MATLAB Fatigue and Structural Integrity Analysis

By

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MATLAB ANALYSIS TOOL DESIGN

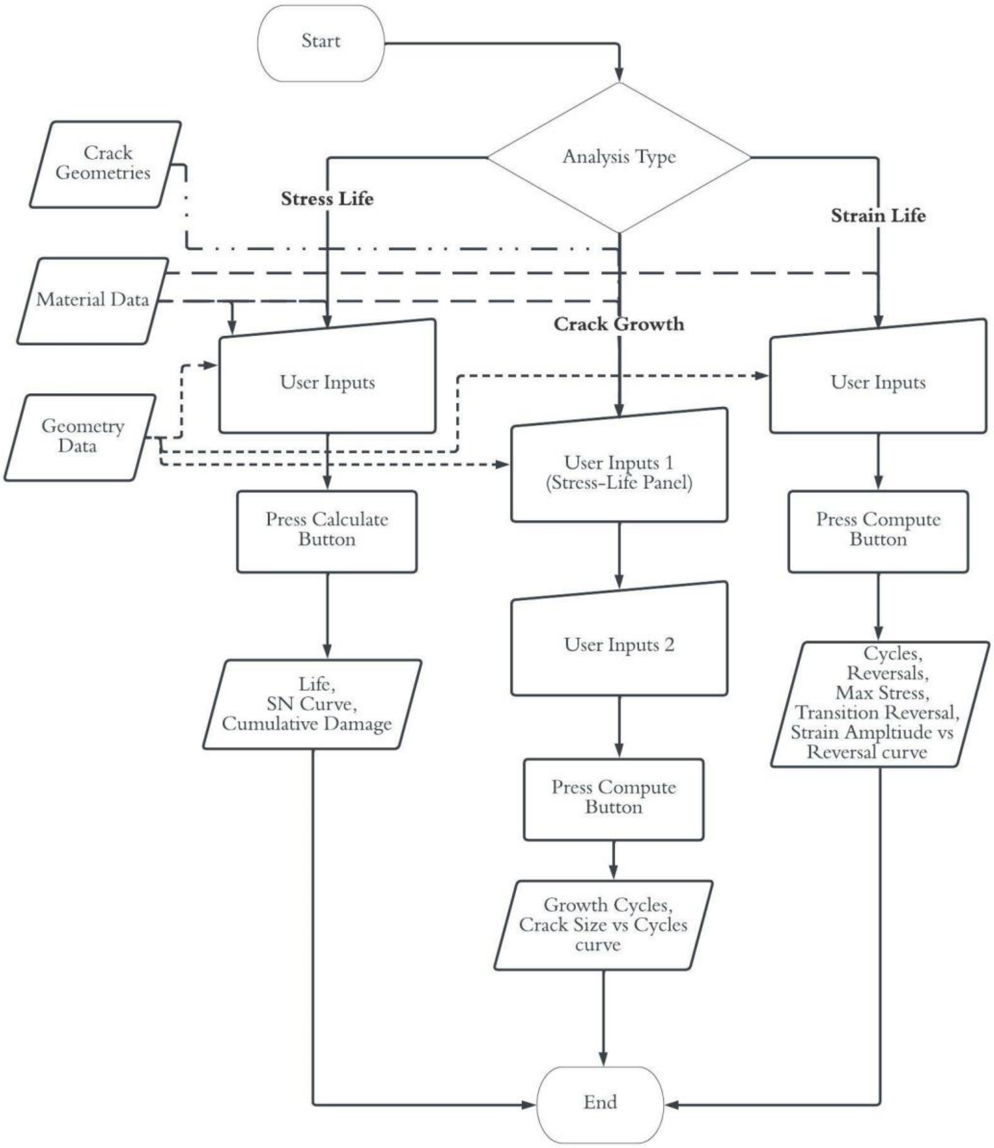

Fig. **Error! No text of specified style in document.**.1. MATLAB Tool Flowchart

Safe-Life (Stress-Life)

Stress-Life analysis as previously stated is one of the legacy methods of determining the fatigue life of a specimen based on the SN curve. The analytical approximation of this is given by Basquin's Law:

$$\mathbf{S_f = A(N_f)^B}$$ Error! No text of

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Algorithm

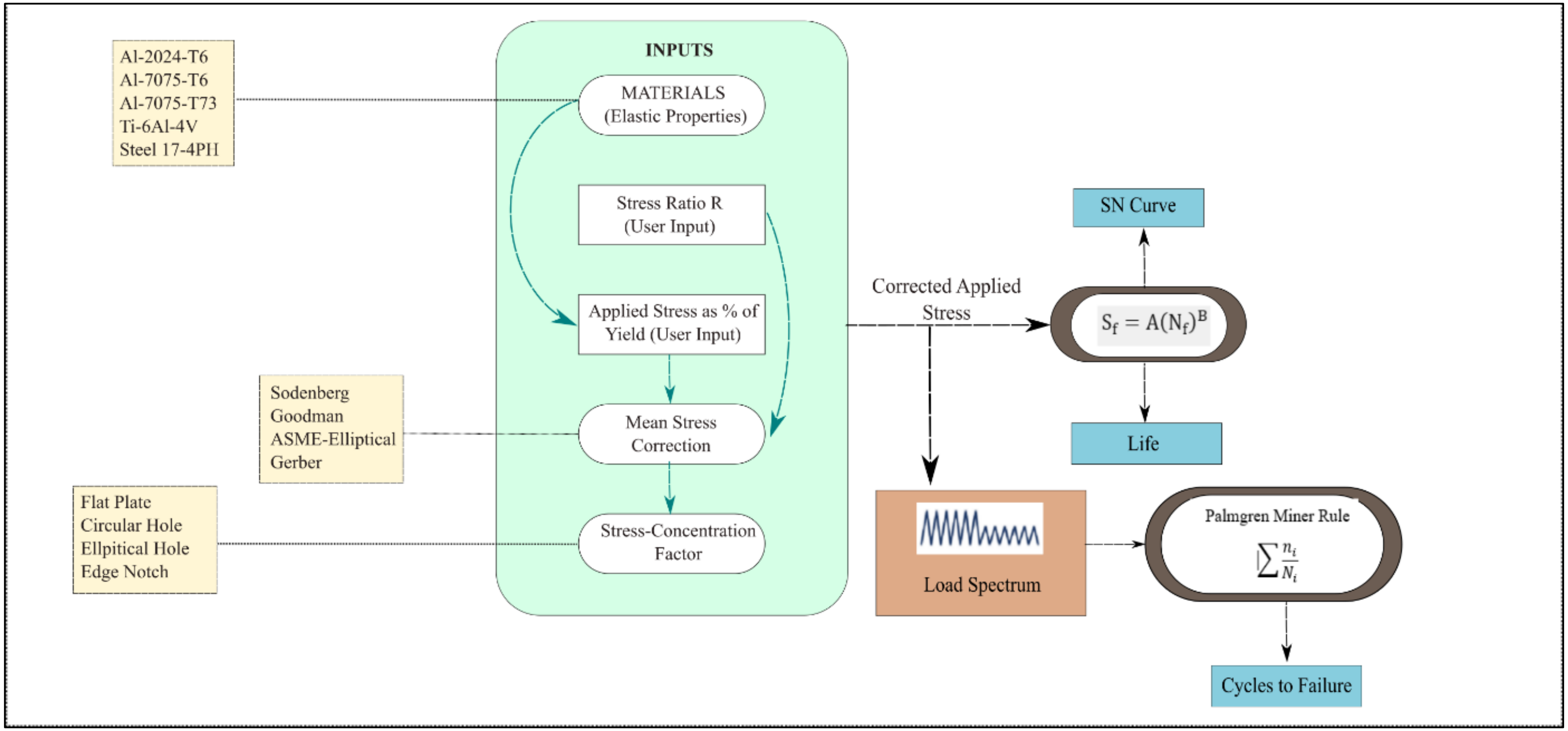

Fig. **Error! No text of specified style in document.**.2. Algorithm for Stress-Life Method

Graphic User Interface (GUI)

