CVRP: From Mathematical Models to a Practical Study Case

Reto MA2001B - Optimización determinista
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***Abstract*—The e-commerce industry has been developing extremely rapid over the last couple of years. This report proposes a solution to the problem of distributing products to customers, in collaboration with Coppel and their e-commerce department. The solution proposed was modeled with two specific goals in mind: customer satisfaction and time efficiency.**

The model was designed taking into account the particular requirements the company has, specifically, for its distribution Center (DC) located in Guadalajara, Jalisco Mexico. The solution proposed applies linear programming models and uses known algorithms and heuristics to solve the capacity vehicle routing problem (CVRP), strongly related with the problem that is solved in this paper.

The generated routes with the methods proposed show to be faster and more optimal than those Coppel uses at the moment, and therefore they demonstrate an improvement in the company's vehicle routing.

***Index Terms*—e-commerce, CVRP, linear programming, vehicle routing**

I. INTRODUCTION

When faced with transportation and distribution problems, finding the best optimal solution can often be solved with distinct methods and algorithms. These sorts of problems are presented in various sectors, such as manufacturing factories, warehouses or delivery routes.

Optimizing these problems allow us to minimize transportation cost, minimize distance route, maximize units distribution or minimize carbon footprint. In the following article we will be evaluating a specific route done on the 18^{th} of January 2022 in Guadalajara, Jalisco Mexico by Coppel with the objective to determine an algorithm that may facilitate the distribution of products to the customers.

Using the database that was provided by Coppel we were able to analyze the shipping route done by Coppel on the 18th of January of 2022. In order to optimize this route it is indispensable to identify parameters and variables that allow us to focus our optimization on a specific attribute and structure our constraints. Optimizing these routes will allow a larger amount of sales and reach a larger amount of people at a quicker rate.

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II. DESCRIPTION OF THE PROBLEM

A. Company description

Coppel is a company that offers commercial and financial services. They are a department store chain that is dedicated to credit and final sales in furniture, electrical appliances, jewelry, clothing, and footwear, among others. Coppel has 1600 sales points, 113,000 collaborators and 87 distribution centers that are located in different states of the Mexican Republic.

This report is focused on the DC located in Av. Dr. Roberto Michel 970, San Carlos, 44440 Guadalajara, Jal. This specific distribution center has 28 vehicles available for e-commerce and 22 routes that all part once and finish at the DC.

B. Objective

Because of the size of the operations that Coppel maintains as a company, the goal is to create the most optimal routes for each vehicle in such a way that all products are delivered from the DC to the delivery points (customers) while certain resources are optimized and certain constraints are respected.

The resources that Coppel wants to optimize when creating the routes in no particular order are:

- Minimize delivery times
- Minimize costs associated to transportation
- Maximize client satisfaction

And the conditions that have to be respected when creating the routes in no particular order are:

- Each truck cannot contain more than 30 articles
- All products must be delivered on time

C. Current assignation

One of the main tasks Coppel has to do is to deliver the products bought online by its customers from a distribution center to their houses in a smaller margin of time in order to compete globally with other businesses.

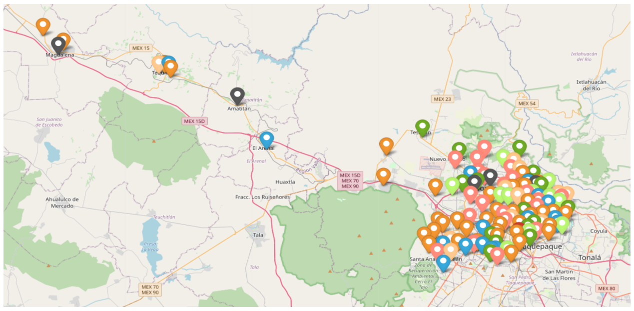

Fig. 1. Coppel´s delivery points from the 18th of January 2022

In figure 1, the customers are shown as points in the map. Coppel has a set of routes that uses for delivering its products. In figure 1, customers are classified by the route in which they are in. This can be appreciated as each color in figure 1 indicates a different route.

The routes that Coppel uses nowadays are designed using circular routing and zigzag assignment. These methods have a starting point at the furthest delivery point and assign the next depending on the method previously established. The circular routing is typically used for rural places such as Tequila and Ixtlán del Río because it clusters the delivery points in circular routes, while the zigzag method alternates from point to point, this is commonly used for more urban zones such as the Guadalajara's metropolitan area and Zapopan.

