UNIVERSITY OF GENOVA

DEPARTMENT OF COMPUTER SCIENCE, BIOENGINEERING, ROBOTICS AND SYSTEM ENGINEERING (DIBRIS)

COMPUTER VISION

LAB SESSION N.6

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Introduction

The purpose of this lab is to build the 8-point algorithm for estimating the fundamental matrix F, which is an important component in computer vision since it represents the geometric relationship between two camera perspectives of a scene. Initially, we will implement and test the 8-point algorithm on a set of predetermined points. Later, we will analyze how point normalization improves this algorithm's efficiency. Finally, we will use the SURF algorithm, a matching point algorithm, to compute point of interest. Eventually, we will use these points in the RANSAC function to estimate the optimal fundamental matrix for stereo pictures.

Chapter I: Eight Point Algorithm

The fundamental matrix F is the unique 3x3 rank 2 matrix built in a way that, for each corresponding pair (x_1, x_2), we have:

$$x_2^\top \cdot F \cdot x_1 = 0 \tag{1.1}$$

This formula forces the vector to satisfy the epipolar constraint. $x_1 = [x_1, y_1, 1]^T$ and $x_2 = [x_2, y_2, 1]^T$ are the projections expressed in homogeneous coordinates of the same 3D point in the plane of the cameras. Therefore, the fundamental matrix F is computed starting from the affinity matrix A, which is based on the epipolar constraint (Equation 1.1) and validates the constraint $A \cdot f = 0$. In matrix form:

$$\begin{bmatrix} x'_1x_1 & x'_1y_1 & x'_1 & y'_1x_1 & y'_1y_1 & y'_1 & x_1 & y_1 & 1 \\ x'_2x_2 & x'_2y_2 & x'_2 & y'_2x_2 & y'_2y_2 & y'_2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_nx_n & x'_ny_n & x'_n & y'_nx_n & y'_ny_n & y'_n & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0} \tag{1.2}$$

The *EightPointsAlgorithm* function first computes the affinity matrix A using the input point correspondences in homogeneous coordinates. Then, it performs the Singular Value Decomposition (SVD) of A and gets as a solution the last column of V (since $A = UDV^\top$). This column is reshaped into a 3×3 matrix in order to create the fundamental matrix F. Finally, the rank of F is constrained to 2 by setting the smallest singular value (σ_3) to zero before recomputing F.

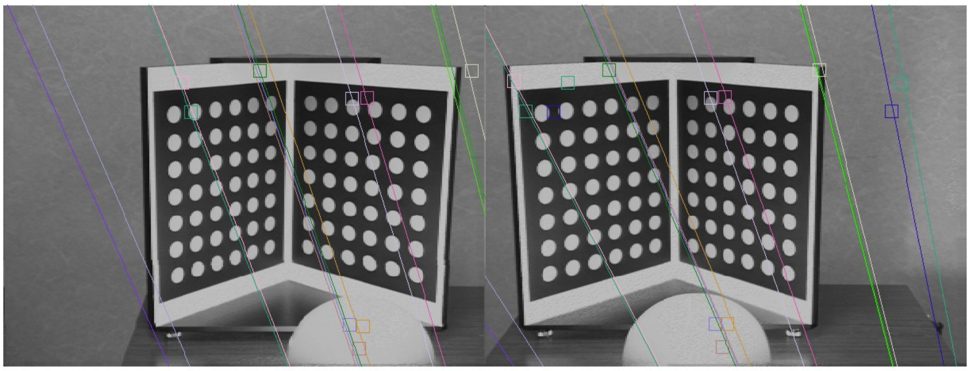

Figure 1.1
Epipolar lines found by EightPointsAlgorithm without points normalisation

The *EightPointsAlgorithmN* function normalizes the input points using the *normalise2dpts* function, applies the *EightPointsAlgorithm*, computes F and eventually denormalizes the result.

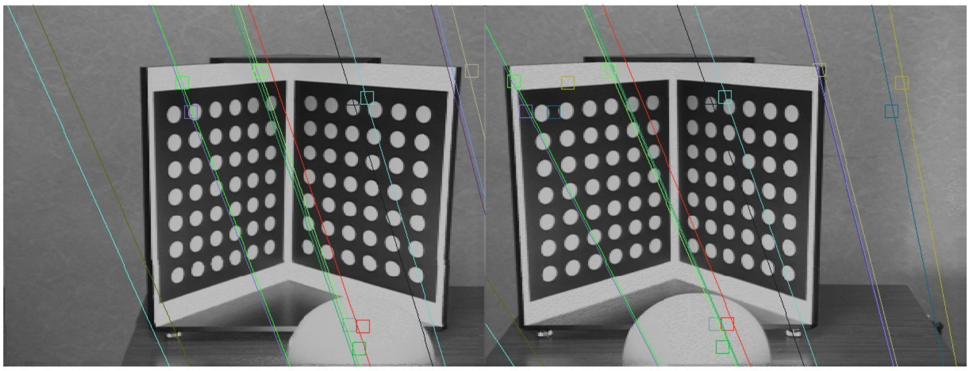

Figure 1.2
Epipolar lines found by EightPointsAlgorithmN with points normalisation

