Lecture 16/06/2016

14. Optical tests of Quantum mechanics - Bell's inequality

- quantum mechanics: statistical theory
  $\Rightarrow$ probabil. statements about ensembles of identically prepared quant. systems

- behavior of single qu. system seems to contradict "common sense"

- Schrödinger (1952): "... we never experiment with just one electron or atom or molecule. In thought experiments we sometimes assume we do; this invariably entails ridiculous consequences."

- Modern Quantum optics
  $\Rightarrow$ realize such "thought experiments" and test "rid. consequences"

14.1. Entangled states + EPR Argument

Entangled state of 2 spin-$\frac{1}{2}$ particles

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2\right) \qquad (14.1)$$

... $|\uparrow_z\rangle_i$ ($|\downarrow_z\rangle_i$) ... spin-up (down) state of particle i

Properties:

- $|\psi^-\rangle$ has total spin $= 0$
- probability to find each particle in $\uparrow_z$ or $\downarrow_z$ is 50%

- perfect anticorrelation: when particle 1 is measured to be in $\uparrow_z$, the probability to obtain the result $\downarrow_z$ for particle 2 is one and vice versa

$$P(\uparrow_z^1 | \downarrow_z^2) = P(\downarrow_z^1 | \uparrow_z^2) = 1$$

- symmetric vs rotations

$$|\psi^-\rangle = (|\uparrow_e\rangle|\downarrow_e\rangle - |\downarrow_e\rangle|\uparrow_e\rangle)\frac{1}{\sqrt{2}} \quad \text{for any vector } e \quad (14.2)$$

- correlations predicted for such entangled states lead Einstein, Podolsky and Rosen to conclude: QM is not a complete description of reality

EPR argument

- QM: measurement of spin of particle 1

  $\Rightarrow$ we can predict with certainty outcome of

$\Rightarrow$ we can predict with certainty outcome of measurement of spin of particle 2 (along the same direction)

- particles can have arbitrarily large distance
  $\Rightarrow$ they can not interact
  $\Rightarrow$ measurement of 1 can not influence the state of 2
  (locality assumption)

- The results of measurement of $\hat{\sigma}_x$, $\hat{\sigma}_y$, $\hat{\sigma}_z$ are "elements of reality" because they can be predicted with certainty without influencing particle 2
  (reality assumption)

$\Rightarrow$ QM is incomplete:

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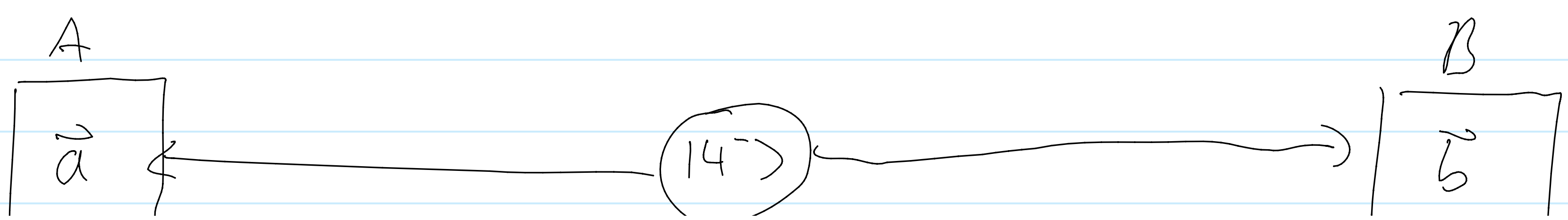
- $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$ do not commute
  $\Rightarrow$ there is no quantum state with well defined values for all three operators
  $\Rightarrow$ there are „elements of reality" that are not described by QM

Note: - EPR only assumed that QM is incomplete, <u>not</u> that it is wrong

- complete theory would contain all values of $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$ (set by so-called hidden variable)

14.2 Bell theorem, CHSH inequality



$$\boxed{\vec a} \longleftarrow |\psi^-\rangle \longrightarrow \boxed{\vec b}$$

Spin measurement of particle 1 along $\vec a$ gives result $A(\vec a) = +1$ for result $\uparrow_{\vec a}$ and $A(\vec a) = -1$ for result $\downarrow_{\vec a}$

