

a : lower bound of interval
 b : upper bound of interval
 n : number of bins, $n \geq 1$
 c : bin width
 r : bin overlap ratio
 \bar{r} : bin non-overlap ratio
 s : bin width to interval length ratio

r_k : bin k -overlap ratio, $k \geq 1$

a_i : bin lower bound, $i = 1, \dots, n$

b_i : bin upper bound, $i = 1, \dots, n$

γ : Ayasdi gain

$$a < b$$

$$0 < c \leq b - a$$

$$0 \leq r < 1$$

$$a = a_1 < \dots < a_n < b$$

$$a < b_1 < \dots < b_n = b$$

$$b_i - a_i = c, \quad i = 1, \dots, n$$

$$b_i - a_{i+1} = rc, \quad i = 1, \dots, n-1$$

$$r_1 = r$$

$$b_i - a_{i+k} = r_k c, \quad i = 1, \dots, n-1, \quad k \geq 1$$

$$a_{i+1} - a_i = b_{i+1} - b_i = (1-r)c, \quad i = 1, \dots, n-1$$

$$a_i = a + (i-1)(1-r)c, \quad i = 1, \dots, n$$

$$b_i = b - (n-i)(1-r)c, \quad i = 1, \dots, n$$

$$c = b_1 - a_1$$

$$= b_1 - a$$

$$= b - (n-1)(1-r)c - a$$

$$b - a = (1 + (n-1)(1-r))c$$

$$= (1 + (1-r)n - (1-r))c$$

$$= (r + (1-r)n)c$$

$$\boxed{b - a = (r + (1-r)n)c}$$

$$\boxed{n = \frac{1}{1-r} \left(\frac{b-a}{c} - r \right) = \frac{1}{1-r} \left(\frac{1}{s} - r \right) = 1 + \left(\frac{1}{s} - 1 \right) \gamma}$$

$$\boxed{c = \frac{b-a}{r + (1-r)n}}$$

$$\boxed{r = \frac{1}{n-1} \left(n - \frac{b-a}{c} \right) = \frac{1}{n-1} \left(n - \frac{1}{s} \right)}$$

$$\boxed{s = \frac{1}{r + (1-r)n} = \frac{1}{1 + (n-1)/\gamma}}$$

$$\begin{aligned}
r_k c &= b_i - a_{i+k} \\
&= (b - (n - i)(1 - r)c) - (a + (i + k - 1)(1 - r)c) \\
&= (b - a) - ((n - i) + (i + k - 1))(1 - r)c \\
&= (r + (1 - r)n)c - (n + k - 1)(1 - r)c \\
r_k &= r + (1 - r)n - (n + k - 1)(1 - r) \\
&= 1 - k(1 - r)
\end{aligned}$$

$$r_k = 1 - k(1 - r) = 1 - k/\gamma$$

$$r_k < 0, \forall k \geq 2 \Leftrightarrow r_2 < 0 \Leftrightarrow r < 0.5 \Leftrightarrow \gamma < 2$$

$$\bar{r} = 1 - 2r = -r_2$$

The bin non-overlap ratio \bar{r} is a sensible indicator only if $r < 0.5$, i.e. $\gamma < 2$.

With $r \geq 0.5$, the inner bins (indices $i = 2, \dots, n - 1$) have no proper elements.

A few parameter settings of interest

γ	r	\bar{r}	r_2
1.25	20.0%	60%	·
1.60	37.5%	25%	·
2.00	50.0%	00%	00%
2.50	60.0%	·	20%

