

# Final Answers

## MATH110 December 2012

### How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the [Math Educational Resources](#).

### Tips for Using Previous Exams to Study: Work through problems

*Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.*

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
  - To answer each question, only use what you could also use in the exam. [Download the raw exam here](#).
  - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
  - If your answer is correct: good job! Move on to the next question.
  - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the [Math Educational Resources](#).
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the [full exam \(click here\)](#).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify [mer-wiki@math.ubc.ca](mailto:mer-wiki@math.ubc.ca). Your feedback helps us improve.

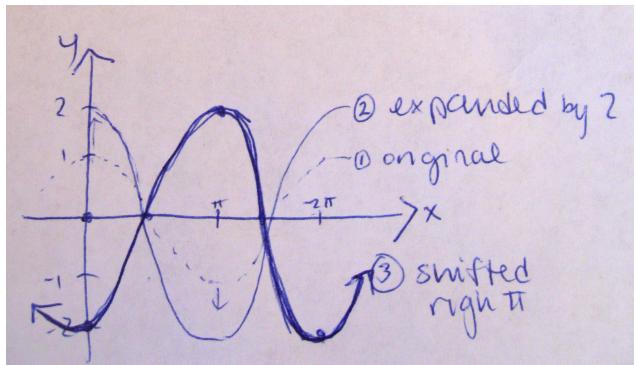
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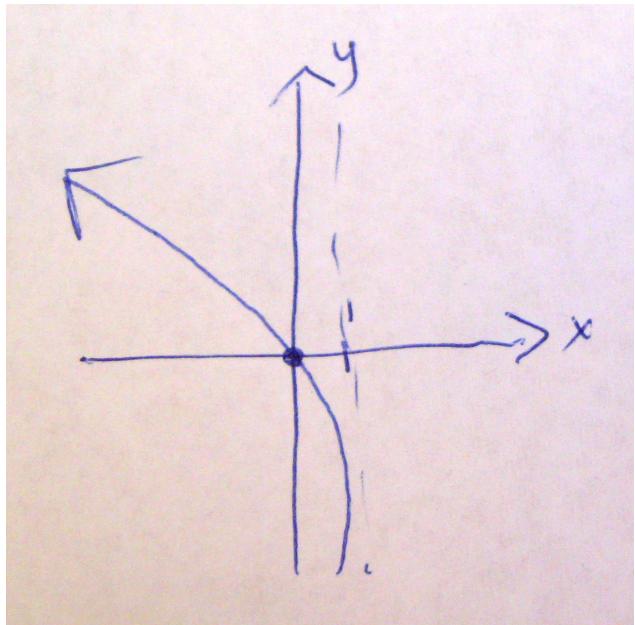


### Question 1 (a)



FINAL ANSWER.

### Question 1 (b)



FINAL ANSWER.

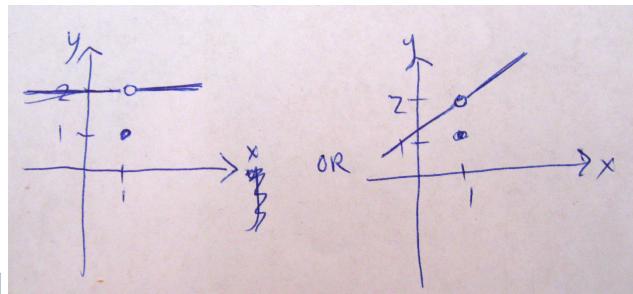
### Question 2 (a)

FINAL ANSWER.  $\lim_{x \rightarrow 0} \frac{1}{x} + (-\frac{1}{x}) = 0$  Which is a perfectly satisfactory limit. Thus, these two functions are a counterexample to this statement, proving it false. To see an interactive illustration of this solution, see this page.

### Question 2 (b)

FINAL ANSWER.  $\lim_{x \rightarrow 2} \frac{1}{2+x} = \frac{1}{4}$  Which matches the right hand side of the original equation. Thus the statement is true.

