Freitag, 6. Mai 2016 13:46

Lecture 11/5/2016

$$G^{(1)}(x_i, x_j) = Tr\left(\hat{S}\, \hat{E}^{(-)}(x_i)\, \hat{E}^{(+)}(x_j)\right) \qquad i,j = 1,2 \qquad (12.38)$$

1st order correlation function

$G^{(1)}(x,x)$ ... special case, corresponds to Intensity arriving from x

$\Rightarrow$ First-order quantum coherence function:

$$g^{(1)}(x_1, x_2) = \frac{G^{(1)}(x_1,x_2)}{\sqrt{G^{(1)}(x_1,x_1)\, G^{(1)}(x_2,x_2)}} \qquad (12.39)$$

Properties:

- $0 \leq |g^{(1)}(x_1,x_2)| \leq 1 \qquad (12.40)$
- 3 possible cases (similar to classical):

  $|g^{(1)}(x_1,x_2)| = 1 \Rightarrow$ fully coherent

  $0 < |g^{(1)}(x_1,x_2)| < 1 \Rightarrow$ partially coherent

  $|g^{(1)}(x_1,x_2)| = 0 \Rightarrow$ fully incoherent
- maximal visibility of interference signal, if $|g^{(1)}(x_1,x_2)| = 1$

Example: single-mode field (plane wave with wave-vector $\vec{k}$)

$$\Rightarrow \hat{E}^{(+)}(x) = i\, \mathcal{E}\, \hat{a}\, e^{i(\vec{k}\vec{r} - \omega t)} \qquad (12.41)$$

with $\mathcal{E} = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}$

case (a): field in Fock-state $|n\rangle$

$$\Rightarrow G^{(1)}(x_1, x_2) = \langle n | \hat{E}^{(-)}(x_1)\, \hat{E}^{(+)}(x_2) | n \rangle \qquad (12.42)$$

$$= \mathcal{E}^2 n\, e^{i(\vec{k}(\vec{r}_2 - \vec{r}_1) - \omega(t_2 - t_1))}$$

$$\Rightarrow |g^{(1)}(x_1, x_2)| = 1 \qquad (12.43)$$

case (b): field in coherent-state $|\alpha\rangle$

$$\Rightarrow G^{(1)}(x_1, x_2) = \langle \alpha | \hat{E}^{(-)}(x_1)\, \hat{E}^{(+)}(x_2) | \alpha \rangle \qquad (12.44)$$

$$= \mathcal{E}^2 |\alpha|^2 e^{i(\vec{k}(\vec{r}_2 - \vec{r}_1) - \omega(t_2 - t_1))}$$

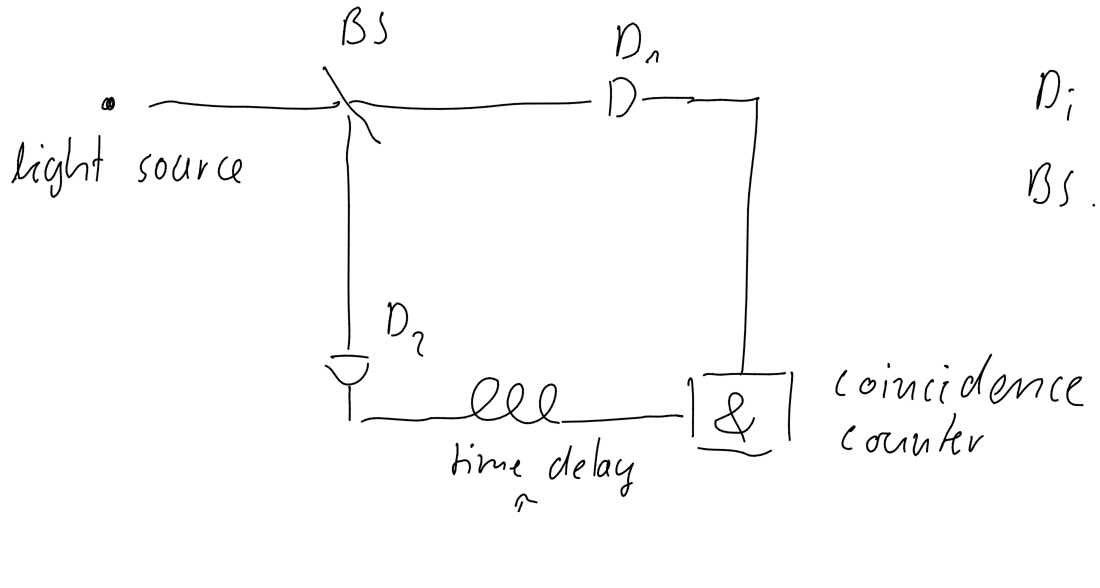
$$\Rightarrow |g^{(1)}(x_1, x_2)| = 1 \qquad (12.45)$$

