Research on Inverted Pendulum 1

Ma Yuxuan

February 2024

1 Physics model

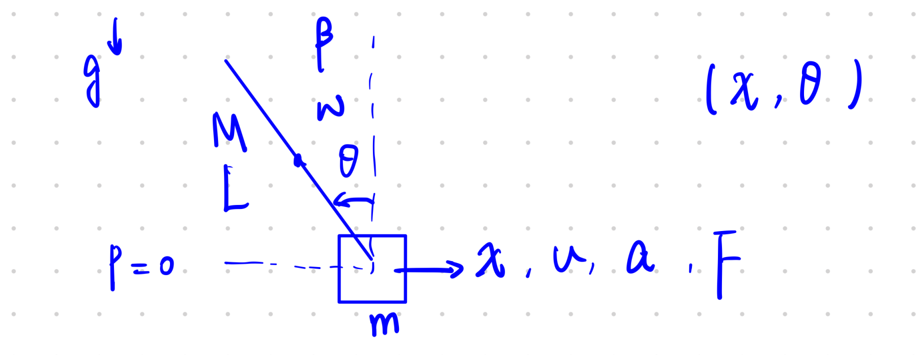

Figure 1: Physics model and parameter definitions

*rational inertia $I = \frac{1}{3}ML^2$

1. Theorem of motion of center of mass

$$x_c = \frac{mx + M(x - \frac{1}{2}L\sin\theta)}{m+M}$$

$$\Rightarrow \quad \dot{x_c} = \frac{m\dot{x} + M(dotx - \frac{1}{2}L\cos\theta\dot{\theta})}{m+M}$$

$$\ddot{x_c} = \frac{m\ddot{x} + M(\ddot{x} - \frac{1}{2}L\cos\theta\ddot{\theta} + \frac{1}{2}L\sin\theta\dot{\theta}^2)}{m+M}$$

$$F = (m+M)\ddot{x_c}$$

$$\therefore F = (m+M)\ddot{x} + \frac{1}{2}ML(\sin\theta\dot{\theta}^2 - \cos\theta\ddot{\theta}) \tag{1.1}$$

2. relative motion
   Take the m as the reference frame:

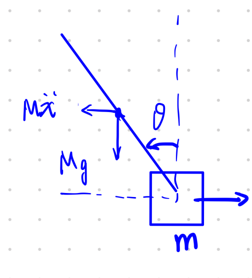

Figure 2: Take the m as the reference frame

$$I\ddot{\theta} = M\ddot{x}\frac{1}{2}L\cos\theta + Mg\frac{1}{2}L\sin\theta \tag{1.2}$$

3. Linearization

$$\theta \to 0$$

★ Problem: what we can do about "$sin\theta\dot{\theta}^2$"?
▲Answer: just ignore.
▲Explanation:

   (a) (not right in Math)
       Linearization at first when calculating x_c

   (b) (maybe right in Physics)
       Let $\dot{\theta} \to 0$

Finally:

$$\frac{d}{dt}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{3Mg}{4m+M} & 0 & 0 \\ 0 & \frac{6(m+M)g}{(4m+M)L} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{4}{4m+M} \\ \frac{6}{(4m+M)L} \end{bmatrix} F$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F$$