Mécanique des Milieux Continus

F. Pétrélis (cours)
T. Jules (TD), theo.jules@ens.fr

TD N^o2: Correction

1 Interprétation géométrique des contraintes

1. Invariants:

$$\mathrm{Tr}(\underline{\underline{\sigma}}) = \sigma_1 + \sigma_2$$
$$\mathrm{Det}(\underline{\underline{\sigma}}) = \sigma_1 \sigma_2$$

2. $\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} = \begin{bmatrix} (\sigma_1+\sigma_2)/2 & 0 \\ 0 & (\sigma_1+\sigma_2)/2 \end{bmatrix} + \begin{bmatrix} (\sigma_1-\sigma_2)/2 & 0 \\ 0 & -(\sigma_1-\sigma_2)/2 \end{bmatrix}$

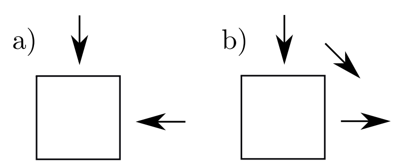


Figure 1: a) Partie symétrique. b) Partie antisymétrique.

3.

$$\underline{n} = \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$$
$$\underline{t} = \begin{pmatrix} -\sin(\alpha) \\ \cos(\alpha) \end{pmatrix}$$
$$\underline{T} = \underline{\underline{\sigma}} \cdot \underline{n} = \begin{pmatrix} \sigma_1 \cos(\alpha) \\ \sigma_2 \sin(\alpha) \end{pmatrix}$$
$$\sigma_n = \underline{T} \cdot \underline{n} = \sigma_1 \cos(\alpha)^2 + \sigma_2 \sin(\alpha)^2 = \frac{(\sigma_1+\sigma_2)}{2} + \frac{(\sigma_1-\sigma_2)}{2}\cos(2\alpha)$$
$$\tau = \underline{T} \cdot \underline{t} = (\sigma_2 - \sigma_1)\sin(\alpha)\cos(\alpha) = (\sigma_2-\sigma_1)\frac{\sin(2\alpha)}{2}$$

4. $\tau^2 + (\sigma_n - \frac{(\sigma_1+\sigma_2)}{2})^2 = (\frac{\sigma_1-\sigma_2}{2})^2$
5. τ sera maximum pour $\alpha = \pi/4$.

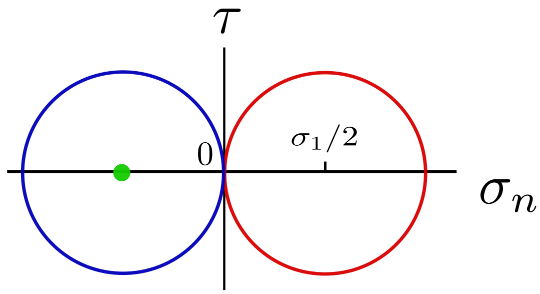
6. 

Figure 2: Rouge: Compression uniaxiale. Bleu: Traction uniaxiale. Vert: Charge hydrostatique

7. Pour un cisaillement pur, on aura:

$$\underline{T} = \begin{pmatrix} \tau \sin(\alpha) \\ \tau \cos(\alpha) \end{pmatrix}$$
$$\sigma_n = \tau \sin(2\alpha)$$
$$\tau = \tau \cos(2\alpha)$$

2 Application du diagramme de Mohr

1. On a $\sigma_1 + \sigma_2 = -24$ MPa et $|\sigma_1 - \sigma_2| = 10$ MPa.
   Donc si on prend $\sigma_1 > \sigma_2$: $\sigma_1 = -7$ MPa et $\sigma_2 = -17$ MPa.
2. Après calculs, on a: $\sigma_n = -9.5$ MPa et $\tau = 4.3$ MPa.
3. Après calculs, on trouve $\alpha'' = 36°$. Cela donne $\tau = 4.8$ MPa.

3 Les critères de rupture

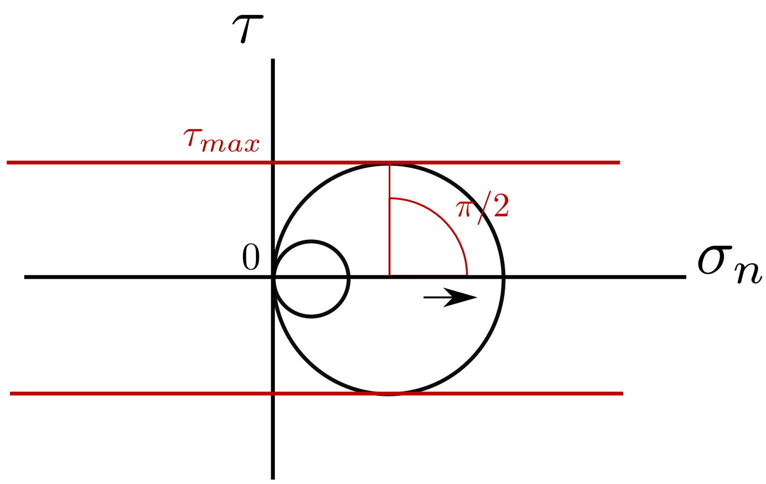


Figure 3: Critère de Tresca.

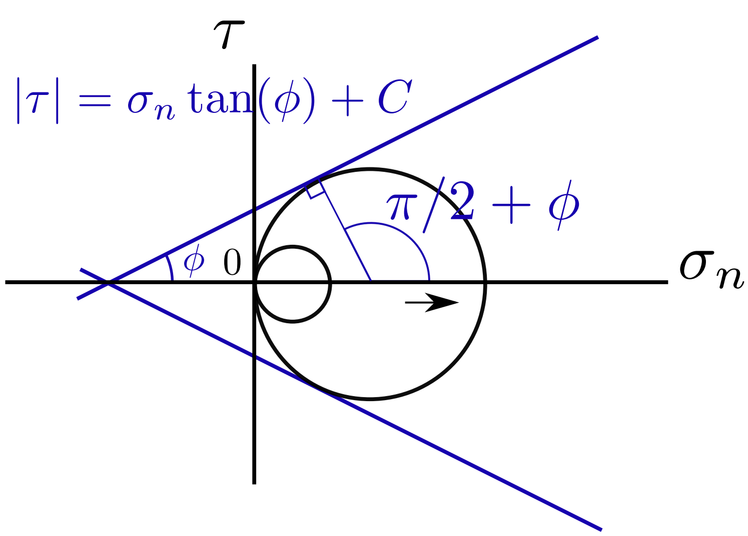


Figure 4: Critère de Mohr-Coulomb.