From (12.42) - (12.45):

first order coherence contains information about coherence length of light but not about statistical properties (Fock state as well as coherent state are fully coherent in first order)

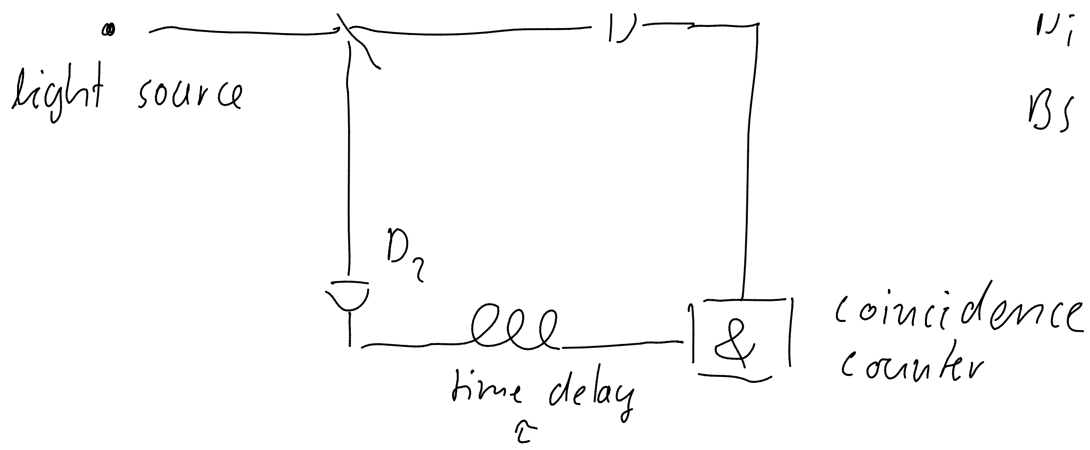
12.3 Higher order coherence functions

Typical setup: Hanbury-Brown Twiss - Experiment:



D_i ... detector

BS ... 50/50 beam splitter



D_i ... detector

BS ... 50/50 beam splitter

Note:

- correlation measurement of intensities, not fields
- setup measures delayed coinc. rate, i.e. event in D_1 at $t = t+\tau$ and in D_2 at t
- for $\tau < \tau_0$ (coherence time) measurement contains info about statistics of the light

Normalized probability to obtain a coincidence count:

$$\gamma^{(2)}(x_1, x_2) = \frac{\langle I(x_1)\, I(x_2)\rangle}{\langle I(x_1)\rangle \langle I(x_2)\rangle}$$ (2nd order coherence function)

$$= \frac{\langle E^*(x_1)\, E^*(x_2)\, E(x_2)\, E(x_1)\rangle}{\langle |E(x_1)|^2\rangle \langle |E(x_2)|^2\rangle} \qquad (12.46)$$

with $I(x_i)$ ... intensity at the detectors and
$\langle \ldots \rangle$ ... time average

Note: - a field is (classicaly) coherent up to 2nd order, when $|\gamma^{(1)}(x_1, x_2)| = 1$ and $|\gamma^{(2)}(x_1, x_2)| = 1$ (12.47)

