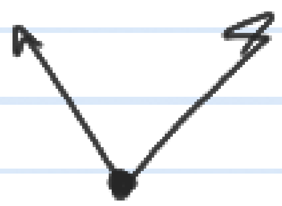
Ф.3 зад 14



$\frac{2}{3}$ туристи $\frac{1}{6}$ верен отг

$\frac{1}{3}$ местни 1

$$\mathbb{P}(\text{верен} \mid \text{наляво}) = \mathbb{P}(\text{правилн}) = \frac{2}{3}\cdot\frac{1}{6} + \frac{1}{3}\cdot 1 = \underline{\underline{\frac{4}{9}}}$$

Б) $\mathbb{P}(\text{верен} \mid \text{отговорил 2 пъти едно и също})$

$$= \frac{\mathbb{P}(\text{верен} \cap A_2)}{\mathbb{P}(A_2)} = \frac{\mathbb{P}(\text{верен} \cap A_2 \mid \text{турист})\,\mathbb{P}(\text{турист}) + \dots}{\mathbb{P}(A_2 \mid \text{турист})\,\mathbb{P}(\text{турист}) + \dots}$$

52
36

$$= \frac{\frac{2}{3}\cdot\left(\frac{1}{6}\right)^2 + \frac{1}{3}\cdot 1^2}{\frac{2}{3}\cdot\left(\left(\frac{1}{6}\right)^2 + \left(\frac{5}{6}\right)^2\right) + \frac{1}{3}\cdot 1} = \frac{2+36}{2\cdot 26+36} = \frac{38}{88}$$

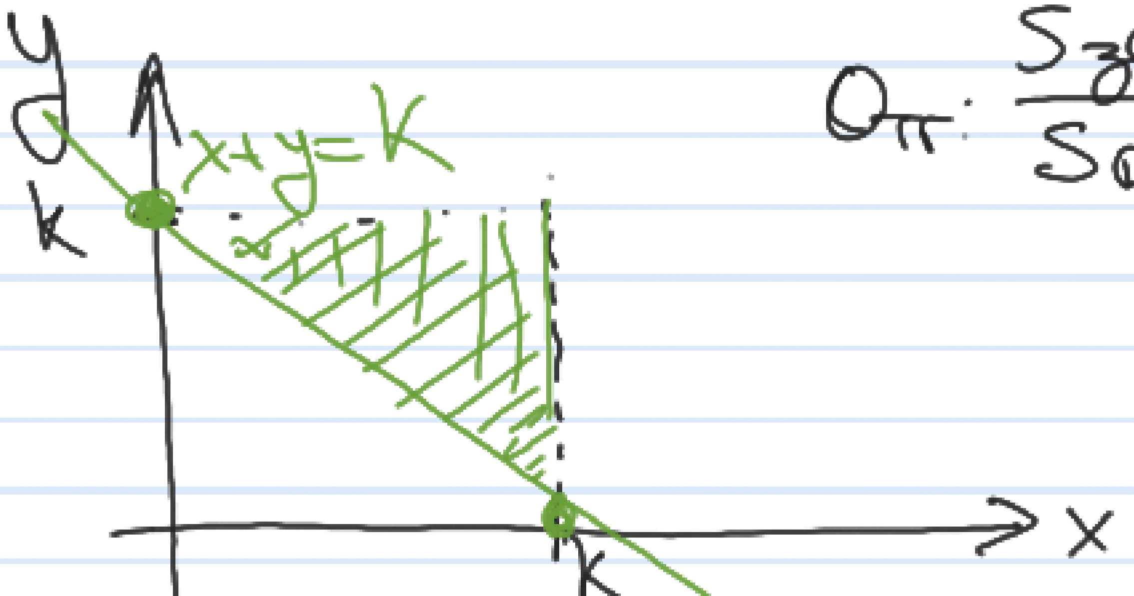
$$= \underline{\underline{\frac{19}{44}}}$$

зад. 4 $\underline{\underline{K}}$

Избираме $x, y \in [0, K]$ (1)

$P(x, y, K$ да образуват $\Delta) = ?$

Р-е: $\begin{cases} 0 \le x, y \le k \\ x+y > K \\ x+k > y \\ y+k > x \end{cases}$?