EightPointsAlgorithm and *EightPointsAlgorithmN* are now compared through the epipolar constraint in the *checkEpipolarConstraint* function. This function demonstrates that the normalized version of the algorithm generates a fundamental matrix F that better satisfies the epipolar constraint; in fact, all of the matrix's elements from Equation 1.1) fall below the established threshold of 10^{-2}. This is correct since normalization reduces numerical errors.

$$F = \begin{bmatrix} 0.00 & 0.00 & -0.003 \\ 0.00 & 0.00 & 0.002 \\ 0.005 & -0.002 & -1.00 \end{bmatrix} \tag{1.3}$$

(a) Without Normalization

$$F = \begin{bmatrix} -0.00 & 0.00 & -0.0003 \\ -0.00 & 0.00 & 0.0042 \\ 0.0003 & -0.0042 & -0.0356 \end{bmatrix} \tag{1.4}$$

(b) With normalization

Figure 1.3
Fundamental matrices F with and without normalization.

Both *EightPointsAlgorithm* and *EightPointsAlgorithmN* have been tested starting from a set of predefined points in order to validate their effectiveness without the influence of other factors. The outcome of *EightPointsAlgorithm* is shown in figure 1.1, where the plot of the epipolar lines are correctly aligned with the points, with a slight margin of error. The *EightPointsAlgorithmN* provides a slightly better result where the epipolar lines fit the points more accurately, as visible in figure 1.2.

From the fundamental matrix F, it is possible to compute also the left and right epipole by performing its SVD decomposition and selecting the last column of the U matrix (left epipole) and the last column of the V matrix (right epipole). The results are shown in Figure 1.4 and Figure 1.5.

Left (0.449, 0.894, 0.001)
Right (-0.409, -0.913, -0.001)

Figure 1.4
Epipoles without point normalization.

Left (0.997, 0.071, 0.000)
Right (-0.998, -0.067, -0.000)

Figure 1.5
Epipoles with point normalization.

Chapter II: Stereo Images

The second part of the lab consists in applying the algorithm developed in the first part starting from a pair of stereo images. In fact, retrieving 3D information, such as structure and distances, from two or more images taken from different viewpoints is a significant problem in computer vision.

A find matches algorithm that uses SURF finds some corresponding points on two stereo images previously uploaded (Figure 2.1); the *find_matches* algorithm detects interesting points (such as corners) and associates them to a vectorial description which is invariant to translation, scale and rotation changes; a sigma value of 20 was chosen for the SURF algorithm, as it is sufficiently large to avoid being overly restrictive in identifying matching points.

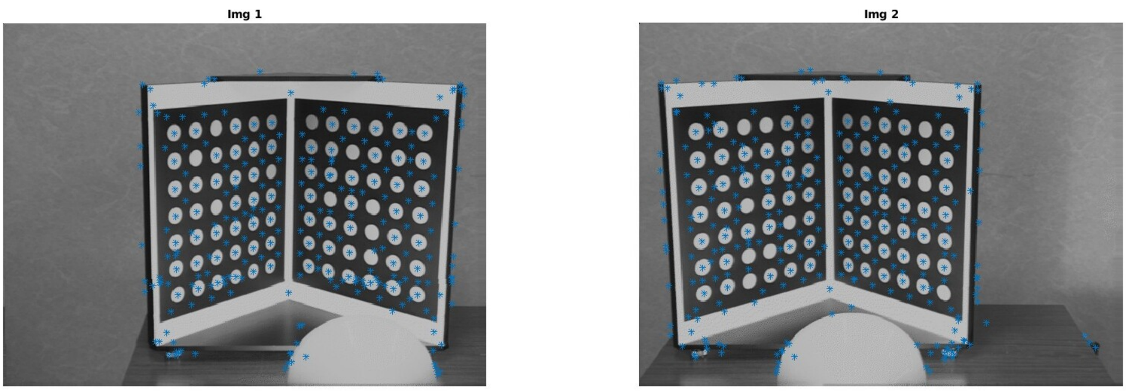

Figure 2.1
Matching point in stereo images

The points found using this algorithm are then transformed into homogeneous coordinates and given as inputs to a RANSAC algorithm called *ransacF*. Here, eight random pairs of points are selected and the *EightPointsAlgorithmN* is iterated to compute the optimal F. In fact, if the number of inliers grows, the best fundamental matrix is updated and if not, a new iteration is performed to a maximum of 1000 iterations. An example of fundamental matrix is shown in Figure 2.2.

$$F = \begin{bmatrix} 0.00 & 0.00 & 0.007 \\ 0.00 & 0.00 & 0.005 \\ -0.003 & -0.002 & -1.15 \end{bmatrix} \tag{2.1}$$

Figure 2.2
Fundamental matrix F with RANSAC algorithm.

