Cheat sheet 2: EIT Pulse Propagation and Storage

Pulse propagation – Static control field[1]

Input – Output relation in frequency domain: $E(\omega, z) = e^{ikz\chi(\omega)}E(\omega, z = 0)$

Optical depth d: $I_{out}(\omega) = I_{in}(\omega)\, e^{-d}, d = kL\, Im\, \chi(\omega)$

Resonant optical depth for $\Omega_c = 0$: $d_0 = \frac{N}{V}\frac{2|\mu_{12}|^2}{\epsilon_0\hbar}\frac{kL}{\gamma_2}$

Group delay:

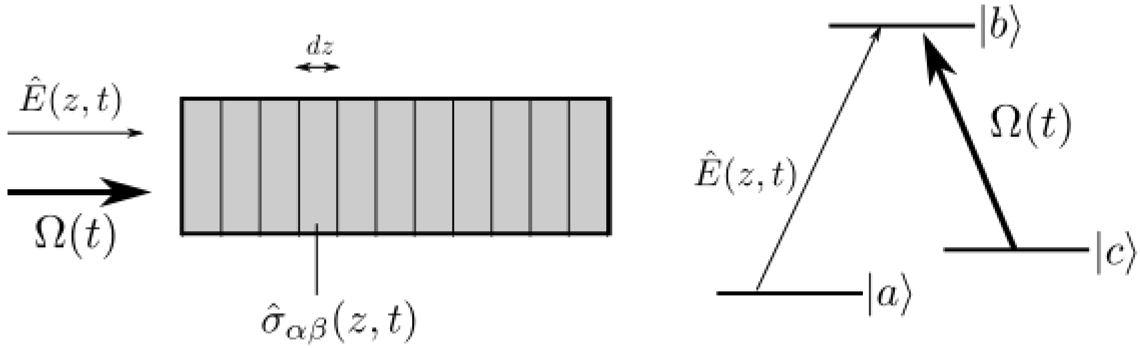
$$\tau_d = L\left(\frac{1}{v_{gr}} - \frac{1}{c}\right) = \frac{d_0}{\gamma_2}\frac{|\Omega_c|^2/\gamma_3^2 - 1}{|1 + |\Omega_c|^2/(\gamma_2\gamma_3)|^2}$$

$$\simeq d_0\left(\frac{\gamma_2}{|\Omega_c|^2} - \frac{\gamma_2^2}{|\Omega_c|^4}\gamma_3\right)$$

Spectral width of the transparency window: $\sigma \sim \frac{1}{\sqrt{d_0}}\,(\gamma_3 + |\Omega_c|^2/\gamma_2)$

Fractional delay: $\frac{\tau_d}{\tau_p} \sim \tau_d\sigma \simeq \sqrt{d_0}\left(1 - \frac{\gamma_2^2\gamma_3^2}{|\Omega_c|^4}\right)$

Dark-State Polaritons[2,3]



Slowly varying operators

$$\hat{E}(z,t) = \sum_k \hat{a}_k(t)\, e^{i(kz - \omega_{ba}t)}, \qquad \hat{\sigma}_{\alpha\beta}(z,t) = \frac{1}{N_z}\sum_{j=1}^{N_z}|\alpha_j\rangle\langle\beta_j|\, e^{-i\omega_{\alpha\beta}t}$$

Interaction Hamiltonian in continuous limit

$$H = -\frac{N}{L}\int_0^L dz\left(\hbar g\sum_k \hat{a}_k(t)e^{ikz}\hat{\sigma}_{ba}(z,t) + \hbar\Omega\,\hat{\sigma}_{bc}(z,t) + H.c.\right)$$

$$g = \mu_{ba}\sqrt{\frac{\omega_{ba}}{2\hbar\epsilon_0 V}}$$

Time evolution in slowly-varying amplitude approximation and for low excitation

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{E}(z,t) = igN\hat{\sigma}_{ba}(z,t)$$

$$\hat{\sigma}_{ba}(z,t) = -\frac{i}{\Omega(t)}\frac{\partial}{\partial t}\hat{\sigma}_{ac}\,(z,t), \quad \sigma_{ac}(z,t) = -\frac{g\hat{E}(z,t)}{\Omega(t)} + \text{terms with } \frac{\partial}{\partial t}$$

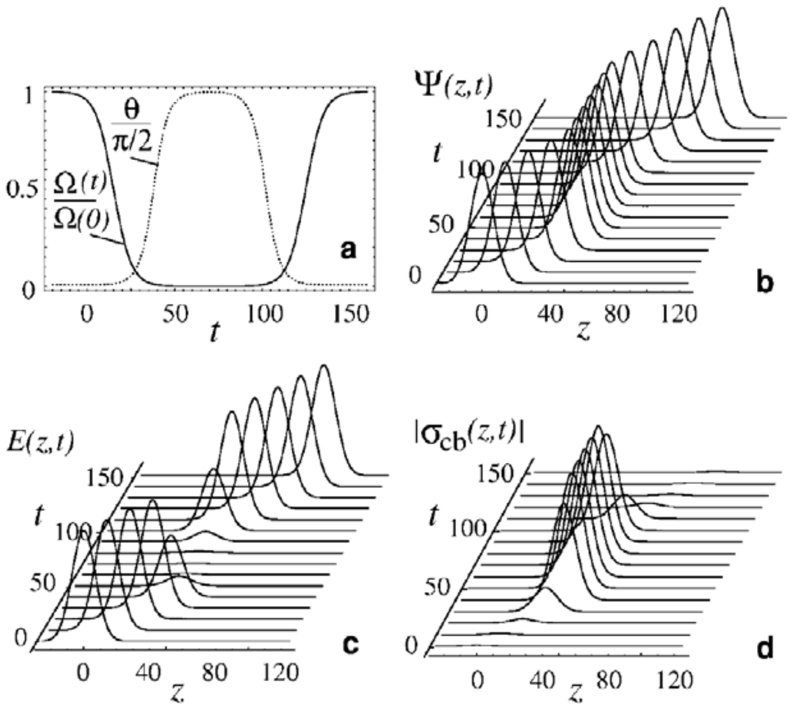
Adiabatic limit

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{E}(z,t) = -\frac{g^2N}{\Omega(t)}\frac{\partial}{\partial t}\frac{\hat{E}(z,t)}{\Omega(t)}$$

Introduce polariton field $\hat{\Psi}(z,t) = \cos\theta(t)\,\hat{E}(z,t) - \sin\theta(t)\,\sqrt{N}\hat{\sigma}_{ab}(z,t)$ with $\tan^2\theta(t) = \frac{g^2N}{|\Omega(t)|^2}$, such that

$$\left(\frac{\partial}{\partial t} + v_{gr}(t)\frac{\partial}{\partial z}\right)\hat{\Psi}(z,t) = 0, \quad \text{where } v_{gr}(t) = c\cos^2\theta(t)$$

Example plot for light storage[2]:



Further reading (optional)

1. Fleischhauer, M., Imamoglu, A. & Marangos, J. Electromagnetically induced transparency: Optics in coherent media. *Rev. Mod. Phys.* **77,** 633–673 (2005).

2. Fleischhauer, M. & Lukin, M. D. Dark-State Polaritons in Electromagnetically Induced Transparency. *Phys. Rev. Lett.* **84,** 5094–5097 (2000).

3. Fleischhauer, M. & Lukin, M. D. Quantum memory for photons: Dark-state polaritons. *Phys. Rev. A* **65,** 022314 (2002).