Отг: $\frac{S_{зел}}{S_{\square}} = \frac{1}{2}$

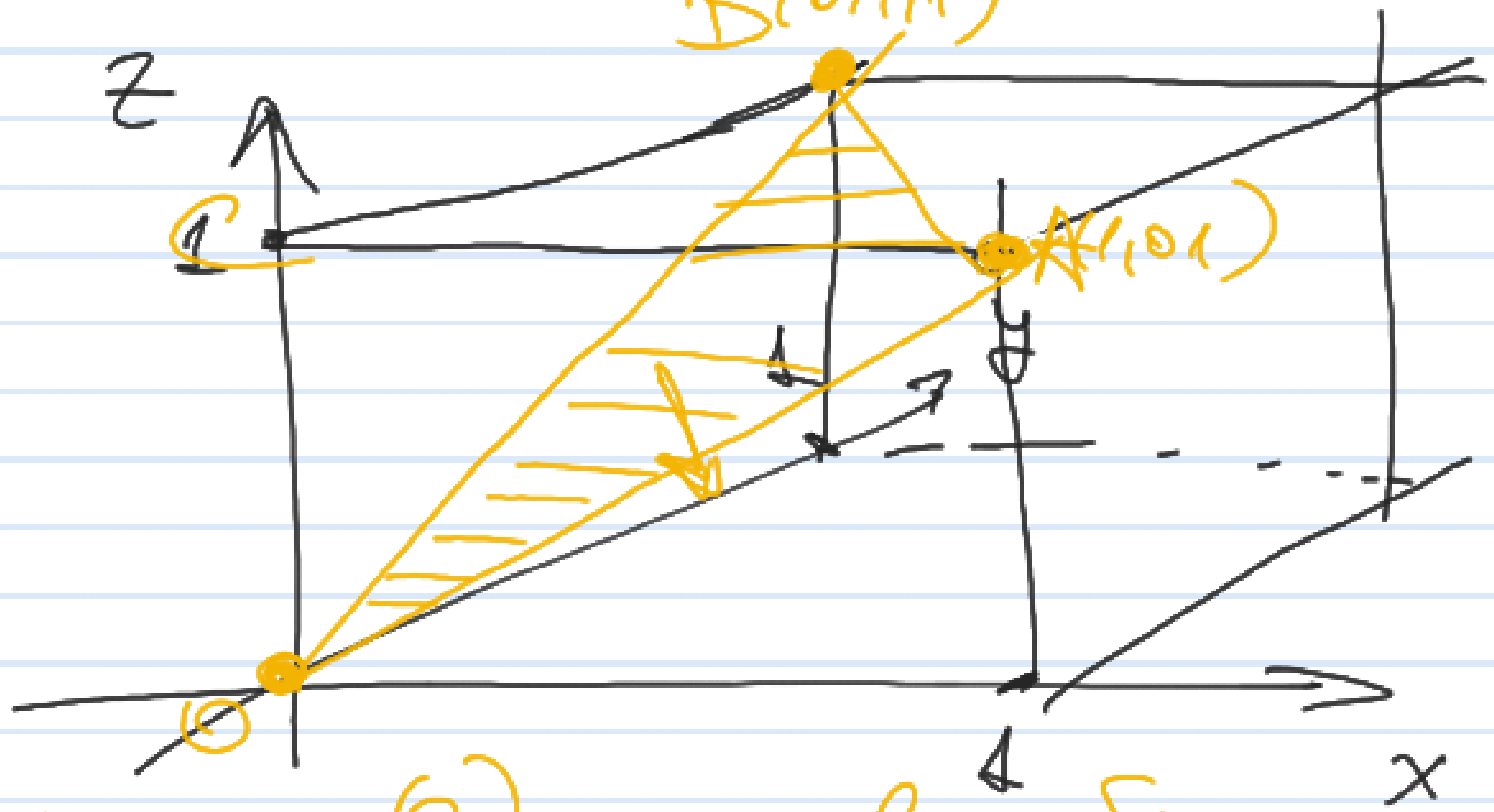
зад. 5 $0 \le x, y, z \le 1$ (1)

$P(x, y, z$ да образуват $\Delta) = ?$

Р-е: (1) $\begin{cases} x+y > z \\ x+z > y \\ y+z > x \end{cases}$ (2)