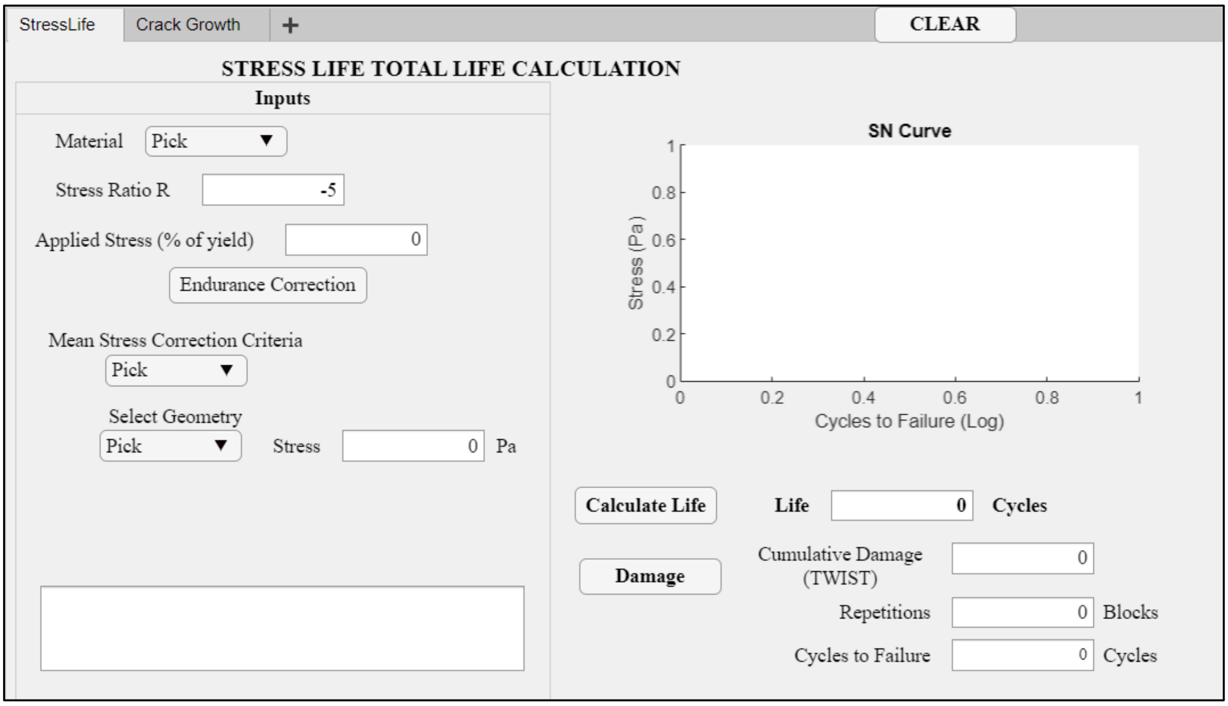

Fig. **Error! No text of specified style in document.**.3. Graphic User Interface for Stress-Life

Inputs

The inputs taken from the user include:

i. Material Selection.

ii. Stress Ratio R.

iii. Applied Stress as a percentage of Yield Stress of Material.

iv. Mean Stress Correction.

v. Stress Correction Factor.

Material Selection

The Material data is coded into MATLAB App designer code view, for the five materials mentioned earlier in the literature review section 4.13.

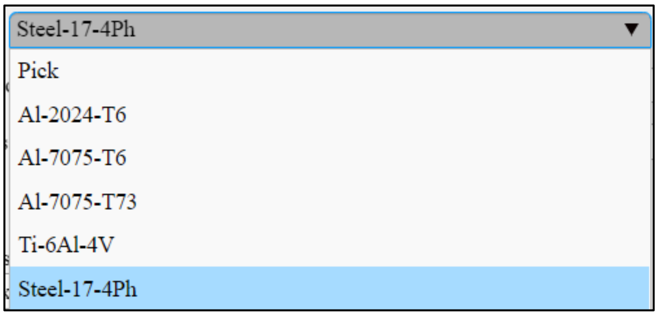

Fig. **Error! No text of specified style in document.**.4. Material Selection (Stress-Life)

Stress Ratio

Stress Ratio is the ratio of Minimum stress to Maximum stress and is taken as the second input.

$$\mathbf{R} = \frac{\boldsymbol{\sigma}_{\mathbf{min}}}{\boldsymbol{\sigma}_{\mathbf{max}}}$$

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Fig. **Error! No text of specified style in document.**.5. Stress Ratio Input (Stress-Life)

Applied Stress

Applied stress is taken as the percentage of the yield stress of the selected material, where you specify in decimal form, the percentage to apply.

Fig. **Error! No text of specified style in document.**.6. Applied Stress Input as Percentage of Yield Strength (Stress-Life)

$$A_{\sigma} = \frac{1 - R}{1 + R}$$ Error! No text of specified style in document..**3**

$$\sigma_{mean} = \frac{A_{\sigma}}{\sigma_a}$$ **Error! No text of specified style in document.**.4

ratio and Applied stress specified are used to determine the mean stress using equations (1) and (2), to apply the desired mean stress correction theory, to get the equivalent fully reversed stress.

Where, σ_a is user input applied stress as a percentage of yield stress.

Mean Stress Correction Theory

Various mean stress correction theories are used to account for the mean stress correction factors and obtain the equivalent fully reversed stress amplitude. The theories used are mentioned in the previous literature review section 4.4.3.

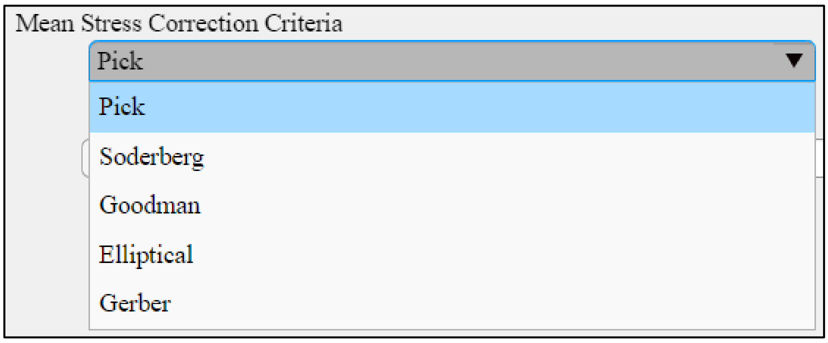

Fig. **Error! No text of specified style in document.**.7. Mean Stress Correction Criteria Input

Reason for Selection

Engineering Components under alternating stresses also experience the effects of mean stresses, operating under positive stress ratios. The mean stress has a more significant impact on crack initiation and fatigue lives in general than any other effect. This often occurs in a component because of residual stresses resulting from either manufacturing processes or left behind at notches overloads of components. Therefore, accountability of the mean stress effects becomes critical[32].

Various analytical models for accounting for the mean stress effects in the prediction of fatigue life are present throughout the literature. The common principle across all these models is that they assume either yield strength σ_y or ultimate strength σ_u and fatigue strength σ_f to be the physical limits of mean stress σ_m[32].

For our tool, we have chosen four of the most frequently used mean stress correction theories. These include:

- Soderberg
- Goodman
- ASME Elliptical
- Gerber

Soderberg

Soderberg's theory is the most conservative of all the theories lying completely below the yield line, therefore it does not apply to the majority of the cases. Its curve is a straight line substantially easy to draw. However, its simplicity is also its major drawback as it fails to cater to the majority of real-world cases[33].

Goodman

Goodman is by far the most frequently used mean stress correction criteria, for two reasons. Its use of the ultimate strength value makes it applicable to most cases experienced in the Aerospace and Engineering field in general. Secondly, its curve being a straight line makes it easier to apply and access. It's primarily a good choice for brittle materials, as there is virtually no crack propagation associated with brittle materials, and therefore Goodman can be applied. Experiments have determined that the majority of the cases lie between Goodman and Gerber curves[33]. Another thing to notice is that both Soderberg and Goodman are unbounded in the presence of negative mean stresses[33]. It is more conservative than Gerber[34].

Gerber

Gerber curve is more of a parabolic curve, which also takes into account the ultimate strength level. Among the above three theories, Gerber is by far the best and is suitable for application to ductile materials, that experience significant crack propagation. However, in addition to its curve being different, it also remains bounded for negative mean stresses[33].

ASME-Elliptical

This theory of mean stress correction is on par with Gerber in terms of passing through experimental data. As its name suggests, its curve is elliptical, and instead of ultimate strength, it incorporates yield strength as the limiting point, which makes it suitable for use in Aerospace owing to its high fidelity and limiting level. Its major drawback is the difficulty in drawing its curve and implementation. Additionally, it incorporates rough yielding checks[34].

For our cases, we stick with Goodman for most of our cases due to its frequent use in practical cases.

Stress Concentration Factor

In this user input, the code asks the user to select any geometry out of the four already present. This geometry in turn corresponds to a stress concentration factor computed using analytical relations.

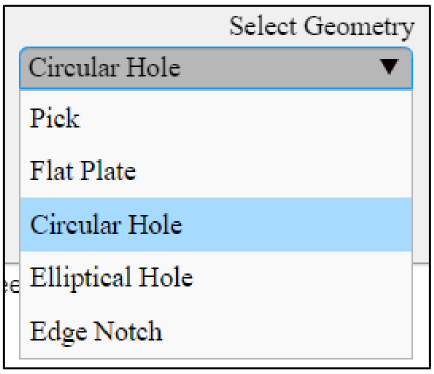

Fig. **Error! No text of specified style in document.**.8. Stress Concentration Factor Inputs

Flat Plate

The case of Flat Plate corresponds to the stress concentration factor of unity as there are no stress raisers present in the geometry.

$$\mathbf{K_f = \frac{\sigma_{max}}{\sigma_{nominal}} = 1}$$

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Circular Hole

The Fatigue Stress Concentration factor for all these cases is taken from the book “Formulas for Stress and Strain, 4th ed by Raymond J. Roark”[35]

$$\mathbf{K_f = \frac{3d}{a + d}}$$ Error! No text of specified style in document..**6**

a – diameter of the hole.

d – width of the plate.