Fig. 2. Route 1 delivery points from the 18th of January 2022

Fig. 3. Route 2 delivery points from the 18th of January 2022

In figure 2 and figure 3, a zigzag assignment is shown. In the figures, the number over the markers represent the order in which the vehicles of Coppel visit them, and a zigzag pattern is generated between the delivery points. It is worth mentioning that the routes displayed in figure 2 and figure 3 are not the exact routes generated by Coppel, since more information would be needed to have more accurate routes.

Instead, these routes were generated by the research team using the zigzag methodology that Coppel stated is currently using. There are a couple of assumptions we have taken into account while replicating Coppel's routes. The first being that unsuccessful deliveries where the driver had to return to the same delivery point are not being used. Another assumption made was that there may be various vehicles for each route, in the case of figure 2 there are two vehicles. The last assumption taken was that the routes generated were respected by the drivers, strictly delivering the packages assigned.

III. Research Questions

To determine the best route assignation strategy for Coppel's e-commerce sector, the following questions were established regarding the parameters of interest:

1) Which heuristic algorithm is better for optimizing time than the heuristics models that Coppel is currently using?
2) Which parameters are relevant to make a realistic approach for this model?

IV. Variables and Parameters

The identification of variables and parameters that will allow to obtain an efficient model as well as formulate the necessary constraints is a fundamental part of building the model and algorithm. The variables and parameters that have been selected for the depot are the following:

- Location

The variables and parameters that have been selected for the vehicles are the following:

- Number of delivery vehicles
- Storage capacity (volume)
- Unit performance (km/litres gasoline)
- Maximum Velocity

The variables and parameters that have been selected for the customers are the following:

- Location
- Number of customers

The variables and parameters that have been selected for each route are the following:

- Distance
- Traffic
- Time

There are variables and parameters that are constant through out the model. These are the following:

- There is just one depot
- Localization of the depot does not change
- Vehicle quantity
- Storage capacity, tank capacity and performance of vehicles do not change

V. Methodology

To elaborate a proper solution and create an appropriate model that represents the problematic at hand, the methodology used is reported below:

1) Research: Recollection of CVRP algorithms and database exploration.
2) Data filtering: Exploration, eliminating inconsistencies and identifying data of interest.
3) Location: Locate each point from the 18th of January 2022 utilizing google API.
4) Optimization: Modeling, calculating and determining each new route.
5) Data analysis: Visualization and conclusions.

VI. Assumptions And limitations

Due to the large number of variables that may exist when obtaining the most optimal routes, it is necessary to assume certain situations to simplify the model in order to be able to arrive at a practically useful solution for Coppel. The assumptions are summarized in:

- All vehicles can go to all customers, virtually all vehicles are considered to have the same spatial dimensions and therefore there are no restrictions for vehicles not being able to access certain streets.
- All products that the vehicles deliver are considered as perfect cubes that can be placed anywhere inside the vehicle as long as the restrictions of volume are respected, therefore restrictions about how products are placed inside the vehicle or if some products require special cares inside the transportation units are not taken into account.
- All vehicles are available to be used and there is not a situations in which some of them are being repaired or similar issues.
- All vehicles depart from the depot when delivery operations begin and they always return to the depot when those operations end.

In addition to the assumptions, there are some important limitations that mus be taken into account. Because this is a complex problem, there various limitations in terms of the number of variables that can be added, because the more variables used, the more difficult it will be to find an algorithm that solves the problem efficiently being practical enough for Coppel's operations. These limitations are summarized in:

- Algorithm Complexity. For solving this problem, it is necessary to have a matrix with the distances from all delivery points to all delivery points. For the number of customers of the 18^{th} of January 2022, this is equal to have a matrix of 457x457 elements. This is not too much in the memory of a computer, but for creating such matrix, it is necessary to call that number of times the Google Maps API. This is expensive and very time consuming. That is why, an optimization has been used. This optimization is based in the fact that if there is an optimal route that connects some points, each of the points in that route will be connected to points that are not too far from the former. It does not make sense that if an optimal route is found, the route contains at least one point that is connected to a point that is pretty far from it, because this will increase the distance and time of the route. For implementing this optimization, when calling to the Google API, just the first 100 neighbors of each point where called, instead of the 457 points. With this optimization, the number of the API calls were reduced from 208,849 to 45,700.
- Stochastic models: There are many variables in Coppel's problem that turn out to be important to generate the most optimal routes to reach the customers, such as the time it takes for an operator to assemble a delivered furniture product, something that can vary depending on many factors. These are problems where there is some uncertainty and with that, the complexity of the problem is increased. For the situation analyzed in this report, the models are limited to deterministic models and the existence of stochastic situations are not considered.

