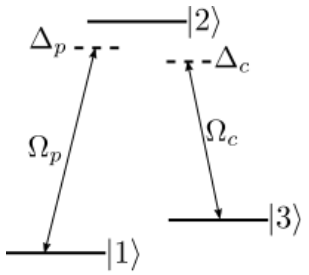
Cheat sheet 1: Basics of EIT – Static case



$$H = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_p & 0 \\ \Omega_p & -2\Delta_p & \Omega_c \\ 0 & \Omega_c & -2(\Delta_p - \Delta_c) \end{pmatrix}$$

Eigenstates and –energies

$$|a_0\rangle = \cos\theta\,|1\rangle - \sin\theta\,|3\rangle \qquad \hbar\omega_0 = 0$$

$$|a_+\rangle = \sin\theta\sin\phi\,|1\rangle + \cos\phi\,|2\rangle + \cos\theta\sin\phi\,|3\rangle \qquad \hbar\omega_\pm = \frac{\hbar}{2}\left(\Delta_p \pm \sqrt{\Delta_p^2 + \Omega_p^2 + \Omega_c^2}\right)$$

$$|a_-\rangle = \sin\theta\cos\phi\,|1\rangle - \sin\phi\,|2\rangle + \cos\theta\cos\phi\,|3\rangle \qquad \tan\theta = \frac{\Omega_p}{\Omega_c} \qquad \tan 2\phi = \frac{\sqrt{\Omega_c^2+\Omega_p^2}}{\Delta_p}$$

Parameters for EIT regime: $\Delta_p = 0, \Delta_p - \Delta_c = 0, \Omega_p \ll \Omega_c \Rightarrow |a_0\rangle \simeq |1\rangle$.

Optical response for weak probe field

1. Master equation: $\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \mathcal{L}\rho$
   ($\mathcal{L}$: Lindblad superoperator for decay/dephasing)
2. Consider perturbative regime: Ω_p small $\Rightarrow \frac{d\rho_{11}}{dt} \simeq 0$ and $\rho_{11} \simeq 1$.

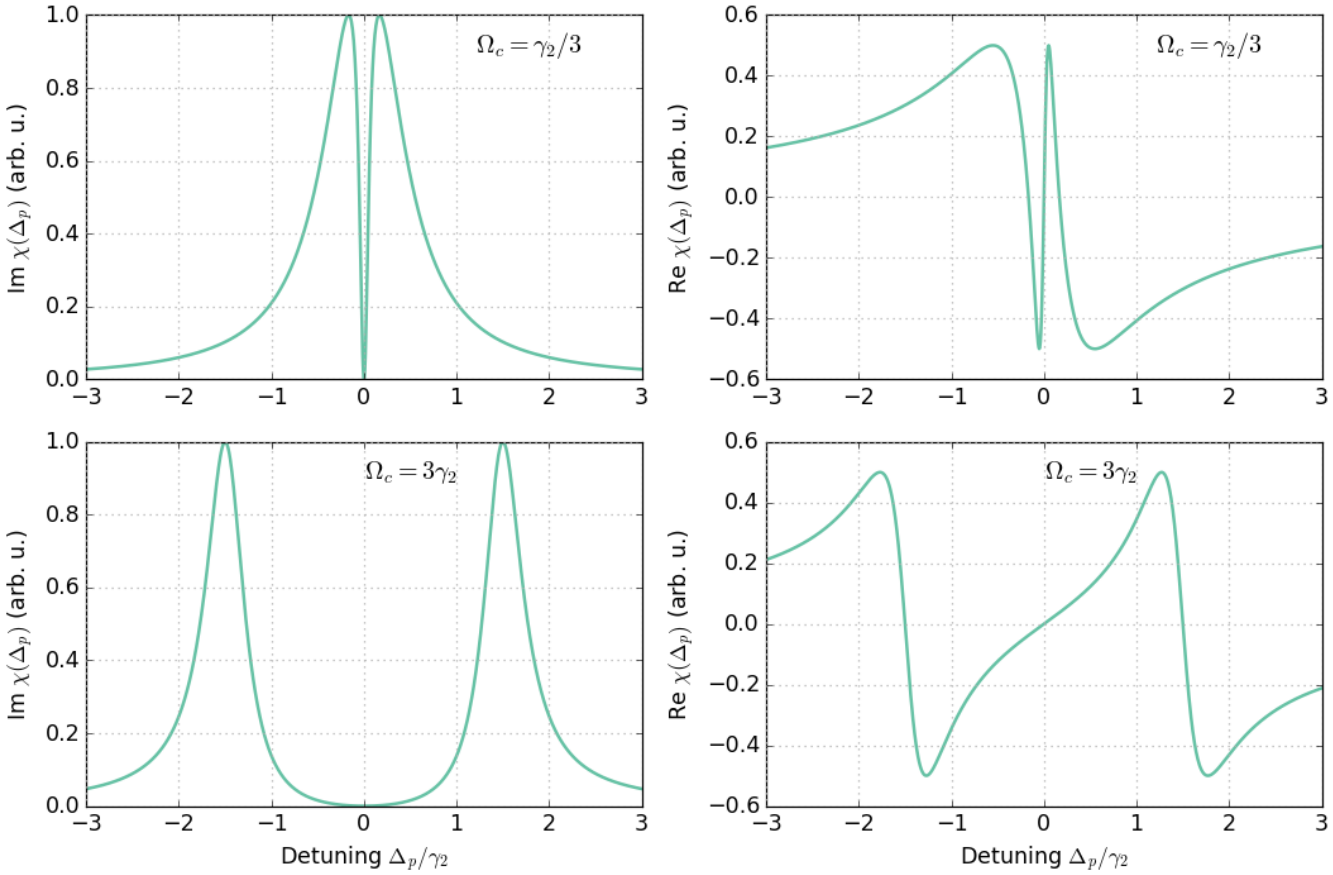
Induced atomic polarization: $P_{12} = \frac{N}{V}\mu_{12}\rho_{12} = \epsilon_0\chi(\omega_p)$

$$\Rightarrow \chi(\omega_p) = \frac{N}{V}\frac{|\mu_{12}|^2}{\epsilon_0\hbar}\left\{\frac{4\delta(|\Omega_c|^2 - 4\delta\Delta) - 4\Delta_p\gamma_3^2}{||\Omega_c|^2 + (\gamma_2 + 2i\Delta)(\gamma_3 + 2i\delta)|^2} + i\frac{8\delta^2\gamma_2 + 2\gamma_3(|\Omega_c|^2 + \gamma_2\gamma_3)}{||\Omega_c|^2 + (\gamma_2 + 2i\Delta)(\gamma_3 + 2i\delta)|^2}\right\}$$

where γ_j: decoherence rate of $|j\rangle$, $\delta = \Delta_p - \Delta_c$

Remember: $Im\,\chi \rightarrow$ absorption, $\quad Re\,\chi \rightarrow$ refractive index

Example plots for $\Delta_c = \gamma_3 = 0$



Further reading (optional)

1. Fleischhauer, M., Imamoglu, A. & Marangos, J. Electromagnetically induced transparency: Optics in coherent media. *Rev. Mod. Phys.* **77,** 633–673 (2005).