Both of these parameters can be entered back into the code.

Elliptical Hole

Also taken from[35],

$$\mathbf{K_f = 1 + 2\left(\frac{a}{b}\right)}$$ Error! No text of specified style in document..**7**

a – semi-major axis of the hole

b – semi-minor axis of the hole

Again, these two parameters can be entered in the code as per requirement.

Edge Notch

It involves the presence of a notch at the edge parallel to the loading direction, such that the notch grows perpendicular to the tensile load direction[35].

$$\mathbf{K_f = 1 + 2\sqrt{\frac{h}{r}}}$$ Error! No text of specified

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h – the height of the notch

r – radius of notch

Endurance Correction

A button for the Correction of Material Endurance strength/limit to cater to various factors affecting endurance strength. Various factors are briefly mentioned in the previous section 4.4.4.

For our cases, we take the following factors into account,

Table **Error! No text of specified style in document.**.1

Endurance Modifiers

Factor	**Value**
C_{load}	0.85
C_{size}	From Equation [d]
C_{surf}	a = 57.7 b = -0.718 in equation [e]
C_{temp}	1
C_{relia}	0.868

Calculate Life

Upon Pressing the Calculate Life button, we get both the cycles to failure at the corrected stress, and an SN curve of the condition.

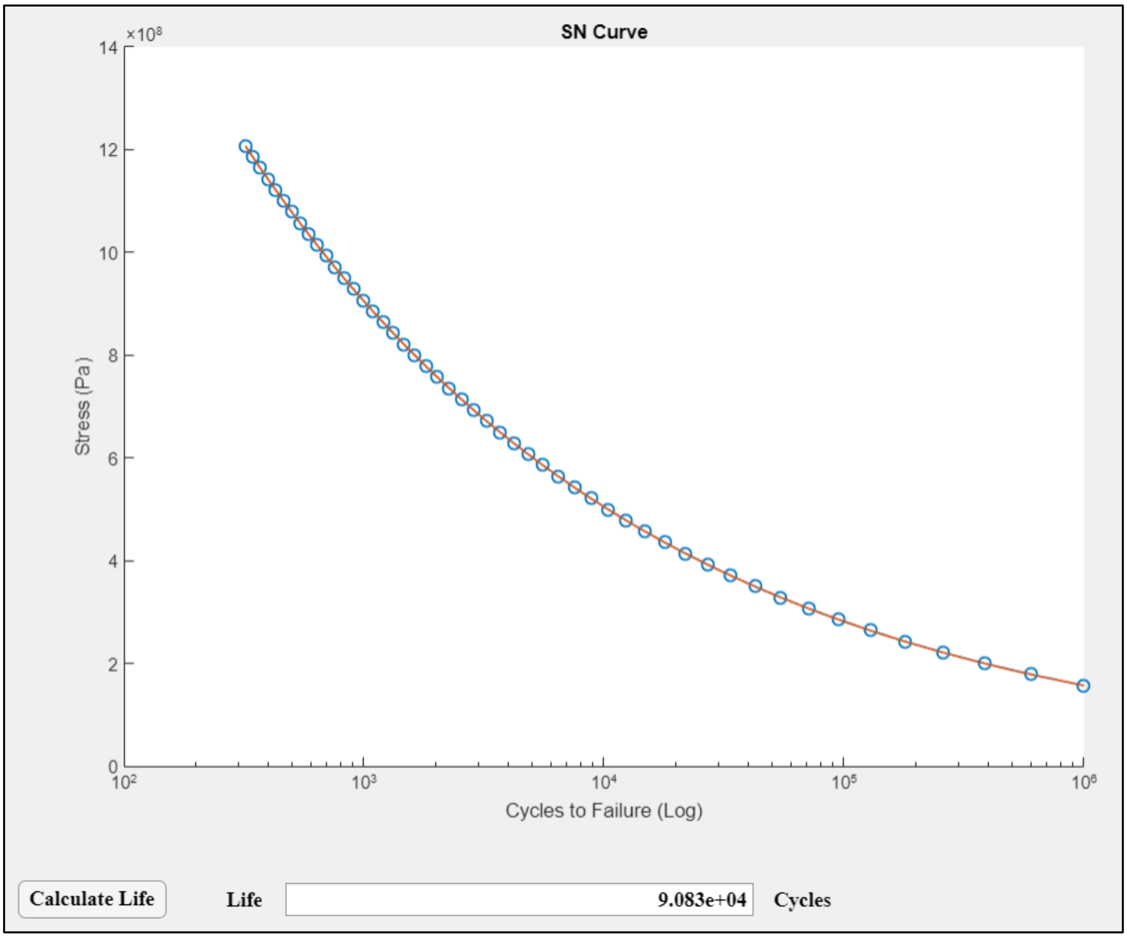

Fig. **Error! No text of specified style in document.**.9. Stress-Life Results

Variable Amplitude Case

For the Variable amplitude case, we use the Miner damage rule to get an estimate of the damage, and from that, we get the block repetitions and cycles to failure. Using eq (1) (2) and (3) we get cycles to failure.

$$\mathbf{D} = \sum \frac{\mathbf{n_i}}{\mathbf{N_i}}$$

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$$\mathbf{N_{rep}} = \frac{\mathbf{1}}{\mathbf{D}}$$

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style in document..**10**

$$\textbf{Cycles to Failure} = \mathbf{N_{rep}} \sum \mathbf{n_i}$$

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For the variable loading case, we employed the TWIST (Transport Wing Load Spectrum) taken from the literature.

Table **Error! No text of specified style in document.**.2

TWIST Load Spectrum

Load Level	N (Cycles per 4000 Flights)	Mean Stress	Stress Amplitude
Airborne Loads			
1	1	1.0	1.600
2	2	1.0	1.500
3	5	1.0	1.300
4	18	1.0	1.150
5	52	1.0	0.995
6	152	1.0	0.840
7	800	1.0	0.685
8	4170	1.0	0.530
9	34800	1.0	0.375
10	358665	1.0	0.222
Ground Loads			

11	1	1.0500	1.5500
12	1	1.0000	1.5000
13	3	0.9000	1.4000
14	9	0.8250	1.3250
15	24	0.7475	1.2475
16	60	0.6700	1.1700
17	420	0.5150	1.0150
18	1090	0.4375	0.9375
19	2211	0.3610	0.8610

Damage

On pressing the Damage Button, we get three outputs: damage, blocks, and cycles.

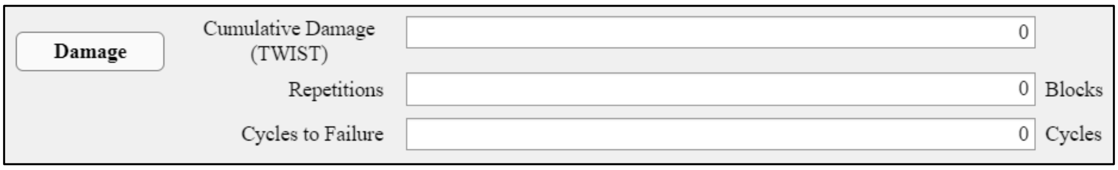

Fig. **Error! No text of specified style in document.**.10. Variable Stress Life Results

Linear Elastic Fracture Mechanics

The Module for Fracture Mechanics employs the principles of Linear-Elastic Fracture Mechanics to predict crack propagation using certain models. Some of these models are mentioned in the Literature Review 4.6.6

