# GR - Cosmology TDs

Marco Biroli

September 10, 2020

# Chapter 1

# TD 1

## 1.1 Transverse Doppler effect and aberration.

1. ...
2. ...
3. ...

## 1.2 Relativistic invariants and conserved quantities.

1. (a) The four position is given by $\vec{x} = \begin{pmatrix} ct \\ \mathbf{x} \end{pmatrix}$ and we have that the proper time differential is given by: $\mathrm{d}t = \gamma \mathrm{d}\tau$. Hence the four velocity is given by:

$$\vec{u} = \frac{\mathrm{d}\vec{x}}{\mathrm{d}t} = \gamma \begin{pmatrix} c \\ \mathbf{u} \end{pmatrix}$$

   (b) We have the simple relation $p^\mu = mu^\mu$. Which gives:

$$\vec{p} = \begin{pmatrix} \gamma mc \\ \gamma m\mathbf{u} \end{pmatrix} = \begin{pmatrix} \frac{E}{c} \\ \gamma m\mathbf{u} \end{pmatrix}$$

   (c) The invariant built from the momentum is given by:

$$p_\mu p^\mu = \eta_{\mu,\nu} p^\mu p^\nu = -m^2c^2$$

   Which gives:

$$m^2c^4 = E^2 - |\vec{p}|^2 c^2$$

2. (a) The conserved physical quantities are the energy of the whole system and the momentum of the center of mass. Hence in other words the four momentum is conserved.

   (b) The photon will lose energy and hence it will have a lower frequency $\nu' < \nu$. The four-momenta are given by:

$$p_{e0} = (m, 0, 0, 0), p_{p0} = (E_0, 0, 0, E_0)$$

   And after the collision by:

$$p_{e1} = (E'', 0, p_y, p_z), p_{p1} = (E', 0, -E' \sin\theta, -E' \cos\theta)$$

   (c)