- for $\vec{r}_1 = \vec{r}_2$:

$$\gamma^{(2)}(x_1, x_2) = \gamma^{(2)}(\tau) = \frac{\langle I(t)\, I(t+\tau)\rangle}{\langle I(t)\rangle^2} \qquad (12.48)$$

Properties:

- $\gamma^{(2)}$ is not limited (in contrast to $\gamma^{(1)}$):

- $g^{(2)}$ is not limited (in contrast to $g^{(1)}$):

$$g^{(2)}(\tau=0) = \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2} \qquad (12.49)$$

Cauchy's inequality: $\langle I(t)^2 \rangle \geq \langle I(t) \rangle^2$ (12.50)

$$\Rightarrow 1 \leq g^{(2)}(0) < \infty \qquad (12.50)$$

↑ no upper limit!

for $\tau \neq 0$ one can show:

$$0 \leq g^{(2)}_{(\tau)} < \infty \qquad (12.51)$$

- Maximum at $\tau = 0$:

Cauchy-Schwarz inequ: $\langle I(t)\, I(t+\tau) \rangle \leq \langle I(t)^2 \rangle$ (12.52)

$$\Rightarrow \boxed{g^{(2)}(\tau) \leq g^{(2)}(0)} \qquad (12.53)$$

- (12.50) and (12.53) limit $g^{(2)}$ for classical (!) fields

Examples:

- Plane wave in z-direction (12.20)

$$\Rightarrow \langle E^*(t)\, E^*(t+\tau)\, E(t+\tau)\, E(t) \rangle = E_0^4 \qquad (12.54)$$

$$\Rightarrow g^{(2)}(\tau) = 1$$

- in general: for every field with konstant (non-fluctuating) intensity

$$I(t) = I(t+\tau) = I_0 \qquad (12.55)$$

$$g^{(2)}(\tau) = 1$$

$$I(t) = I(t+\tau) = I_0 \qquad (12.55)$$

$$g^{(2)}(\tau) = 1$$

- Now light-source that consists of large number of independent, collisionally broadened atoms:

One can show:

$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2 \qquad (12.56)$$

From $0 \le |g^{(1)}(\tau)| \le 1$:

$$1 \le g^{(2)}(\tau) \le 2 \qquad (12.57)$$

For light with Lorentz-type spectrum one gets

$$|g^{(1)}(\tau)| = e^{-\frac{|\tau|}{\tau_0}} \qquad (12.58)$$

$$\Rightarrow g^{(2)}(\tau) = 1 + e^{-2\frac{|\tau|}{\tau_0}} \qquad (12.59)$$

<u>Note:</u>

- For $\tau \to \infty$: $g^{(2)}(\tau) = 1$
- For $\tau \to \infty$: $g^{(2)}(0) = 2$
- if light measured at detector D_1 would be independent of D_2 one would expect $g^{(2)}(\tau) = 1$.

  But one expects (12.59) where, e.g., $g^{(2)}(0) = 2g^{(2)}(\infty)$

  $\Rightarrow$ Photons arrive pairwise for $\tau = 0$ and independent for $\tau \to \infty$.

  $\Rightarrow$ case $\tau = 0$ is called "photon bunching"

- from (12.59): measurement of $g^{(2)}(\tau)$ allows one to determine ...

measurement of $\gamma^{(1)}(\tau)$ allows one to determine coherence time τ_0 of the light field.