Algorithm

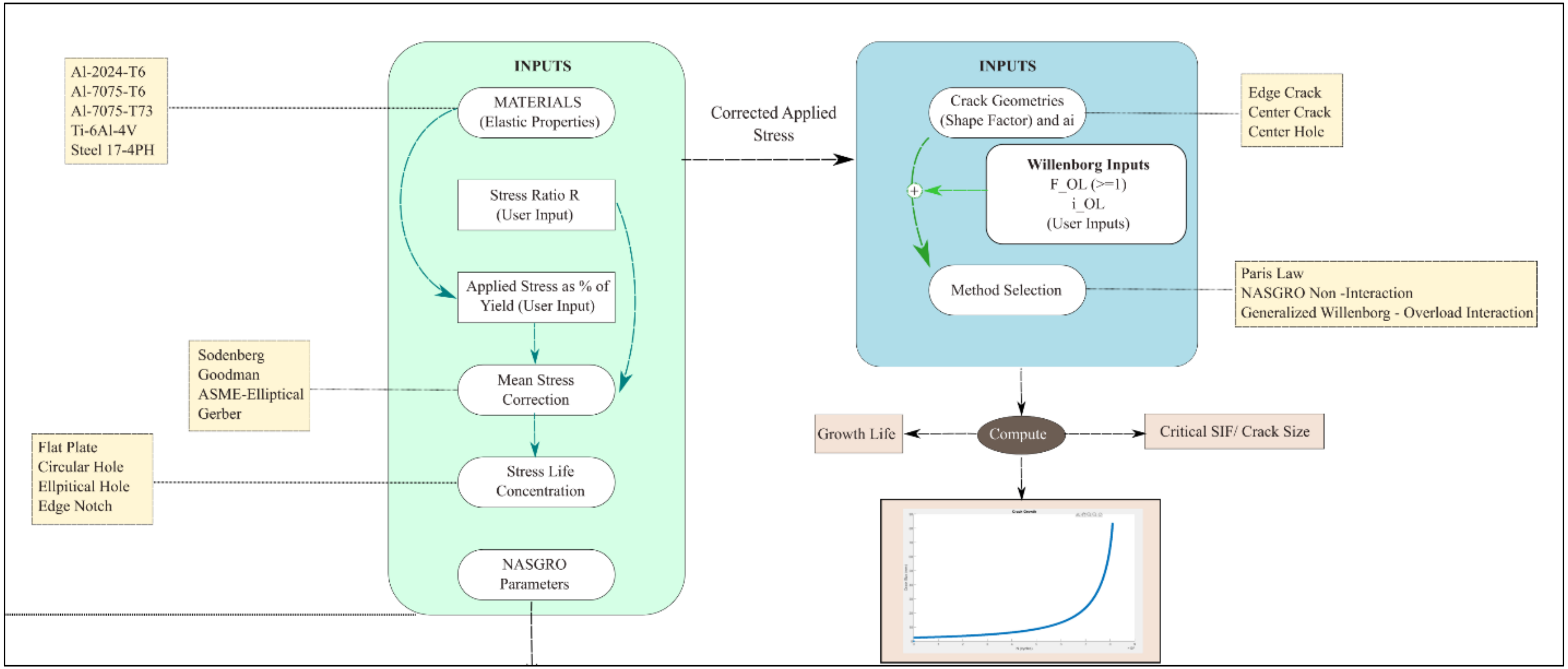

Fig. **Error! No text of specified style in document.**.11. Algorithm for Fracture Mechanics

Graphic User Interface (GUI)

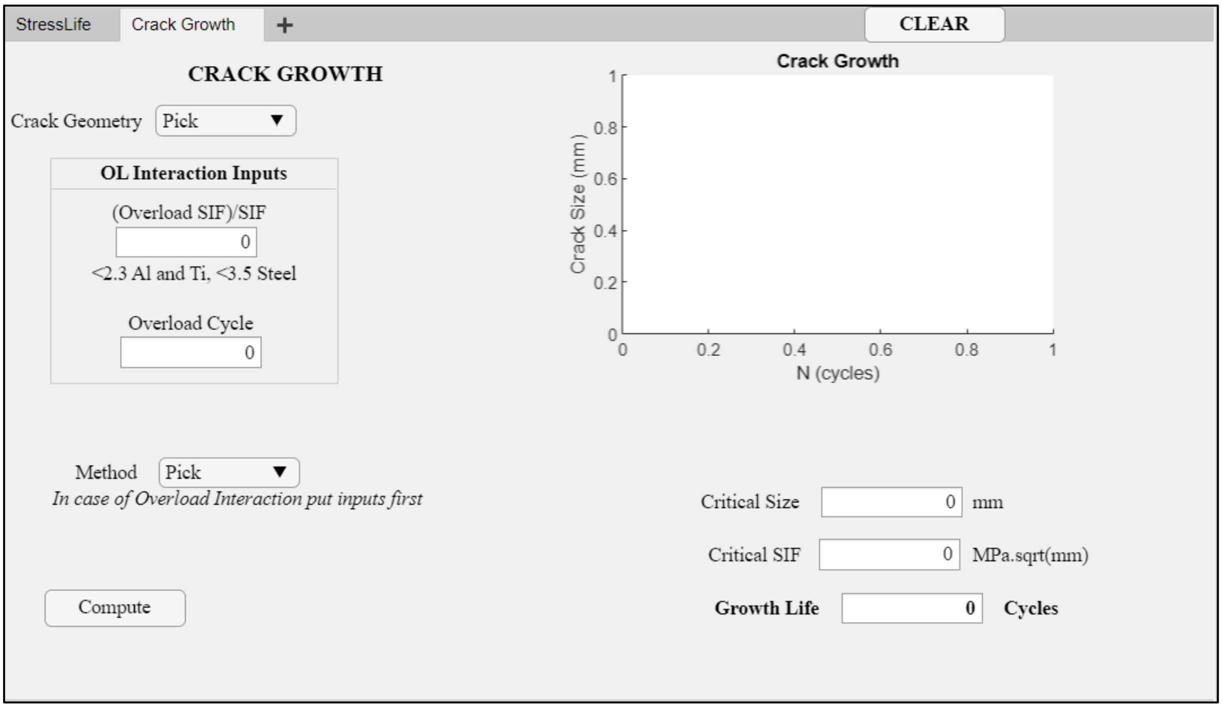

Fig. **Error! No text of specified style in document.**.12. Graphic User Interface for Fracture Mechanics

Inputs

The User Inputs required for operation are:

i. Material Selection.

ii. Stress Ratio R.

iii. Applied Stress as a percentage of Yield Stress of Material.

iv. Mean Stress Correction.

v. Stress Correction Factor.

vi. Crack Geometry Selection.

vii. Model Selection.

The first five inputs are transferred over from the stress-life module and are the same as explained in section 5.1.3.

Material Selection

The materials mentioned in section 4.13 are used. In addition to the elastic data, the NASGRO parameters are also required for operation.

Crack Geometry Selection

For this User-Input the operator picks up the shape of the initial crack upon which the crack propagation models are to be later applied. In addition to the value (or function) of the shape factor, the user also selects the initial crack size taken from [22]. The crack geometries primarily picked for our analysis include:

i. Edge Crack.
ii. Center Crack.
iii. Center Hole.

Edge Crack

For the Edge Crack[36],

$$\mathbf{Y = 1.12}$$ Error! No text of specified style in document..**12**

While the initial crack size is,

$$\mathbf{a_i = 2.540\ mm}$$ Error! No text of specified style in document..**13**

Center Crack

Center Crack refers to a crack at the center of the specimen[36],

$$\mathbf{Y = sqrt\left(\frac{1}{\cos\left(pi * \frac{a}{W}\right)}\right)}$$

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W – width of the plate.

a – crack size at a given instance

Therefore, for the center crack, the crack shape factor varies with crack size.

The initial crack size is,

$$\mathbf{a_i = 1.270\ mm}$$

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Center Hole

This refers to a crack present at the periphery of the hole present in the specimen. Its shape factor exists in tabular form as shown below[36],

Table **Error! No text of specified style in document.**.3

Shape Factor for Center Hole [36]

a/R	**f(a/R)**
1.01	0.3256
1.02	0.4514
1.04	0.6082
1.06	0.7104
1.08	0.7843

1.10	0.8400
1.20	0.9851
1.25	1.0168
1.30	1.0358
1.40	1.0536
1.80	1.0495

To implement this, an equivalence relation is obtained through regression analysis given as,

$$\mathbf{Y = -0.8194 * \left(\left(\frac{a+R}{R}\right)^{-14.93}\right) + 1.049}$$

Error! No text of specified style in document..**16**

R – Radius of the hole.

a – crack size at a given instance.

The initial crack size is,

$$\mathbf{a_i = 2.540\ mm}$$

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Model Selection

This User Input corresponds to the selection of the crack propagation model, which the user wants to utilize for analysis. Several empirical relations are used to model the crack propagation behavior associated with the fracture of the specimen[4.6.6]. For our tool, we used the following three models:

- Paris Law
- NASGRO Equation
- Generalized Willenborg Model

The details of these are mentioned in section 4.6.6

Reason for Selection

The reasons behind the selection of these three specific models are as follows:

- Start with a basic model such as Paris law for modeling the crack propagation behavior.
- Observe the variation in the results by utilizing a more elaborate model corresponding to the NASGRO equation, which in addition to including the effects of threshold SIF (at which crack propagation begins), and critical SIF (at which crack propagation terminates), also takes into account the influence of stress ratio of the loading, along with plasticity induced crack-closure effects via the Newman function f.
- Additionally, to understand the effects of switching from a monotone load to a variable load, specifically an overload on the crack propagation. This is achieved through the Generalized Willenborg Model.
- The reason why other models were not chosen, is because of the difficulty in coding these models, and the less intuitive nature in terms of comprehending various effects; one example of this is the strip yield model, while it’s a much high fidelity compared to the above mentioned, it’s primarily concerned with plasticity effects, and modeling these is quite an arduous task.