(similar for particle 2 measured along $\vec b$)

$$\Rightarrow A(\vec a)\, B(\vec b) = \begin{cases} +1 & \text{for } \uparrow_a \uparrow_b \text{ or } \downarrow_a \downarrow_b \\ -1 & \text{for } \uparrow_a \downarrow_b \text{ or } \downarrow_a \uparrow_b \end{cases} \qquad (11.3)$$

$\Rightarrow$ expectation value

$$C(\vec a, \vec b) = P(\uparrow_a \uparrow_b) + P(\downarrow_a \downarrow_b) - P(\uparrow_a \downarrow_b) - P(\downarrow_a \uparrow_b) \qquad (14.4)$$

<u>Local hidden variable theories</u> (LHVT)

$A(\vec a)$, $B(\vec b)$ depend on set of hidden variables λ

$$\Rightarrow A(\vec a, \lambda) = \pm 1 \qquad B(\vec b, \lambda) = \pm 1 \qquad (14.5)$$

$$\Rightarrow \quad A(\vec{a},\lambda) = \pm 1 \qquad B(b,\lambda) = \pm 1 \qquad (14.5)$$

important: $A(\vec{a}, \cancel{\vec{b}}, \lambda)$, $B(\vec{b}, \cancel{\vec{a}}, \lambda)$ $\Leftrightarrow$ locality

$$\Rightarrow \quad C_{hv}(\vec{a},\vec{b}) = \int A(\vec{a},\lambda)\cdot B(\vec{b},\lambda)\, \rho(\lambda)\, d\lambda \qquad (14.6)$$

with $\rho(\lambda)$ ... probability density of λ

$$\int \rho(\lambda)\, d\lambda = 1 \qquad (14.7)$$

Quantum mechanics:

$$C_{qm}(\vec{a},\vec{b}) = \langle \psi | (\vec{\sigma}\cdot\vec{b})^{(2)} (\vec{\sigma}\cdot\vec{a})^{(1)} | \psi \rangle \qquad (14.8)$$

For $|\psi\rangle = |\psi^-\rangle$: $\quad C_{qm}(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b} \qquad (14.9)$

CHSH inequality (Clauser, Horne, Shimony, Holt)

consider the expression

$$S = C(a,b) + C(\vec{a}', b) + C(\vec{a}, \vec{b}') - C(\vec{a}', \vec{b}') \qquad (14.10)$$

with $A = A(\vec{a}, \lambda)$, $A' = A(\vec{a}', \lambda)$, ...

$$\Rightarrow \quad S_{hv} = \int d\lambda \, \rho(\lambda) \left[AB + A'B + AB' - A'B' \right]$$

$$= \int d\lambda \, \rho(\lambda) \underbrace{\left[A(B+B') + A'(B-B') \right]}_{= \pm 2} \qquad (14.11)$$

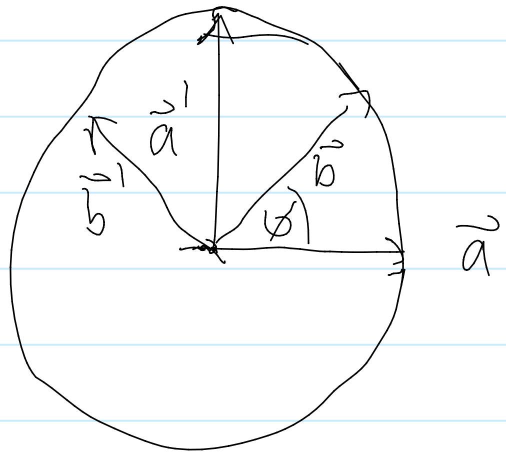
$$\Rightarrow \quad \boxed{-2 \leq S_{hv} \leq +2} \qquad (14.12)$$

CHSH – type Bell's inequality

From eqn (14.9):

$$S_{qm} = -\vec{a}\vec{b} - \vec{a}'\vec{b} - \vec{a}\vec{b}' + \vec{a}'\vec{b}' \qquad (14.13)$$

For $\theta = 0$, $\theta' = \frac{\pi}{4}$, $\phi = \frac{\pi}{8}$, $\phi' = -\frac{\pi}{8}$



$\Rightarrow$ maximum value

$$\boxed{S_{qm} = -2\sqrt{2}} \qquad (14.14)$$

$\Rightarrow$ $|S_{qm}| > |S_{hv}|$!!

$\Rightarrow$ Quantum mechanics locality or reality assumption (or both)