$x+y-z=0$

0	1	1
1	0	1



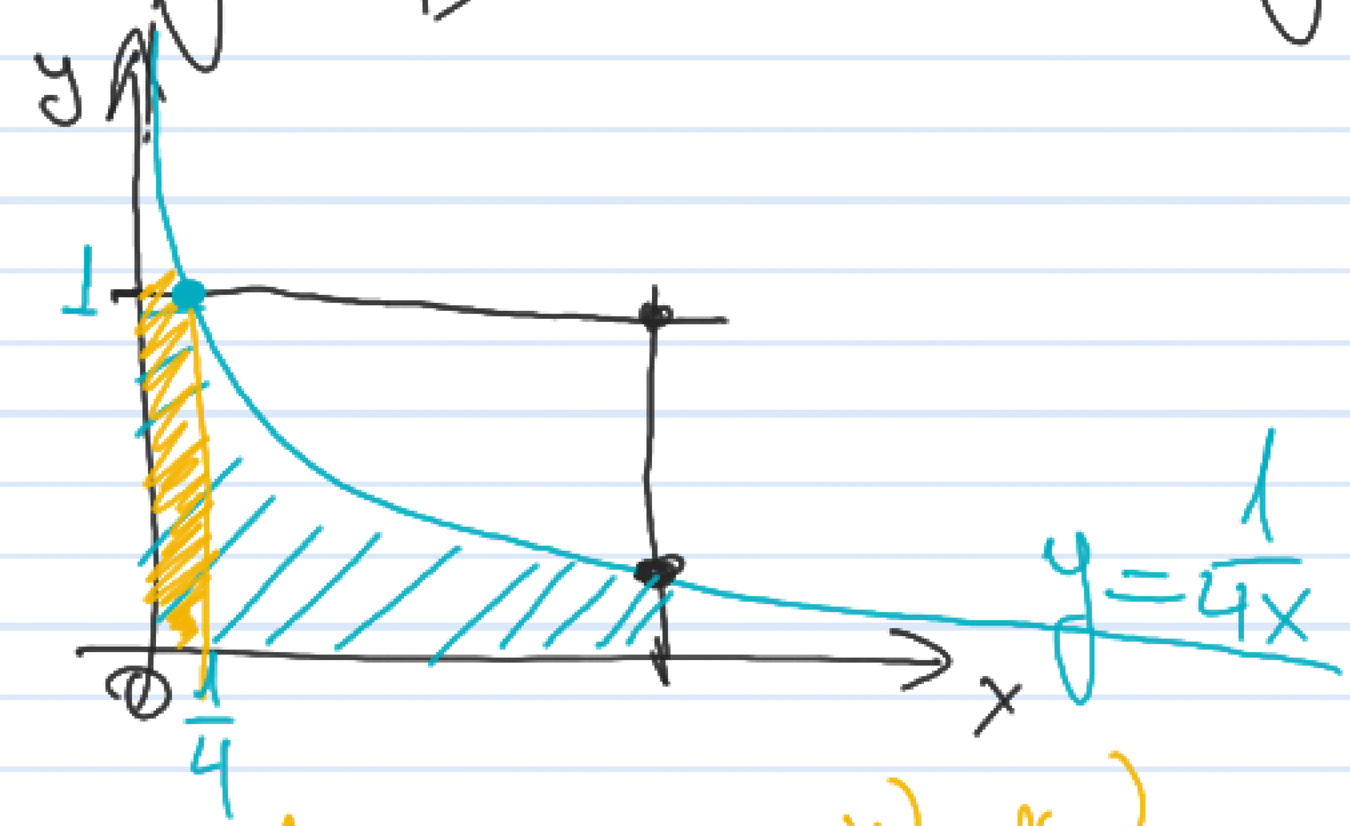
Ограничение (2) премахва обем от $\frac{1}{6}$ — пирам. OABC

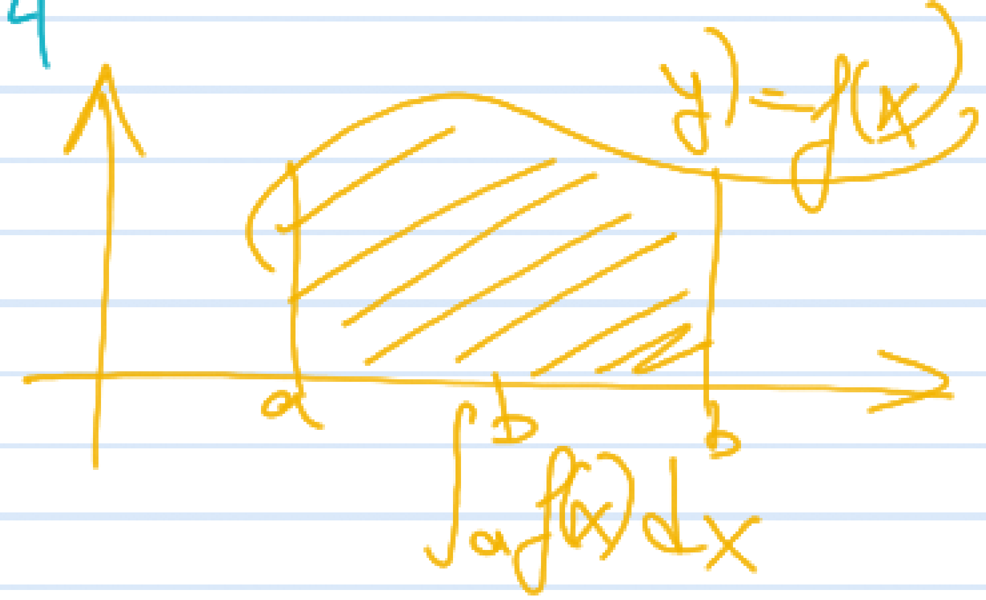
Anal другите две от махат също по $\frac{1}{6}$

Отг: $\dfrac{1-3\cdot\frac{1}{6}}{1} = \dfrac{1}{2}$

зад. 7 $x, y \in (0,1]$

$P\left(xy \le \frac{1}{4}\right) = ?$ $\quad y \le \dfrac{1}{4x}$





Отговор: $\dfrac{1}{4} + \displaystyle\int_{1/4}^{1} \frac{1}{4x}\,dx$

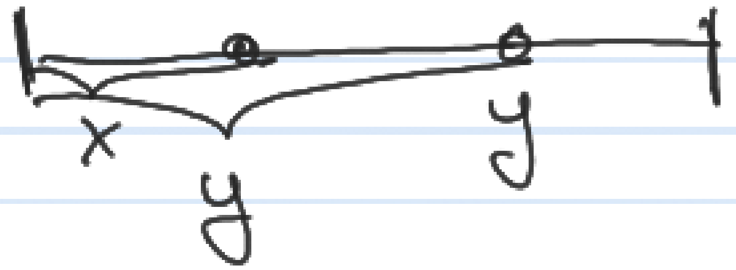
$= \dfrac{1}{4}\left(1 + \ln x \Big|_{1/4}^{1}\right)$

$= \dfrac{1}{4}(1+\ln 4)$.