Paris Law

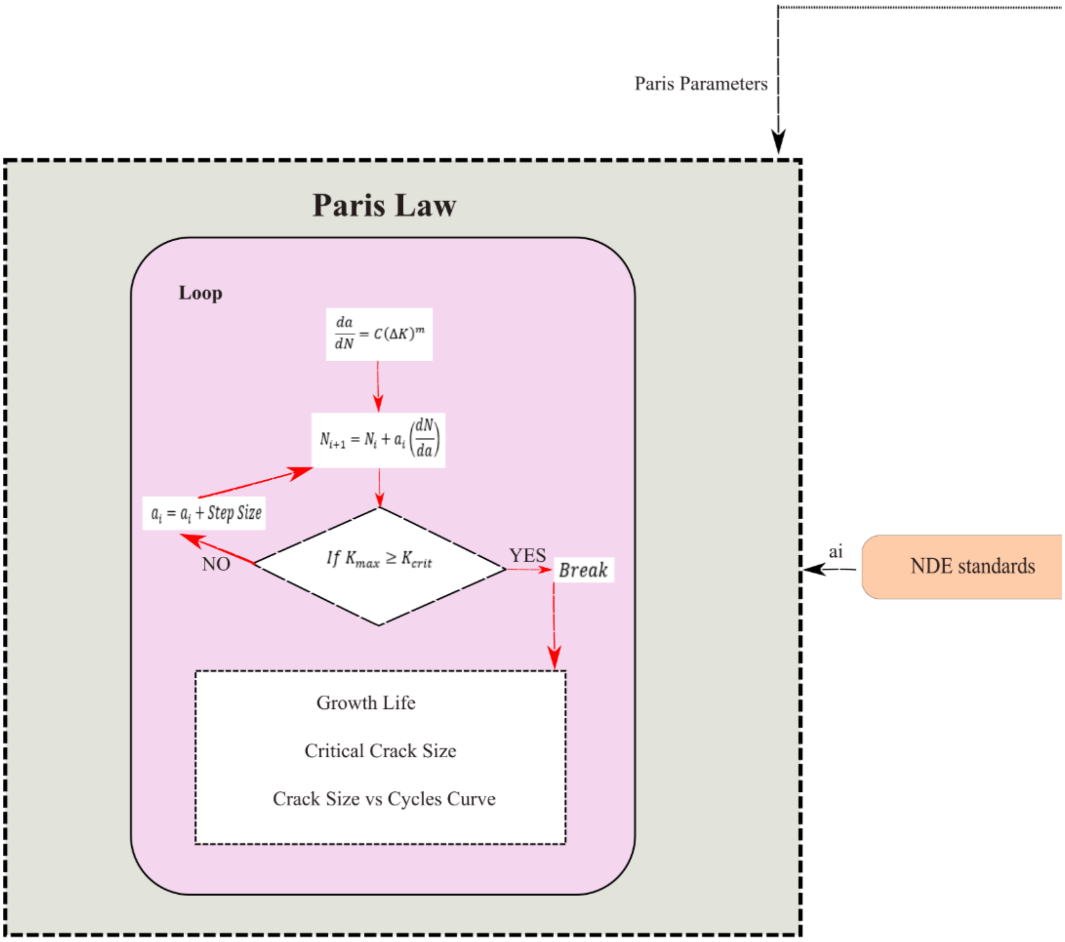

Fig. **Error! No text of specified style in document.**.13. Algorithm for Paris Law

This algorithm is a recursive algorithm comprising a loop. Within the loop, the rate of change of crack size with cycles is computed, from which the increment in cycles is determined. The initial crack size is taken from the NDE standards, and the increase in crack size is augmented into that initial crack size.

NASGRO Equation

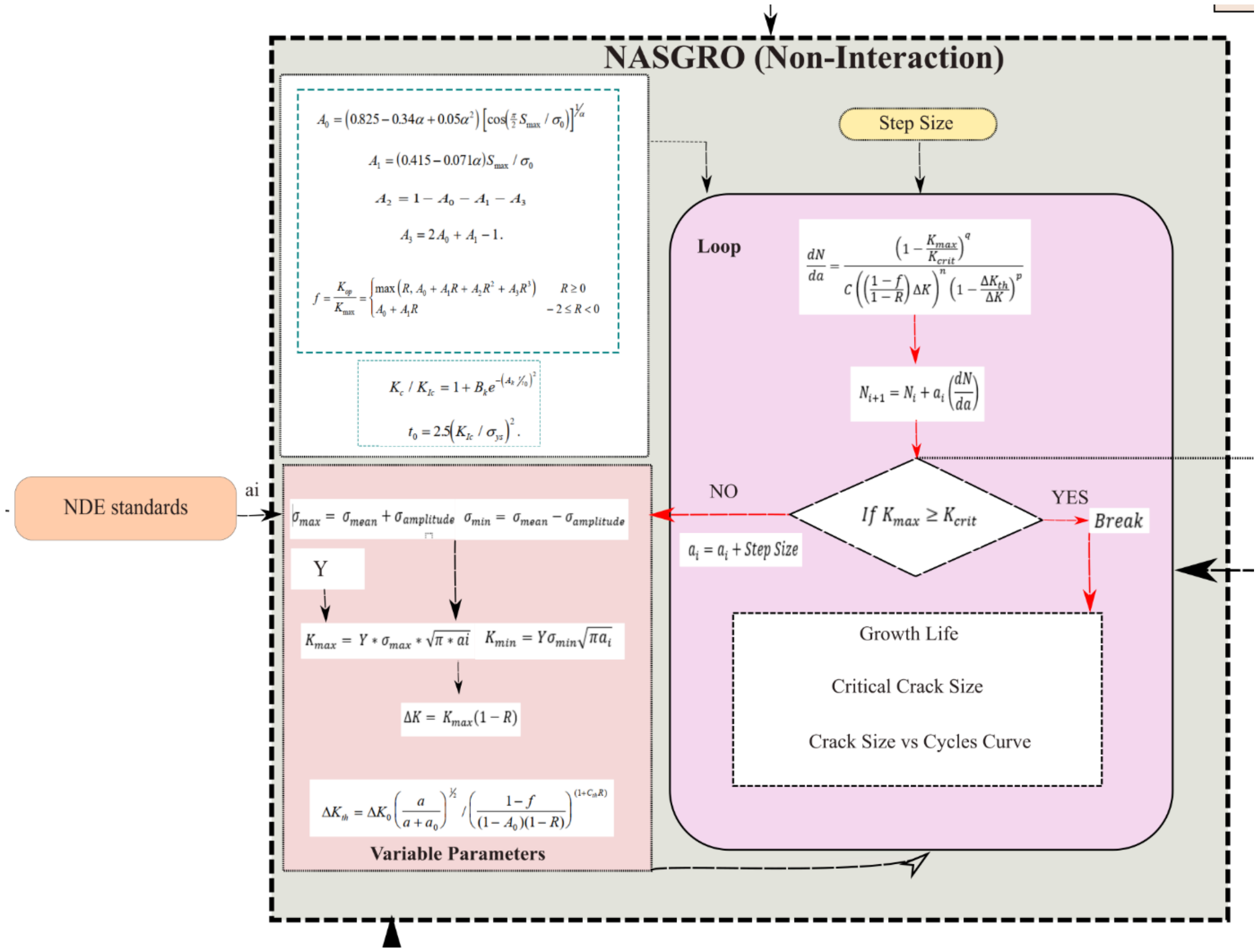

Fig. **Error! No text of specified style in document.**.14. Algorithm for NASGRO Equation

NASGRO Equation model is mentioned in the tool as the No-Interaction model.

This time the loop is much more extensive compared to Paris law, but the external inputs are still just the initial crack size and equation parameters associated with the materials. The basic premise of the loop is still the same i.e., the calculation of $\frac{da}{dN}$ using the equation, and then computing the cycles to failure. The process terminates when the maximum SIF becomes equal to or exceeds the critical SIF (Fracture Toughness). Additionally, during each loop turn, both the change in SIF (ΔK) and the change in threshold SIF (ΔK_{th}) are computed as the crack grows. Furthermore, using the

NASGRO parameters associated with the material, the Newman crack-closure function is also computed which remains constant throughout the run.

