2.1 When expressed in natural units the lifetime of the W boson is approximately $\tau \approx 0.5\,\mathrm{GeV}^{-1}$. What is the corresponding value in S.I. units?

2.4 A particle of mass 3 GeV is travelling in the positive z-direction with momentum 4 GeV; what are its energy and velocity?

2.6 For the decay $a \to 1 + 2$, show that the mass of the particle a can be expressed as

$$m_a^2 = m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2 \cos\theta),$$

where β_1 and β_2 are the velocities of the daughter particles ($\beta_i = v_i/c$) and θ is the angle between them.

2.7 In a collider experiment, Λ baryons can be identified from the decay $\Lambda \to \pi^- p$, which gives rise to a displaced vertex in a tracking detector. In a particular decay, the momenta of the π^+ and p are measured to be 0.75 GeV and 4.25 GeV respectively, and the opening angle between the tracks is 9°. The masses of the pion and proton are 139.6 MeV and 938.3 MeV.

(a) Calculate the mass of the Λ baryon.
(b) On average, Λ baryons of this energy are observed to decay at a distance of 0.35 m from the point of production. Calculate the lifetime of the Λ.

2.8 In the laboratory frame, a proton with total energy E collides with proton at rest. Find the minimum proton energy such that process

$$p + p \to p + p + \bar{p} + \bar{p}$$

is kinematically allowed.

2.11 Tau-leptons are produced in the process $e^+e^- \to \tau^+\tau^-$ at a centre-of-mass energy of 91.2 GeV. The angular distribution of the π^- from the decay $\tau^- \to \pi^-\nu_\tau$ is

$$\frac{dN}{d(\cos\theta^*)} \propto 1 + \cos\theta^*,$$

where θ^* is the polar angle of the π^- in the tau-lepton rest frame, relative to the direction defined by the τ (tau) spin. Determine the laboratory frame energy distribution of the π^- for the cases where the tau-lepton spin is (i) *aligned with* or (ii) *opposite to* its direction of flight.

2.12 For the process $1 + 2 \to 3 + 4$, the Mandelstam variables s, t and u are defined as $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$. Show that

$$s + u + t = m_1^2 + m_2^2 + m_3^2 + m_4^2.$$

2.13 At the HERA collider, 27.5 GeV electrons were collided head-on with 820 GeV protons. Calculate the centre-of-mass energy.

2.14 Consider the Compton scattering of a photon of momentum $\mathbf{k}$ and energy $E = |\mathbf{k}| = k$ from an electron *at rest*. Writing the four-momenta of the scattered photon and electron respectively as k' and p', conservation of four-momentum is expressed as $k + p = k' + p'$. Use the relation $p'^2 = m_e^2$ to show that the energy of the scattered photon is given by

$$E' = \frac{E}{1 + (E/m_e)(1 - \cos\theta)},$$

where θ is the angle through which the photon is scattered.