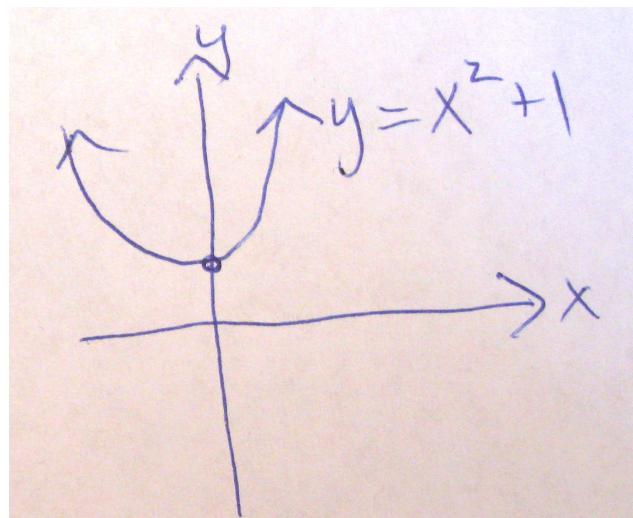
### Question 2 (c)



FINAL ANSWER.

Because the conclusion of the statement is not necessarily true, the statement is false.

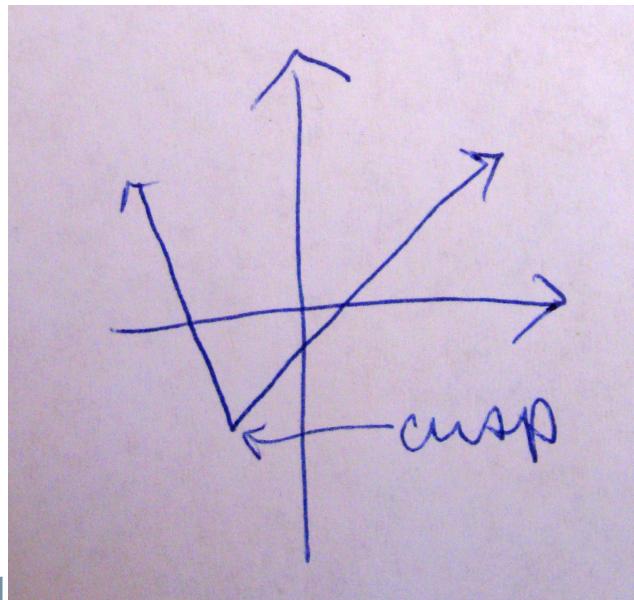
### Question 2 (d)



FINAL ANSWER.

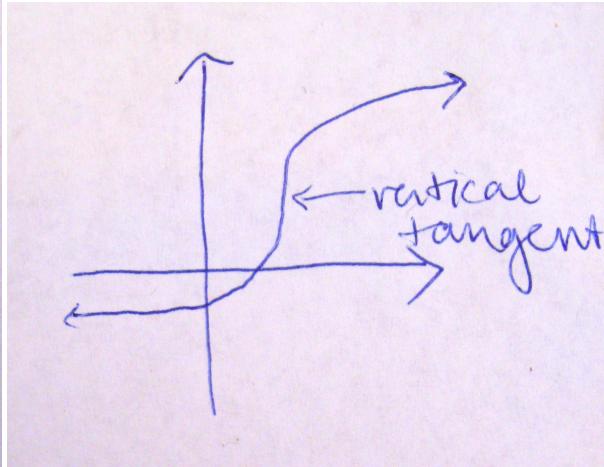
Either of these would be a counterexample, so the statement is false.

### Question 2 (e)



**FINAL ANSWER.**

a function can be continuous, without necessarily being differentiable. Thus the statement is false.