Generalized Willenborg Model

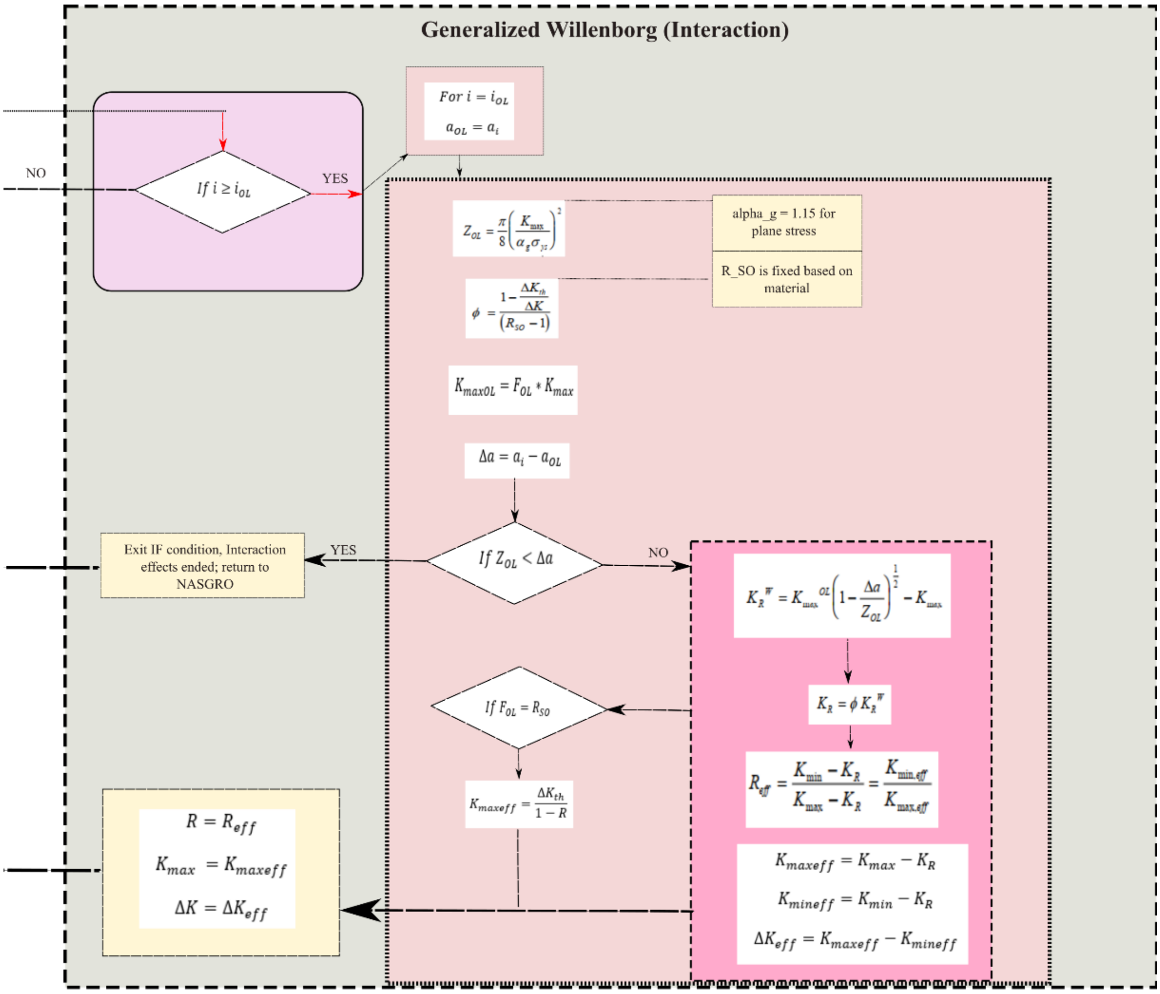

Fig. **Error! No text of specified style in document.**.15. Algorithm for Generalized Willenborg (1)

The algorithm for Generalized Willenborg is just a modification of the NASGRO Equation algorithm, to study the influence of a single overload applied. There are two additional inputs applied for this model to operate as shown in Fig. 5.16. Additionally, this is referred to as the Overload Interaction model in the tool GUI.

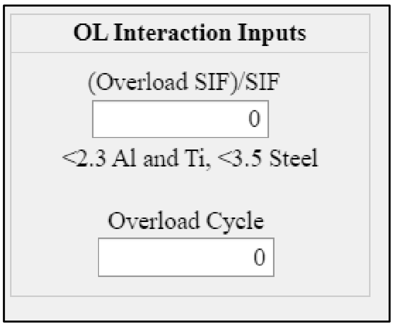

Fig. **Error! No text of specified style in document.**.16. Overload GUI Inputs

The first input refers to the magnitude of overload, given as the ratio $F_{OL} = \frac{K_{OL}}{K_{max}} \geq 1$, This value however should be < 2.3 for Aluminum and Titanium alloys, while < 3.5 for Steel alloys[37]. These values correspond to the shutoff Ratio or the ratio at which crack propagation completely stops[37]. The second input referred to as the Overload Cycle refers to the value of the loop variable 'i_{OL}' at which overload occurs

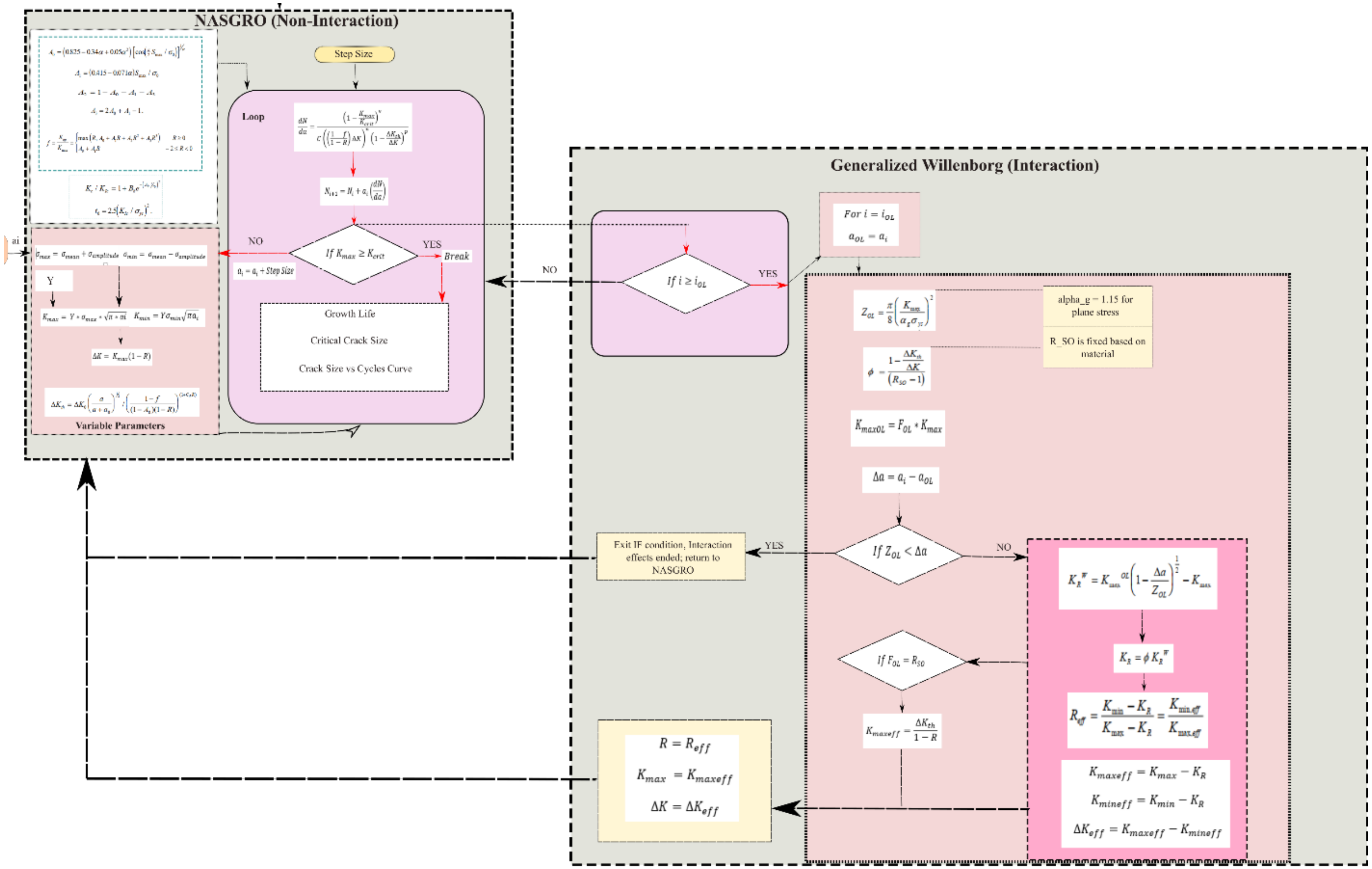

Fig. **Error! No text of specified style in document.**.17. Algorithm for Generalized Willenborg (2)

During the NASGRO loop run, if i = i_{OL} then the overload is encountered. This overload interaction continues till the change in crack size exceeds the size of the plastic zone created by the overload; during which,

$$\mathbf{R = R_{eff} = \frac{K_{min} - K_R}{K_{max} - K_R}}$$

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$$\mathbf{K_R = \varphi K_R^W}$$

Error! No text of specified style in document..**19**

Here,

$$\boldsymbol{\varphi} = \mathbf{\frac{1 - \frac{\Delta K_{th}}{\Delta K}}{R_{SO} - 1}}$$

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$$\mathbf{K_R^W = K_{max}^{OL}\left(1 - \frac{\Delta a}{Z_{OL}}\right)^{\frac{1}{2}} - K_{max}}$$

Error! No text of specified style in document..**21**

Z_{OL} – denotes the size of the plastic zone caused by the overload given by,

$$\mathbf{Z_{OL} = \frac{\pi}{8}\left(\frac{K_{max}}{\alpha_g \sigma_y}\right)^2}$$

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σ_y – yield strength of the material.

α_g – Factors denoting either plane stress or plane strain condition (For plane stress it is taken to be 1.15 [21]).

After the net crack size change exceeds the overload plastic zone size, the algorithm returns to running the basic NASGRO loop.

Safe-Life (Strain-Life)

Similar to Stress-Life, Strain-Life is also one of the historical methods used to predict the life of a component. However, it is much more involved than stress-life that just comprises monitoring the cycles to failure corresponding to a given stress level. In strain, we not only keep track of part life because of elastic deformation, but also one resulting due to plastic local deformation in the structure. The following equation gives the life of the component,

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

Error! No text of specified style in document..**23**

This is known as the Coffin-Manson Relationship and is used to plot a strain-life curve, in contrast to the stress-life curve in the Stress-Life approach. Additionally, we monitor not the cycles to failure but the reversals to failure of the component. 'One Cycle to failure is equivalent to Two reversals to failure'. For in-depth understanding refer to Section 4.5.