VII. Mathematical Model

This particular problem can be modeled and represented using the vehicle routing problem with capacity (CVRP) which is a combinatorial optimization problem that allows to find the most efficient route based on a vehicle adding capacities limitation and demands for the customers. This problem tries to minimize the distance traveled by the vehicles but with some easy modifications to the problem, it can optimize time traveled, that is the goal of Coppel.

This informal CVRP description can be stated mathematically as a multi commodity network flow problem with the following variables:

- v Vehicles
- C Vehicle capacity
- C_i Costumer demand
- d_{ij} Transportation distance
- n Number of nodes

- $X_{ijk} \begin{cases} 1 \; the \; arc \; from \; node \; i \; to \; j \; driven \; by \; k \\ 0 \; there \; is \; no \; travel \end{cases}$

The following expression represents the objective function. In this case the objective function tries to minimize the time traveled by the vehicles:

$$min \sum_{k=1}^{v} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} X_{ijk} \tag{1}$$

The following expression represents that each vehicle leaves the node it enters:

$$\sum_{i=1}^{n} X_{ijk} = \sum_{i=1}^{n} X_{jik} \cdot \forall j \in \{1, ..., n\} \forall k \in \{1, ..., v\} \tag{2}$$

The following expressions ensures that each node is entered once:

$$\sum_{k=1}^{v}\sum_{i=1}^{n} X_{ijk} = 1 \cdot \forall j \in \{2, ..., n\} \tag{3}$$

The following expression represents the capacity constraint:

$$\sum_{i=1}^{n}\sum_{j=2}^{n} C_j X_{ijk} \leq C \cdot \forall k \in \{1, ..., v\} \tag{4}$$

The following expressions ensures that there are no cycles disconnected from the depot:

$$\sum_{k=1}^{v}\sum_{i\in S}\sum_{j\in S, I\neq J} X_{KiJ} \leq |S| - 1 \cdot \forall S \in \{1, ..., n\} \tag{5}$$

In addition to these constraints, other considerations regarding specific limitations that Coppel has such as the time a driver can work were added. These constraints were not implemented in the mathematical model, but they are implemented in the algorithm that generates the routes. These constraints are:

1) Drivers cannot exceed eight working hours.
2) Drivers cannot spend more than two hours traveling between points.
3) Each vehicle has at maximum twenty-eight deliveries.

VIII. Model optimizations

The model described previously is an easy approach for solving this problem. As it describes a complex problem using a linear programming approach with just five equations. The disadvantage of such model is that it cannot be used for solving practical problems with a big number of customers.

This is due to the large number of combinations, something that the model described does not take into account. This disadvantage does not mean that it cannot be used for solving the problem, but it is necessary to add some optimizations to the model in order to be practical. One of this optimizations is to use the Dijkstra's Shortest Path Algorithm which determines the shortest path from a node to another using modeling network.

IX. Results

After implementing the modeled described previously, it was demonstrated that the methods used in this paper are more efficient and optimal than the heuristic Coppel currently uses for creating its routes. The model and the algorithms proposed gave the following results:

Our proposed assignment takes at most eight hours to be completed, uses 19 vehicles (one for each route) and takes 108 accumulated hours to be completed taking into account traffic. The summary of these results can be seen in table 1.

This is a better solution and assignment that the one Coppel uses at this moment. The current assignment that Coppel uses takes 22 vehicles and takes the same amount of accumulated hours to be completed but without traffic and with a constant velocity of 100km/hr. With this analysis, it can be said that the routes proposed in this paper are better than the ones Coppel uses currently, in addition there are more variables that are being optimized in the model proposed, such as the number of vehicles and number of hours a driver have to drive.

