

Lecture 23/06/2016

16 Quantum communication16.1. Quantum key distribution16.1.1 Cryptography

Alice wants to send secret message to Bob:

=> modern cryptography

publicly known algorithm for encoding ( $\hat{E}$ )  
and decoding ( $\hat{D}$ ) + shared (secret) key ( $k$ )

$$\hat{E}_k(M) = C \rightarrow \hat{D}_k(C) = M$$

$M$  ... plaintext message

$C$  ... encrypted message

- Examples (see slides, Caesar + one time pad)

One time pad:

- Message and key have same length
- add key to message
- Claude - Shannon (1949): is absolutely secure,  
if key is fully random and only used  
once

=> Communication is secure if there is a secure  
way to exchange / generate a secret key

Solution:

- public-key-method: security not proven

- public-key-method: security not proven
- Quantum-key distribution

### 16.1.2 Quantum-key-distribution

No-cloning theorem:

it is impossible to perfectly copy an unknown quantum state

Proof: We assume there is an operation  $\hat{U}$  which copies an arbitrary quantum state:

$$\hat{U} |\phi\rangle |0\rangle \rightarrow |\phi\rangle |\phi\rangle$$

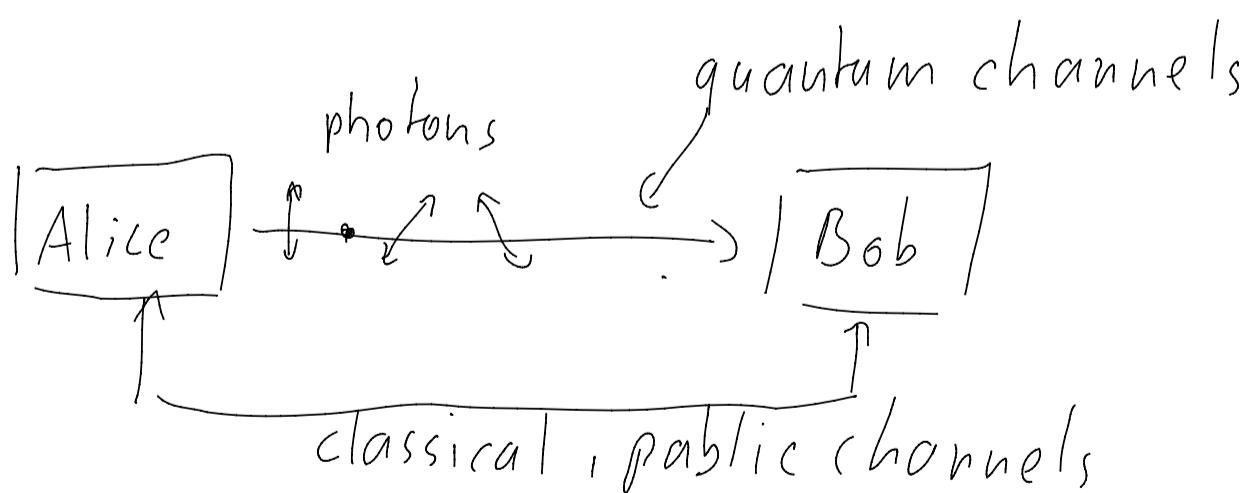
$$\hat{U} |4\rangle |0\rangle \rightarrow |4\rangle |4\rangle$$

$$\Rightarrow \hat{U} [|\phi\rangle + |4\rangle] |0\rangle \rightarrow |\phi\rangle |\phi\rangle + |4\rangle |4\rangle$$