Algorithm

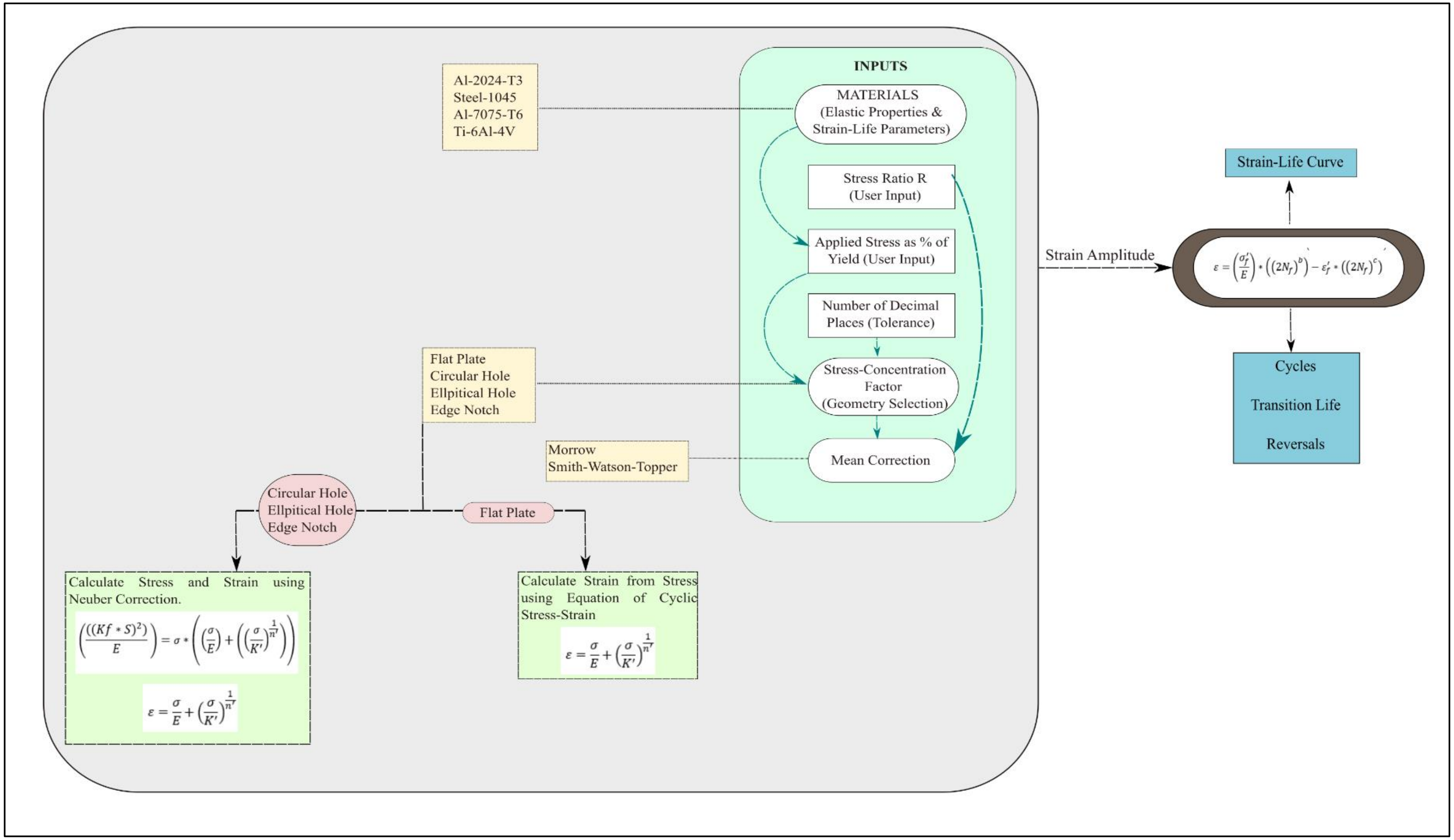

Fig. **Error! No text of specified style in document.**.18. Algorithm for Strain-Life

Graphic User Interface (GUI)

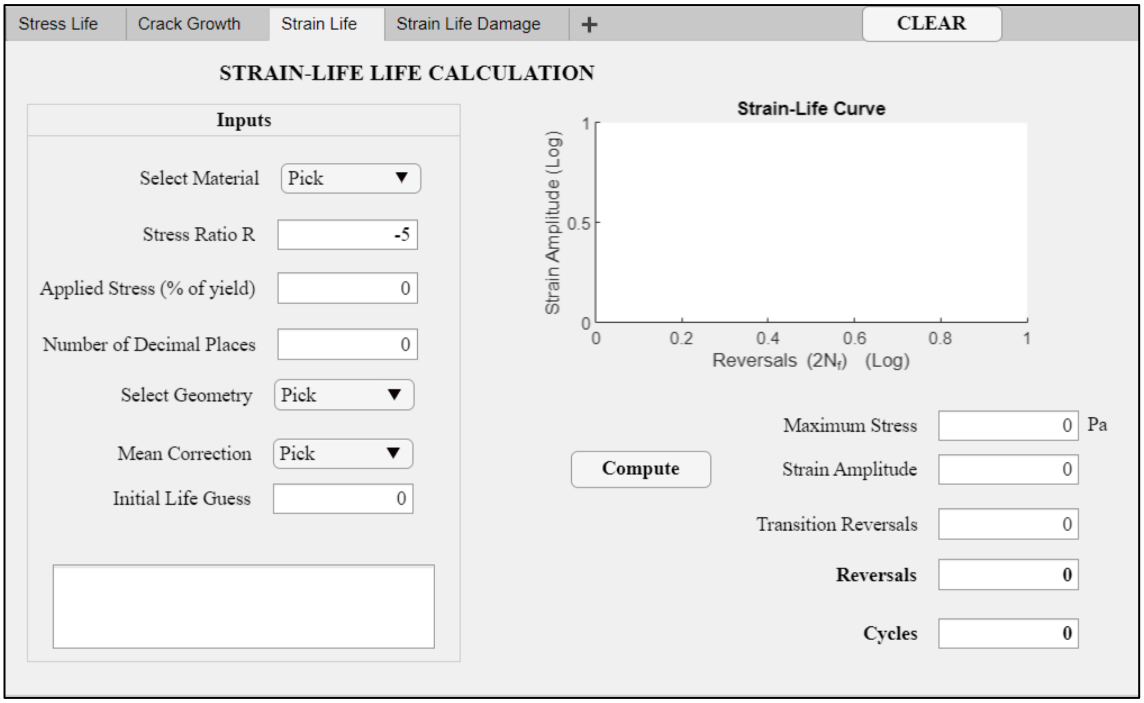

Fig. **Error! No text of specified style in document.**.19. Graphic User Interface for Strain-Life

Inputs

The User Inputs required for the operation are,

i. Material Selection.

ii. Stress Ratio.

iii. Applied Stress as Percent of Yield Stress of the material.

iv. The number of decimal places.

v. Geometry Selection.

vi. Mean Stress Correction.

vii. Initial Life Guess.

Material Selection

The Material data is coded into MATLAB App designer code view, for the five materials mentioned earlier in the literature review section 4.13.6.

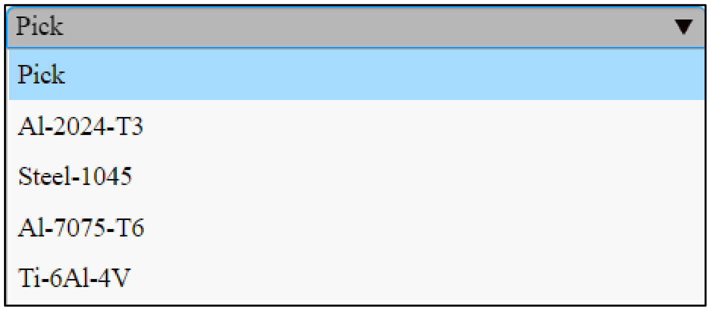

Fig. **Error! No text of specified style in document.**.20. Material Selection (Strain-Life)

Stress Ratio

$$\mathbf{R} = \frac{\boldsymbol{\sigma}_{\mathbf{min}}}{\boldsymbol{\sigma}_{\mathbf{max}}}$$

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Fig. **Error! No text of specified style in document.**.21. Stress Ratio Input (Strain-Life)

Applied Stress

Similar to the Stress-Life case, applied stress is taken as the percentage of the yield stress of the selected material, where you specify in decimal form, the percentage to apply.