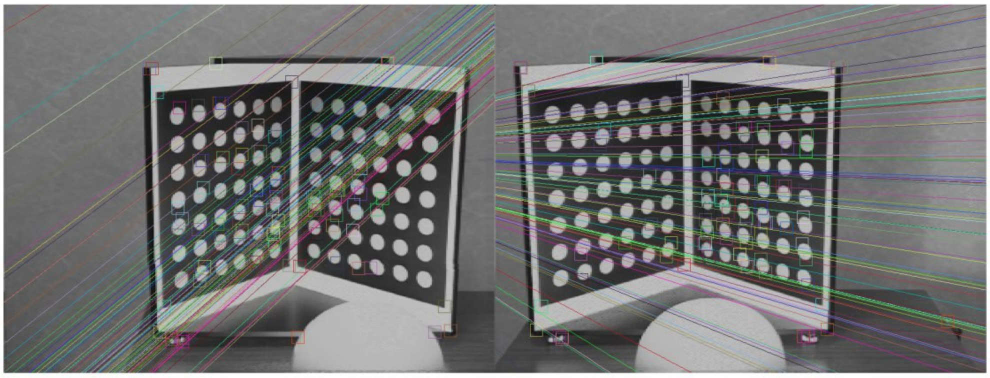

Figure 2.3
Epipolar lines found by ransacF and with point normalization

We used this fundamental matrix F to compute the epipolar lines (as shown in Figure 2.3) and the epipoles (Figure 2.4).

Left (-0.960, 0.281, 0.002)	**Right** (0.995, 0.097, 0.000)

Figure 2.4
Epipoles with RANSAC algorithm.

Finally, we used two hand taken stereo images to test the RANSAC algorithm. Ideally, the epipolar lines should pass through the matching points computed with only slight errors due to approximation. However, the RANSAC function used has some implementation problems. As a result, the calculated fundamental matrix F is not the ideal one, causing the epipolar lines to converge in a single point. Despite this, when the fundamental matrix is properly predicted, the result we obtain are satisfying, since the points in the two images matches exactly and the epipolar lines are correctly aligned, too. This is visible in Figure 2.5.

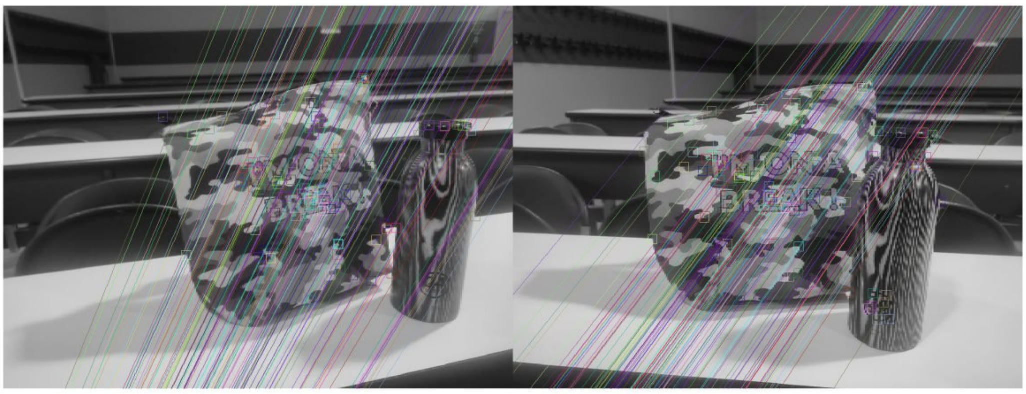

Figure 2.5
Epipolar lines computed by `ransacF` on manually captured images.

Conclusions

The 8-point algorithm is an important technique for calculating the fundamental matrix, which expresses the epipolar constraint between points in two camera perspectives. Normalizing the input points increases the numerical stability and the accuracy of the algorithm; in fact, the epipolar constraint $A \cdot f = 0$ is better satisfied this way. The efficiency improves even more using points of interest, such as corners.
The *EightPointsAlgorithm* also finds application in other algorithms, such as in the RANSAC algorithm, where it can be used iteratively to determine the optimal fundamental matrix; this shows its full potential and its versatility.