### Question 2 (f)

**FINAL ANSWER.** Thus  $y = \sqrt{x}$  has no horizontal tangent lines, so the statement is true.

### Question 3 (a)

**FINAL ANSWER.** So the value of  $f'(9)$  is  $37/3$ .

### Question 3 (b)

**FINAL ANSWER.**  $f'(x) = \frac{-b}{x^2} + \frac{-2c}{x^3} + \frac{-3d}{x^4}$

### Question 3 (c)

**FINAL ANSWER.** You can leave this answer as is.

### Question 3 (d)

**FINAL ANSWER.**  $f'(\theta) = (\cos \theta)/2$  completing the question.

### Question 4

**FINAL ANSWER.** Because  $f(1) < 0 < f(2)$ , we can now use the IVT to conclude that there exists a number  $c$  in the interval  $(1,2)$  such that  $f(c) = 0$ .

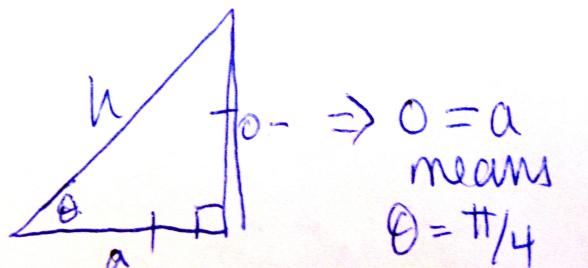
### Question 5

**FINAL ANSWER.** So  $f'(x) = \frac{-2}{x^3}$

### Question 6

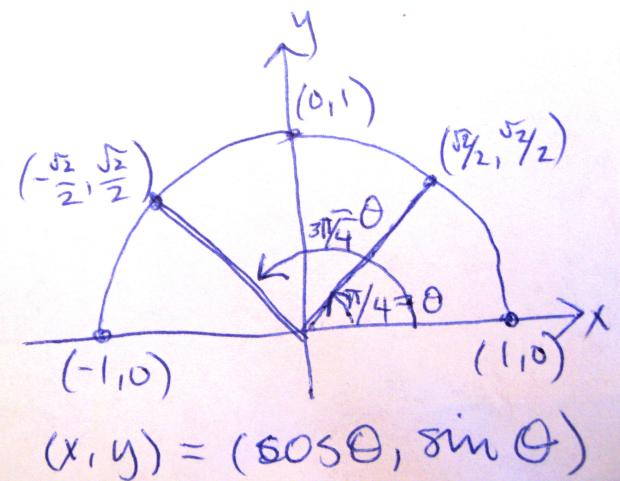
FINAL ANSWER. So  $f(x)$  will be differentiable when  $a = 1$  and  $b = -1/2$  = will be differentiable when  $a = 1$  and  $b = -1/2$

### Question 7 (a)



FINAL ANSWER.

### Question 7 (b)



### Question 8

FINAL ANSWER. This is a sufficient answer, but if you simplify to  $y=mx + b$  form, you find  $L(x) = -100x + 20$

### Question 9 (a)

FINAL ANSWER. If the claim is true for  $n = k$  (that is,  $2^1 + 2^2 + \dots + 2^k = 2^{k+1} - 1$ ), then the statement must be true for  $n = k + 1$  (meaning  $2^1 + 2^2 + \dots + 2^k + 2^{k+1} = 2^{k+2} - 1$ ). This fact basically states that if the claim holds for some integer, it is also true for the consecutive integer.

### Question 9 (b)

**FINAL ANSWER.** We can continue this process until we get to  $n = 6$ , which is the value of  $n$  shown in the statement of the question.(Note that you can prove the above statement is true simply by doing the calculations - but this does not use the inductive reasoning steps described in part a).

### Question 9 (c)

**FINAL ANSWER.** Hence we have proved that if the claim holds for  $n = k$ , it also holds for  $n = k + 1$ , proving Fact 2.

### Question 10

**FINAL ANSWER.** Thus, the rate at which the area of the circle is increasing is  $180,000\pi$  km/h.