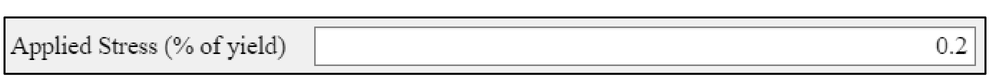

Fig. **Error! No text of specified style in document.**.22. Applied Stress Input (Strain-Life)

Both the Stress ratio and Applied stress specified are used to determine the mean stress using equations (1) and (2), to apply the desired mean stress correction theory, to get the equivalent fully reversed stress.

$$\mathbf{A_{\sigma} = \frac{1 - R}{1 + R}}$$

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$$\boldsymbol{\sigma_{mean} = \frac{A_{\sigma}}{\sigma_a}}$$

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Where, σ_a is user input applied stress as a percentage of yield stress.

Number of Decimal Places

Since this algorithm is based on the numerical method of Newton-Raphson, we need to specify the extent of accuracy we required in the result.

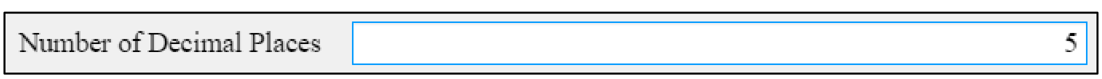

Fig. **Error! No text of specified style in document.**.23. Number of Decimal Places Input (Strain Life)

If ‘n’ is the decimal place, then the level of accuracy chosen is,

$$\in = \mathbf{5 \times 10^{-(n+1)}}$$

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When the difference between two consecutive values determined using newton Raphson is equal to less than ∈ then the iterations stop, and we get the final result within the desired level of accuracy.

Geometry Selection

Geometry Selection just refers to the Selection of the type of plate, with the appropriate stress concentration factor. These are mentioned in Section 5.1.3.5.

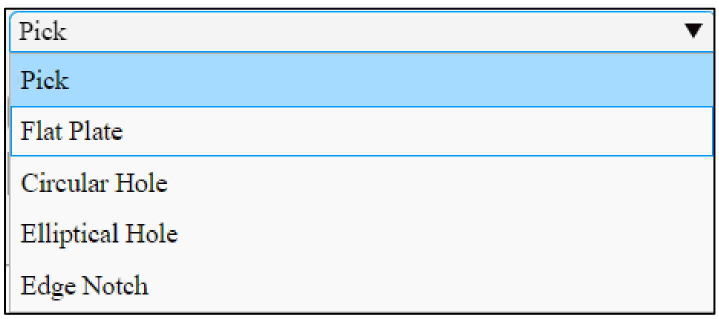

Fig. **Error! No text of specified style in document.**.24. Stress Concentration Factor Inputs (Strain-Life)

Mean Stress Correction

Similar to the case of Stress-Life, the net life of a component is also affected by the level of mean stress in loading in the case of Strain-Life, and thus these effects need to be accounted for to get a more realistic estimate of the failure life.

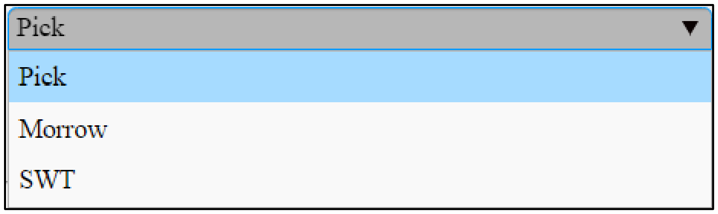

Fig. **Error! No text of specified style in document.**.25. Mean Stress Correction Criteria Input (Strain-Life)

Reason for Selection

The Strain-based approach to fatigue life utilizes Basquin and Coffin-Manson relationship. However, as previously stated it doesn’t take into account the effect of mean stress. Similar to Stress-Life we employ different types of relationships to account for these effects[38].

Morrow

Morrow's approach to mean-stress effect relies on the notion that mean stress mostly affects during the early stages of loading i.e., for higher fatigue life, and in such case, life is dominated by elastic strain effects (Strain Amplitude). It further adds that with as the number of cycles progress, the mean stress regressively goes down to zero. It predicts that mean stress has a significant effect in the case of long-life components dominated by elastic strain[38].

SWT

This proposes that instead of mean stress affecting life itself, it is the interaction of maximum tensile stress and the strain amplitude that account for the mean stress effect[38].

Initial Life Guess

As previously stated, strain-life calculation in our MATLAB model is based on the Newton-Raphson numerical approach; Therefore, we require an initial value of life to trigger the solution process, and here we have provided that as user input.

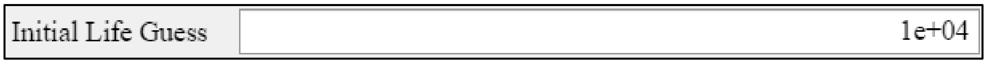

Fig. **Error! No text of specified style in document.**.26. Initial Life Guess Input (Strain-Life)

Note: These values of Initial life guess need to be a reasonable value, otherwise, an inappropriate value might lead to a singularity, and crash the solution process.

Newton Raphson Method

Newton-Raphson method is a numerical approach for solving equations, which cannot be solved directly. It is given by,

$$\mathbf{x_n = x_{n-1} - \frac{f(x_{n-1})}{f(x_{n-1})'}}$$

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In the above equation,

x_n – is the nth solution of the variable.

x_{n-1} – is the (n-1)th solution of the variable.

$f(x_{n-1})$ – is the value of the equation at (n-1)th solution of the x.

$f(x_{n-1})'$ - is the derivative of $f(x_{n-1})$

Results

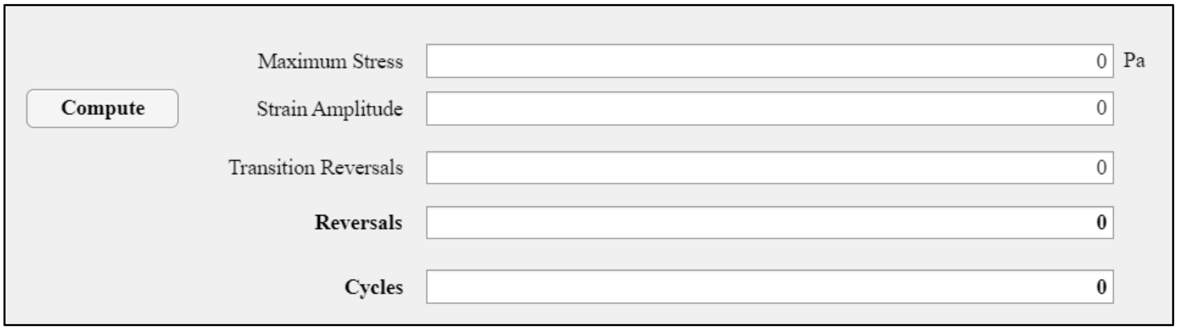

Fig. **Error! No text of specified style in document.**.27. Strain-Life Results

The results computed for the Strain-Life Analysis are:

- Maximum Stress – Max stress that occurs on the body, will be equal to applied stress in case of no notch; However, for a notch, this stress is computed using Neuber correction Section 4.5.5.
- Strain-Amplitude – Strain occurring corresponding to the Stress Amplitude computed using Ramberg-Osgood Relation Section 4.5.3.
- Transition Reversals – Reversals at which Elastic strain life is equal to Plastic strain life.
- Reversals – Reversal to Failure.
- Cycles – On-Half of Reversals.

Variable Amplitude case

Similar to Stress-Life Case, In the Variable amplitude case, we use the Miner damage rule to get an estimate of the damage, and from that, we get the block repetitions and cycles to failure. Using eq (1) (2) and (3) we get cycles to failure.

$$\mathbf{D} = \sum \frac{\mathbf{n_i}}{\mathbf{N_i}}$$

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$$N_{rep} = \frac{1}{D}$$

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$$\text{Cycles to Failure} = N_{rep} \sum n_i$$

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For the variable loading case, we employed the TWIST (Transport Wing Load Spectrum) taken from the literature Table 5.2.

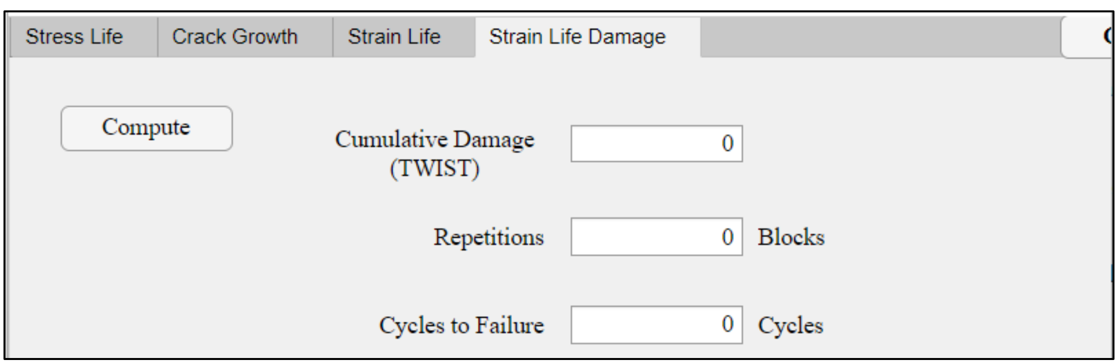

Fig. **Error! No text of specified style in document.**.28. Variable Strain-Life Results