( $\hat{U}$  linear operator)

$$\neq (|\phi\rangle + |4\rangle) (|\phi\rangle + |4\rangle)$$

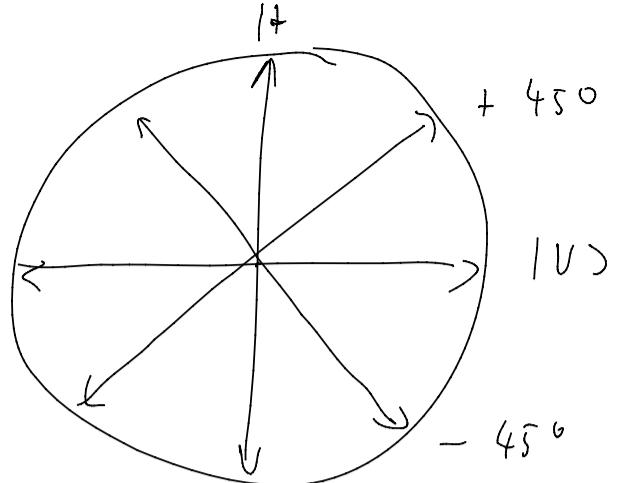
### BB84 - Protocoll (Bennett - Brassard)



- Alice and Bob want to create a secret key

=> Alice sends Bob a sequence of photons that are randomly prepared in one of the 4 pol. states  $|H\rangle, |V\rangle, |+45^\circ\rangle, |-45^\circ\rangle$

start:  $|H\rangle, |V\rangle, |+45^\circ\rangle, |-45^\circ\rangle$



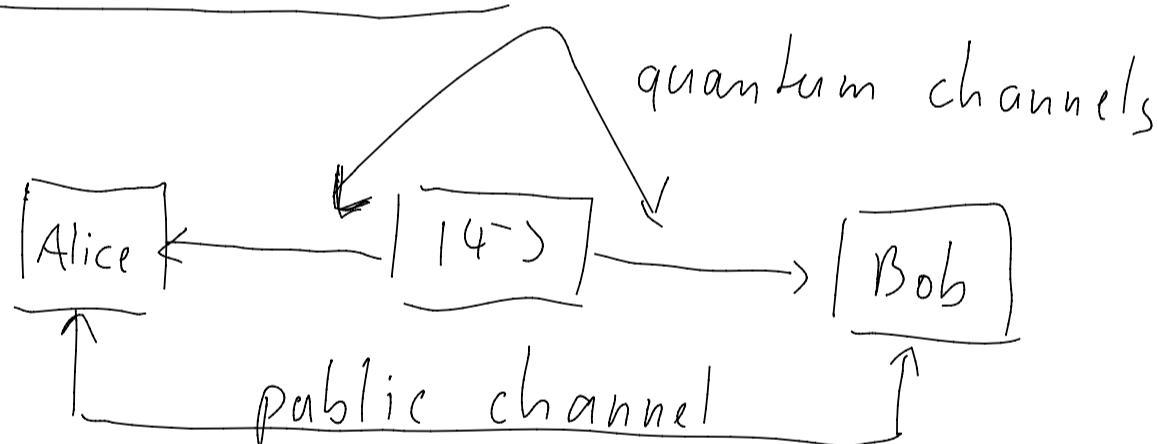
- For each photon Bob chooses randomly the measurement basis  $H/V$  or  $\pm 45^\circ$
- After the measurements:
  - Bob tells Alice in which bases he measured each photon via public channel (newspaper, telephone, internet...).
  - Alice tells Bob, in which cases they coincide with the basis in which Alice's photon was prepared
- => For these cases Bob's polarization measurement results are identical to the state Alice prepared
- => They share a common "string" of photon-polarizations
- => assign bit value "0" to  $|H\rangle, |+45^\circ\rangle$  and "1" to  $|V\rangle, |-45^\circ\rangle$
- => secret key

Checking for Eavesdropper (Eve)

E... I... L... i... b... 1... N... n... h... L... r...

- Eve has to intercept the photons from Alice, measure them and then send them to Bob
    - => introduces errors, i.e. Bob's results are not identical to the state prepared by Alice
  - to check for Eve, Alice and Bob compare a subset of the key (publicly) for errors
  - error rate too high => Eve tried to intercept communication
    - => key is rejected

# Ekerl - protocol



Run UV DLE, HICE + Bob rank a subset of photons and check if their measurement results Bell's inequality

Error correction, Privacy amplification  
(see slides)