$\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1}$ за $n \neq -1$

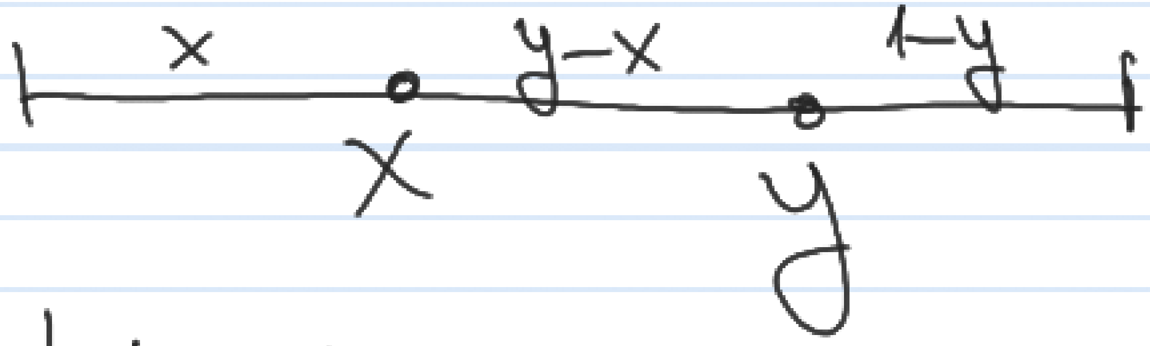
$\displaystyle\int \frac{1}{x}\,dx = \ln x, \quad \int e^x\,dx = e^x$

зад.8 отсечка на 3 части

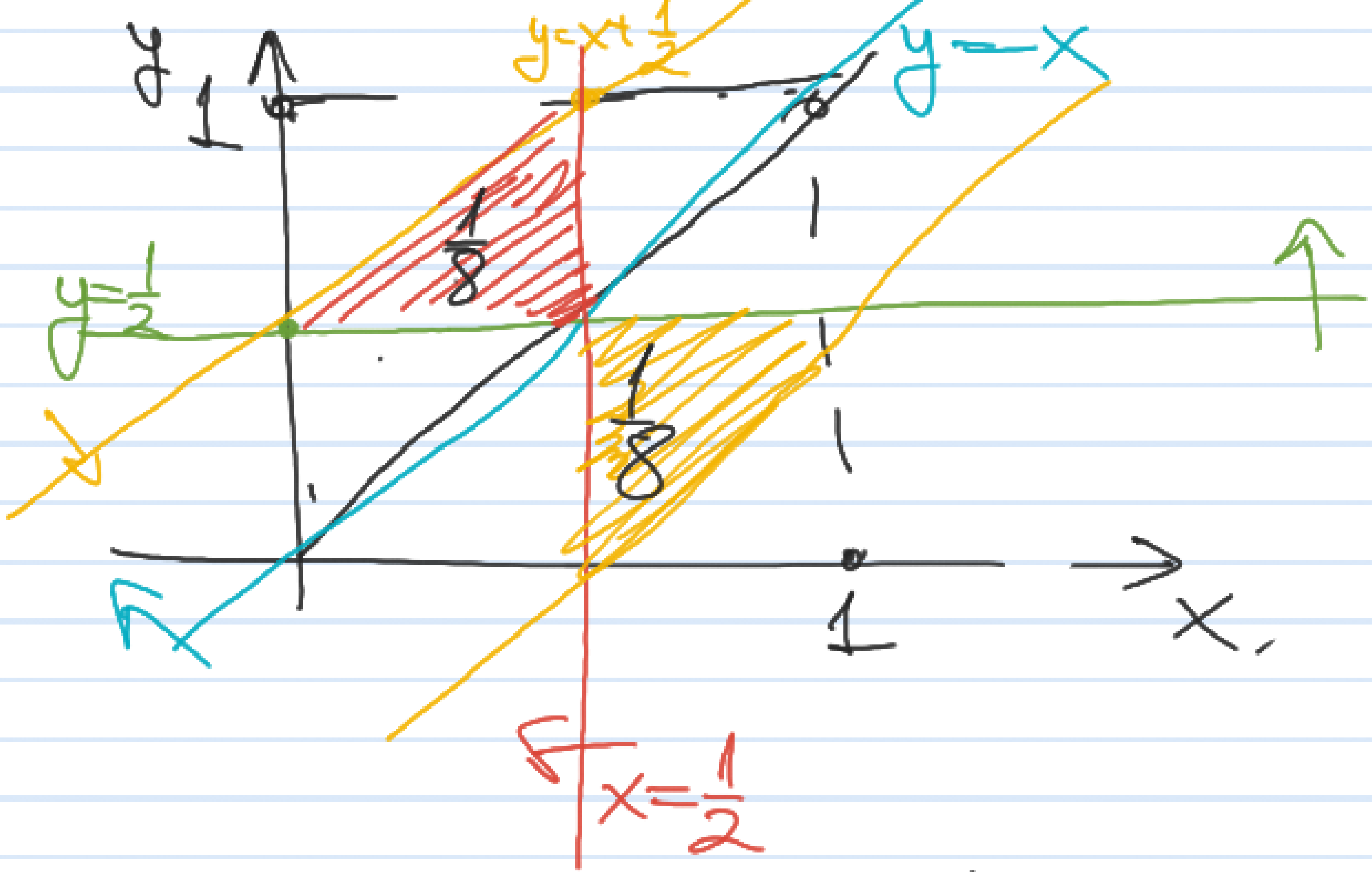


x, y са случ. м/у 0 и 1

P(да можем да обр. Δ от 3-те части)