Quantummechanical description:

- Transition probability for absorption of 2 photons (see 12.32)

$$|\langle f|\hat{E}^{(+)}(x_2)\,\hat{E}^{(+)}(x_1)\,|i\rangle|^2 \qquad (12.60)$$

- Summing over all final states

$$\langle i|\,\hat{E}^{(-)}(x_1)\,\hat{E}^{(-)}(x_2)\,\hat{E}^{(+)}(x_2)\,\hat{E}^{(-)}(x_1)\,|i\rangle \qquad (12.61)$$

- generalizing to mixed states gives second order quantum correlation function

$$G^{(2)}(x_1,x_2) = \mathrm{Tr}\left[\hat{\rho}\,\hat{E}^{(-)}(x_1)\,\hat{E}^{(-)}(x_2)\,\hat{E}^{(+)}(x_2)\,\hat{E}^{(+)}(x_1)\right] \qquad (12.62)$$

$\Rightarrow$ second-order quantum coherence function:

$$g^{(2)}(x_1,x_2) = \frac{G^{(2)}(x_1,x_2)}{G^{(1)}(x_1,x_1)\,G^{(1)}(x_2,x_2)} \qquad (12.63)$$

Note:

- order of operators $\hat{E}^{(-)}, \hat{E}^{(+)}$ is important (normal order)
- $g^{(2)}(x_1,x_2)$ is proportional to the probability to detect a photon at $\vec{r}_1$ at time t_1 and a second photon at $\vec{r}_2$ and time t_2.
- one defines a quantum field as coherent

- one defines a quantum field as coherent up to 2nd order if $|g^{(1)}(x_1, x_2)| = 1$ and $g^{(2)}(x_1, x_2) = 1$.

- for a fixed position $g^{(2)}$ only depends on $\tau = t_1 - t_2$:

$$g^{(2)}(\tau) = \frac{\langle E^{(-)}(t)\, E^{(-)}(t+\tau)\, E^{(+)}(t+\tau)\, E^{(+)}(t) \rangle}{\langle E^{(-)}(t)\, E^{(+)}(t) \rangle \langle E^{(-)}(t+\tau)\, E^{(+)}(t+\tau) \rangle} \quad (12.64)$$

- for a single mode field (12.41) we obtain

$$g^{(2)}(\tau) = \frac{\langle a^+ a^+ a\, a \rangle}{\langle a^+ a \rangle^2} = \frac{\langle \hat{n}(\hat{n}-1) \rangle}{\langle \hat{n} \rangle^2}$$

$$= 1 + \frac{\Delta n^2 - \langle \hat{n} \rangle^2}{\langle \hat{n} \rangle^2} \quad (12.65)$$

which is independent of τ

here $\Delta n = \sqrt{\langle n^2 \rangle - \langle n^2 \rangle}$ is the photon number fluctuation

Examples:

- coherent state $|\alpha\rangle$:

$$g^{(2)}(\tau) = 1 \quad (12.66)$$

- multimode thermal state

$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2 \quad (12.67)$$

$\Rightarrow$ $1 \leq g^{(2)}(\tau) \leq 2$ (as in classical case)

For Lorentzian spectrum:

$$g^{(1)}(\tau) = e^{-i\omega_0\tau - |\tau|/\tau_0} \qquad (12.68)$$

$$\Rightarrow g^{(2)}(\tau) = 1 + e^{-2\frac{|\tau|}{\tau_0}} \qquad (12.69)$$

$\Rightarrow g^{(2)}(0) = 2$ (as in classical case)

$\Rightarrow$ photon bunching

<u>However</u>:

- Now it is also possible to obtain $g^{(2)}(\tau) < 1$ (impossible for classical case)
- from (12.65): $g^{(2)}(\tau) < 1$ if

$$\Delta n^2 < \langle \hat{n} \rangle \qquad (12.70)$$

$\Rightarrow$ this is the case for Fock states ($\Delta n^2 = 0$)

$$\Rightarrow g^{(2)}(\tau) = g^{(2)}(0) = \begin{cases} 0 & \text{for } n = 0, 1 \\ 1 - \frac{1}{n} & \text{for } n \geq 2 \end{cases}$$

- Interpretation: Quantum mechanical violation of Cauchy inequality (negative probabilities)

- the (classical impossible) case $g^{(2)}(0) < g^{(2)}(\tau)$ is called "photon anti-bunching"
  $\Rightarrow$ photons arrive with "fixed" time delay at the detectors

$\Rightarrow$ Slides . . . .