Homework 4 Robotics Manuel Merar

1) Integrating Multiple Sensor Readings

$z_t \in \{R, B, N\}$ $X \in \{X_B, X_R, X_N\}$

$P_r[X = X_N] = 0.5, P_r[X = X_R] = P_r[X = X_B] = 0.25$

1. $P(x_i | z_1, z_2, \ldots, z_n) = \eta \left(\prod_{j=1}^{n} P(z_j | x_i) \right) P(x_i)$

$P(Z_R) = P(Z_R | X_R) P(X_R) + P(Z_R | X_B) P(X_B) + P(Z_R | X_N)(X_N)$
$= (0.6)(0.25) + (0.1)(0.25) + (0.1)(0.5)$
$= 0.225$

$P(Z_N) = (0.1)(0.25) + (0.2)(0.25) + (0.8)(0.5) = 0.475$

$\eta = 1 / (P(Z_R)^2 P(Z_N)) = 1/((0.225)^2 (0.475)) = 41.585$

$P(X_R | Z_R, Z_R, Z_N) = \eta\, P(Z_R | X_R)^2 P(Z_N | X_R)\, P(X_R)$
$= (41.585)(0.6)^2 (0.1)(0.25) = 0.37427$

$P(X_B | Z_R, Z_R, Z_N) = (41.585)(0.1)^2 (0.2)(0.25) = 0.02079$

$P(X_N | Z_R, Z_R, Z_N) = (41.585)(0.1)^2 (0.8)(0.5) = 0.1663417$

The sum of $P(X_R | Z_R, Z_R, Z_N)$, $P(X_B | Z_R, Z_R, Z_N)$, & $P(X_N | Z_R, Z_R, Z_N)$ must equal to one, so we need to normalize the values.

$0.37427 + 0.02079 + 0.1663417 = 0.5614017$

$P(X_R | Z_R, Z_R, Z_N) = 0.37427 / 0.5614017 = 0.6666$
$P(X_B | Z_R, Z_R, Z_N) = 0.02079 / 0.5614017 = 0.0370323$
$P(X_N | Z_R, Z_R, Z_N) = 0.1663417 / 0.561497 = 0.296297$

2. The posterior would not be different because we are assuming the sensor readings are independent events resulting scaling due to multiplication. The order of which the product is found does not matter.

2) Action Uncertainty

$U = P \rightarrow$ push X is the state of the door $X \in \{O, C\} \rightarrow$ open, closed

$Pr(X_0 = C) = 0.9$ $Pr[X_0 = O] = 0.1$

$Pr(X_t = O \mid X_{t-1} = O, U_t = P) = 0.9$

$Pr(X_t = C \mid X_{t-1} = O, U_t = P) = 0.1$

$Pr(X_t = O \mid X_{t-1} = C, U_t = P) = 0.6$

$Pr(X_t = C \mid X_{t-1} = C, U_t = P) = 0.4$

$Pr(X_1 = O \mid U_1 = P) = (0.9)(0.1) + (0.6)(0.9) = 0.63$

$Pr(X_1 = C \mid U_1 = P) = (0.1)(0.1) + (0.4)(0.9) = 0.37$

$Pr(X_2 = O \mid U_1, U_2) = (0.9)(0.63) + (0.6)(0.37) = \boxed{0.789}$

$Pr(X_2 = C \mid U_1, U_2) = (0.1)(0.63) + (0.4)(0.37) = 0.211$