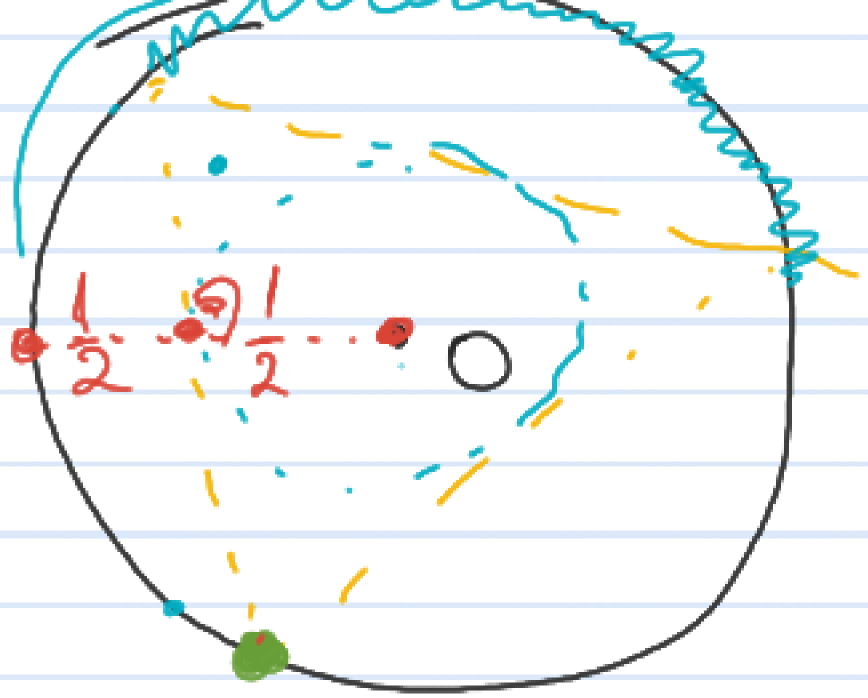


$$\begin{cases} y > x \\ x+(y-x) > 1-y \iff y > \frac{1}{2} \\ x+(1-y) > y-x \iff 2y < 2x+1 \\ (\cancel{y}-x)+(1-\cancel{y}) > x \iff x < \frac{1}{2} \end{cases}$$