Times and number of deliveries of the optimized routes		
Route	Number of deliveries	Time (hours)
Route 1	15	7.3
Route 2	2	4.8
Route 3	28	4
Route 4	28	3.5
Route 5	28	3.8
Route 6	28	5
Route 7	28	3.8
Route 8	28	4.8
Route 9	28	4.6
Route 10	28	5.6
Route 11	28	6.9
Route 12	28	5.11
Route 13	28	5.5
Route 14	28	7.5
Route 15	28	7.9
Route 16	15	6.4
Route 17	7	6
Route 18	28	7.8
Route 19	26	7.9
Total	457	108.21

TABLE I
Table of times and deliveries of each route proposed

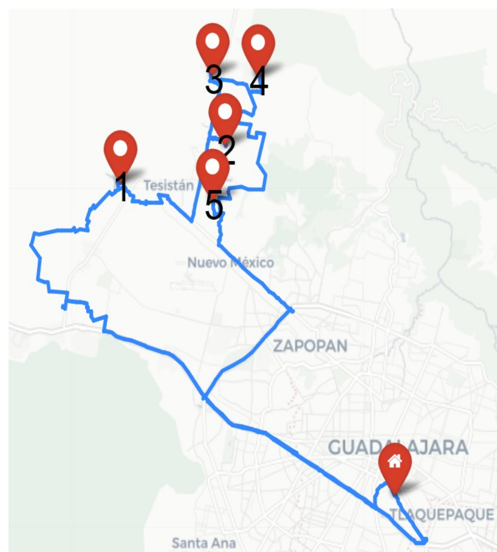

Fig. 4. Route 1 generated using the model proposed

Figure 4 shows an example of the proposed routes. The numbers over the markers represent the order in which a vehicle visit them. Comparing with figure 2, the order in which the points are visited are different. While Coppel uses a zigzag methodology, the routes proposed try to optimize the time, and therefore, the route markers are in order.

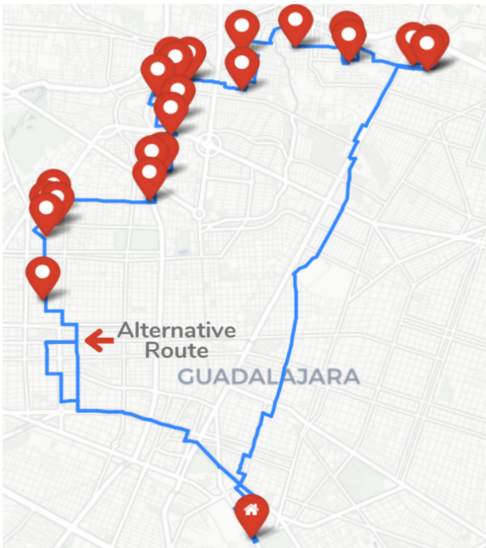

Fig. 5. Route 2 generated using the model proposed

Figure 5 shows another example of the proposed routes. In this case the markers were omitted to have a better visualization on the image, but it is important to clarify that each point is visited in order starting from left to right.

X. Conclusions

Due to the large number of variables that exist in the problem and the specific requirements of Coppel, it is suggested to utilize a model based on a variation of the CVRP problem. This model allows a very simple linear programming formulation that will optimize time and minimize cost (due to the the number of vehicles used).

Even though this model is capable of including several variables, it is not a completely accurate model of Coppel's situation, since there are plenty of variables that would complicate the model and that were discarded because of its low significance with respect to the others. It is also important to note that even though the model was made for a specific day, the algorithm, being deterministic, can be easily used in any other day by simply changing the values of the input variables, however it is necessary to consider that the performance of the algorithm will be compromised by the number of customers that must be visited by the fleet of vehicles.

There are many improvements that can be made to the proposed model starting with adding more variables that bring it closer to the real situation, however, the proposed model does not intend to strictly replicate Coppel's situation, instead, it intends to provide a practical solution that can be used to optimize Coppel's resources and increase its sales.

Because of the points stated above, it is recommended to Coppel the use of the methods and models analyzed in this report to generate its routes and that as can be seen in the results, can be of great benefit for the operations held by Coppel including the reducing of the time in which its vehicles reach its customers, that for this company remains as one of its main priorities.

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