big\{ \tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega_1 t) + \tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega_2 t) + \underbrace{\cos[(\omega_1-\omega_2)t]}_{\text{DFG}} + \underbrace{\cos[(\omega_1+\omega_2)t]}_{\text{SFG}} \Big\}$$

Freq. Doubling

Note:

- Only processes with momentum conservation (phase matching) will be relevant
- $\chi^{(2)}$ material dependent. In particular, $\chi^{(2)} = 0$ for centrosymmetric materials (e.g. glass)
- $\chi^{(2)} \sim 10^{-21} \frac{C}{V^2}$ for good non-linear crystals.

Hamiltonian:

$$H(t) = \int_0^L dz\, \chi^{(2)} E_p^{(+)}(z,t)\, E_s^{(-)}(z,t)\, E_i^{(-)}(z,t)$$

with $E_j^{(+)}(z,t) = \int d\omega_j\, A(\omega_j)\, a_j(\omega_j)\, e^{i(k_j z - \omega_j t)}$

and classical pump.

Consider pulsed pump, Fourier limited, such that pump field is essentially 0 outside interval $[0,T]$, such that we can approximate by extending limits to infinity:

$$\underbrace{\int_0^T H(t)\,dt}_{\text{First order interaction}} \propto \int_{-\infty}^{\infty} dt \int_0^L dz \int d\omega_p \int d\omega_s \int d\omega_i$$

$$\times\, \alpha(\omega_p)\, a_s^\dagger(\omega_s)\, a_i^\dagger(\omega_i)\, \underbrace{e^{i(\omega_s + \omega_i - \omega_p)t}}_{\substack{\text{Energy conservation} \\ \delta(\omega_s+\omega_i-\omega_p)}}$$

$$\times\, \underbrace{e^{-i(k_s + k_i - k_p)z}}_{\text{phase-matching}}$$

$$+\, H.c.$$

(not a delta function!)

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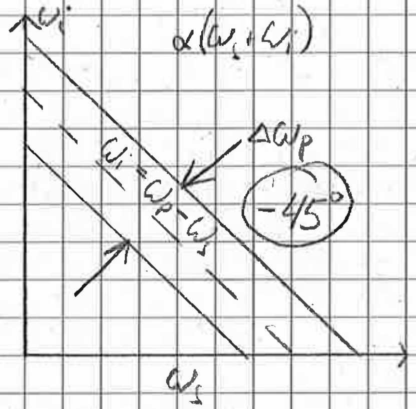
$$\Rightarrow \int_0^T H(t)\,dt \propto \int d\omega_s \int d\omega_i \; \alpha(\omega_s+\omega_i)\,\Phi(\omega_s,\omega_i)\,a_s^\dagger(\omega_s)\,a_i^\dagger(\omega_i) + h.c.$$

where $\alpha(\omega_s+\omega_i)$: pump spectrum

$\Phi(\omega_s,\omega_i) = \mathrm{sinc}\{(k_s+k_i-k_p)L\}$ phase-matching function

Product: "Joint spectral amplitude"

Example:



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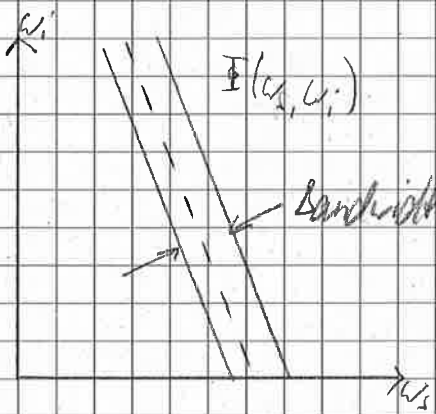

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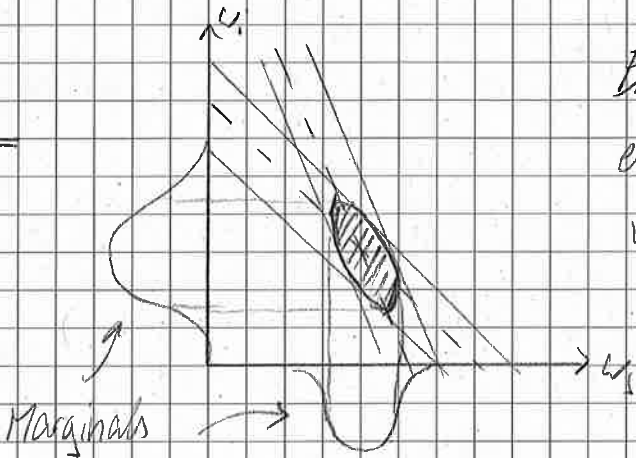

Entangled if ellipsis at angle with axis!

What about CW pump?

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Phase-matching

Due to dispersion, $k_s + k_i \neq k_p$ in general

Tuning:

- Polarization: Type 2: $S \perp I$, Type 1: $S \parallel I$ ($O \parallel P$)
- Temperature: Also T-coefficient of refractive index varies with wavelength!
- Angle
- Periodic poling: Reverse sign of $\chi^{(2)}$ at period Λ

$$\Rightarrow \Delta k = k_s + k_i + \frac{2\pi}{\Lambda} - k_p$$

Sources of polarization-entangled photons (slides)

- Overlapping cones
- Cascaded crystals
- Post-selection
- Mach-Zehnder interferometers
- Sagnac interferometer