

Midterm homework problems

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November 12, 2020

1 Divergence and Laplacian

1. We have the definition of Christoffel symbols:

$$\Gamma_{ij}^k = \frac{\partial \mathbf{e}_i}{\partial x^j} \cdot \mathbf{e}^k$$

Then we have that:

$$\nabla \cdot \mathbf{V} = \partial_i (V^j \mathbf{e}_j)^i = \frac{\partial V^i}{\partial x^i} + \Gamma_{ij}^i V^j = V_{,i}^i + \frac{1}{2} g^{im} (g_{mi,j} + g_{mj,i} - g_{ij,m}) V^j$$