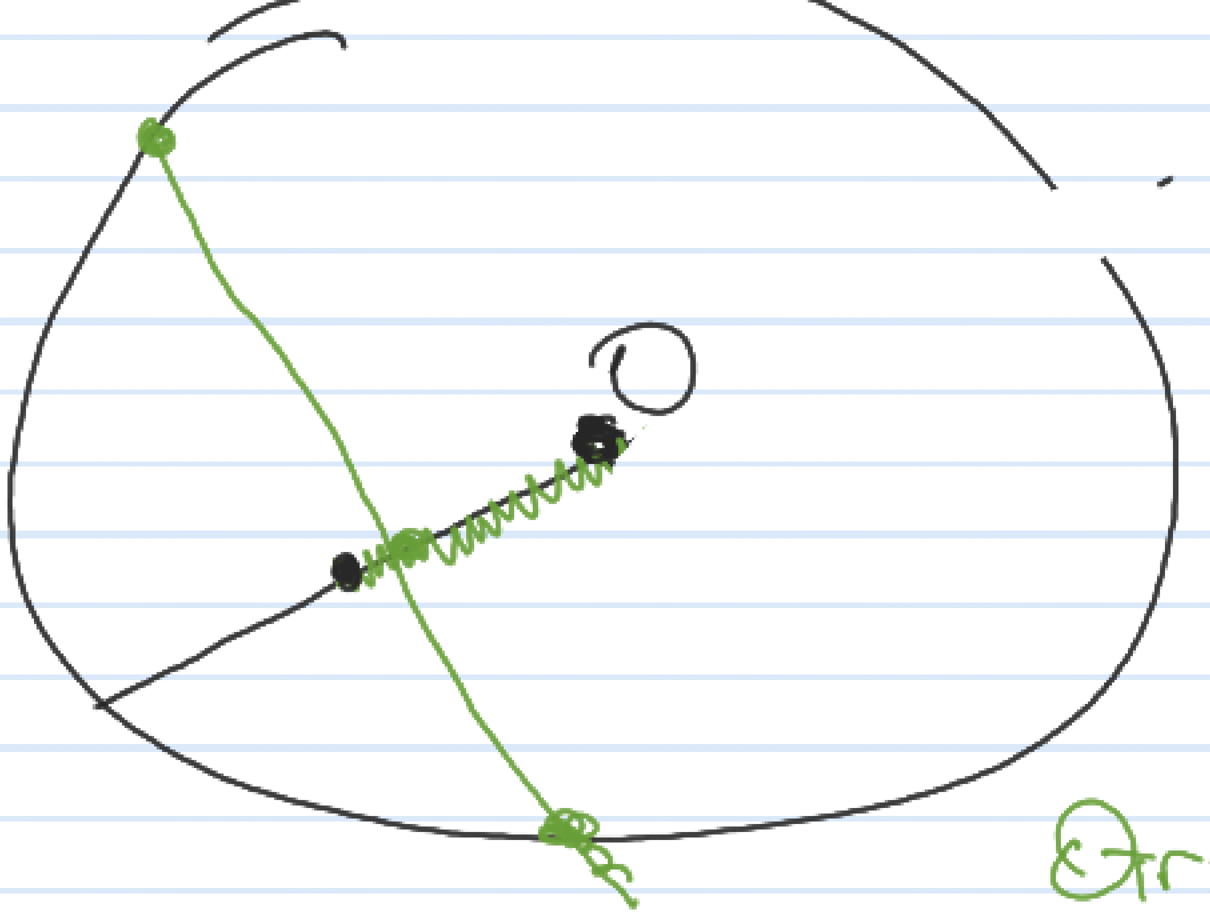


Отг: $\dfrac{2\cdot\frac{1}{8}}{1} = \dfrac{\frac{1}{8}}{\frac{1}{2}}$

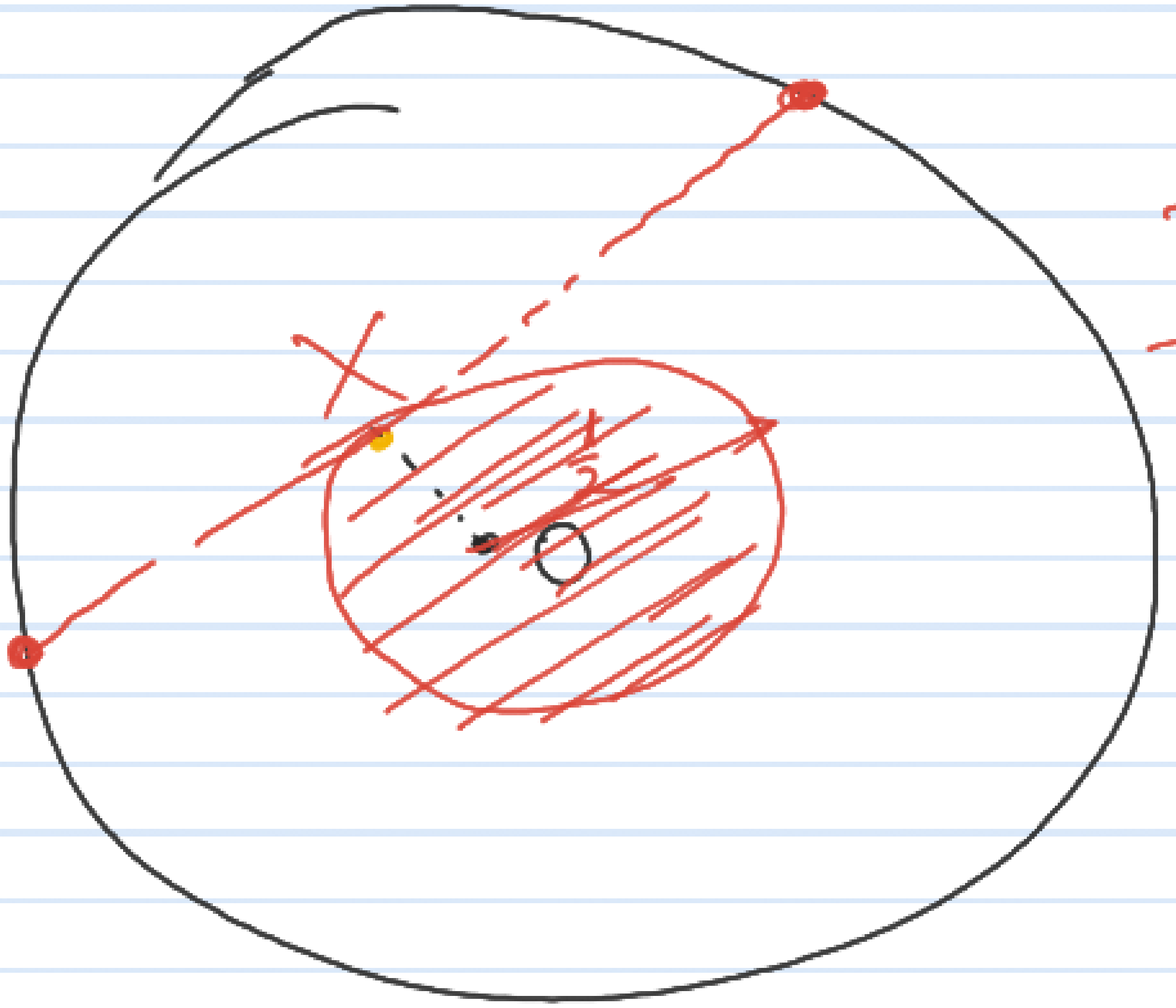
зад. 9



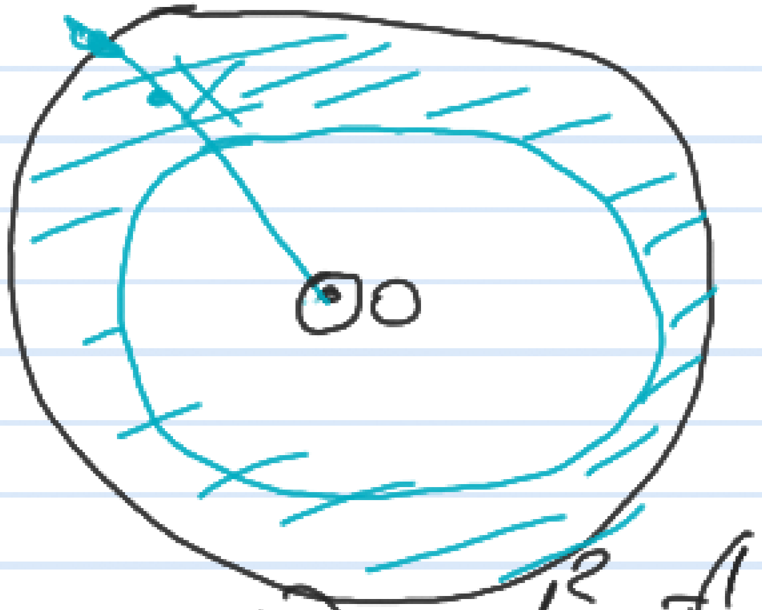
Отг: $\frac{1}{3}$

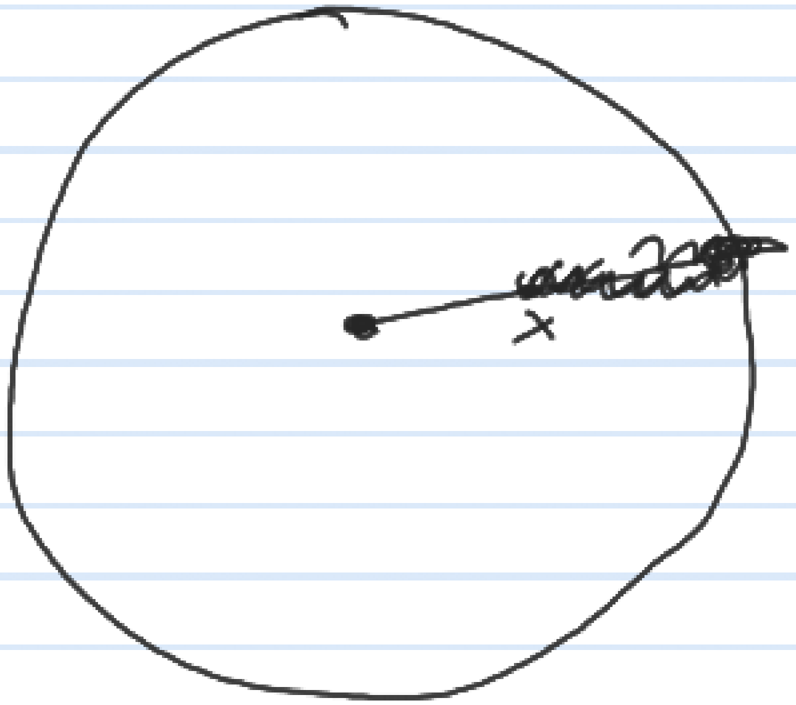


Отг: $\frac{1}{2}$



$$\frac{\pi\left(\frac{1}{2}\right)^2}{\pi 1^2} = \frac{1}{4}$$





$$P\left(OX > \frac{1}{2}\right) = \frac{\pi 1^2 - \pi\left(\frac{1}{2}\right)^2}{\pi 1^2}$$

$$= \frac{3}{4}$$

до 11:30

$(\underset{\text{ел. сб.}}{\Omega}, \underset{\text{сб.}}{\mathcal{A}}, \mathbb{P})$

$\mathbb{P} : \mathcal{A} \to [0,1]$

X (random variable)

$X : \Omega \to \mathbb{R}$

Хвърляме 2 пъти монета

$\Omega = \{(E,E), (T,E), (T,T), (E,T)\}$

$\mathcal{A} = 2^{\Omega}$

$X =$ „брой езита"

$X((E,E)) = 2$, $X((T,T)) = 0$

$X((E,T)) = 1$, $X((T,E)) = 1$

$$\mathbb{P}(X = 1) = \mathbb{P}(\{\omega : X(\omega) = 1\})$$

$$= \mathbb{P}(\{(E,T), (T,E)\}) = \frac{1}{2}$$

дискр. сл. вел. – X приема краен или изб. безкр.

$$\left.\begin{array}{c|c|c|c} X & 0 & 1 & 2 \\ \hline \mathbb{P}(X = \cdot) & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{array}\right\} \text{разпред. на } X$$

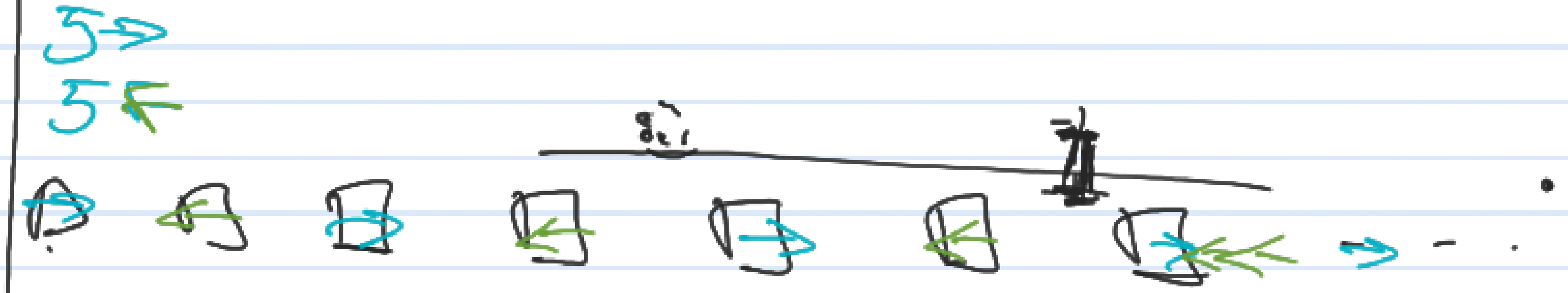
X	x_1	x_2	x_3	...
	p_1	p_2	p_3	...

$$\mathbb{E}X = x_1p_1 + x_2p_2 + \dots$$

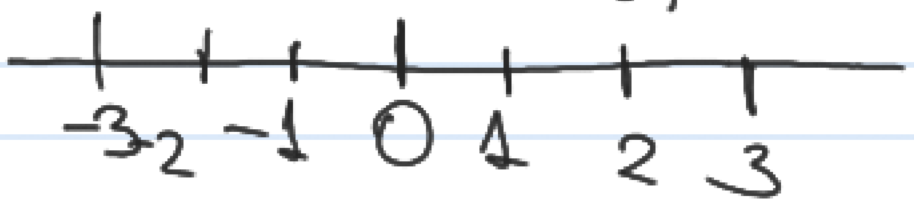
$$\mathbb{D}X = \mathbb{E}(X-\mathbb{E}X)^2$$
$$= \mathbb{E}X^2 - (\mathbb{E}X)^2$$

100	-30
$\frac{1}{2}$	$\frac{1}{2}$

$$\frac{1}{2}\cdot 100 + \frac{1}{2}(-30)$$



§5 зад 1 $\rightarrow$ ези, $\leftarrow$ тура, 10 хв.



X = „крайната позиция"

а) $\mathbb{P}(X=0) = ?$

$$\mathbb{P}(X=0) = \mathbb{P}(5 \text{ ези и } 5 \text{ тура})$$
$$= \frac{\binom{10}{5}}{2^{10}} = \binom{10}{5}\left(\frac{1}{2}\right)^{10}$$

б) $\mathbb{P}$(да е на 2 крачки от началото) $=$

$$\mathbb{P}(X=2 \text{ или } X=-2) = \mathbb{P}\left(\begin{matrix}6\rightarrow, 4\leftarrow \text{ или} \\ 6\leftarrow, 4\rightarrow\end{matrix}\right)$$

$= \left(\frac{1}{2}\right)^{10} \binom{10}{6} + \left(\frac{1}{2}\right)^{10} \binom{10}{4}$

$= 2 \cdot \left(\frac{1}{2}\right)^{10} \binom{10}{6}$

в) $P(X=5) = 0$

$m \rightarrow$

$n \leftarrow$

$\begin{cases} m+n=10 \\ m-n=5 \end{cases}$

$\Rightarrow 2m = 15$ – н.р. в $\mathbb{N}_0$.

$X \sim Ber(p)$

X следва разпр. на Бернули спар p, ако $P(X=1) = p$

$P(X=0) = 1-p$

X	0	1
	$1-p$	p

$\mathbb{E}X = 0 \cdot (1-p) + 1 \cdot p = p$

$DX = \mathbb{E}X^2 - (\mathbb{E}X)^2$

$= 0^2 \cdot (1-p) + 1^2 \cdot p - p^2$

$= p - p^2$

$X \sim Bin(n, p)$ – # успехи

n опита с вер. за успех p

* 10 опита с вер за успех $\frac{1}{5}$

$$P(X=3) = \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^7 \cdot \binom{10}{3}$$

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✓ ✓ ✓ ✗ ✗ ✗ ✗ ✗ ✗ ✗

X	0	1	2 …	n

$$P(X=k) = p^k (1-p)^{n-k} \binom{n}{k}$$

$1-p=q$

* $E(X+Y) = EX + EY$

$$EX = \sum_{k=0}^{n} k \cdot p^k (1-p)^{n-k} \binom{n}{k}$$

n

$$X \overset{d}{=} X_1 + X_2 + \dots + X_n$$

1, ако е усп.

0, ако не е

т.е. $X_i \sim Ber(p)$

$$\mathbb{E}X = \mathbb{E}(X_1 + \dots + X_n)$$

$$= \mathbb{E}X_1 + \dots + \mathbb{E}X_n$$

$$= n \cdot p$$

$$\mathbb{D}X = n \cdot \mathbb{E}X_1 = np(1-p)$$