Barycentric Coordinates

Welcome to the **Barycentric Coordinates** homepage! This website follows our minisymposia given at the 10th SIAM Conference on Geometric Design & Computing and the 2011 SIAM Conference on Geometric & Physical Modeling. It provides links to online resources, including slides and papers.

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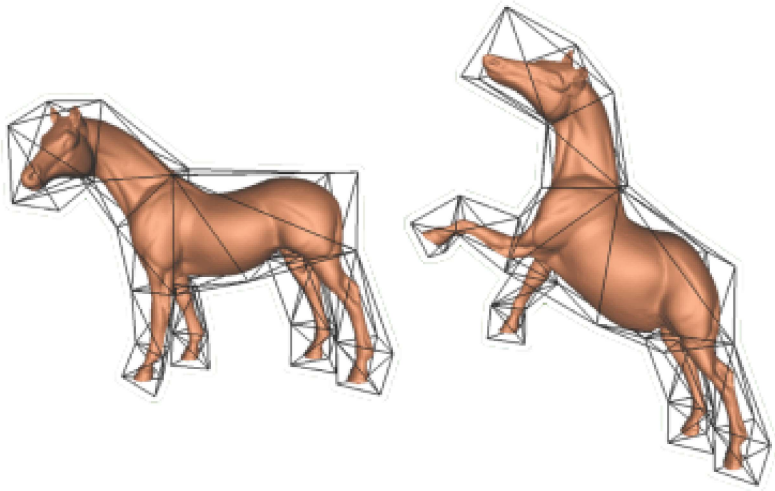

of Cambridge

Raif Rustamov, Drew University

Scott Schaefer, Texas A&M University

N. Sukumar, UC Davis

Slides

Minisymposium on **Barycentric Coordinates and Transfinite Interpolation**

- Lecture 1: Generalized Barycentric Coordinates by Kai Hormann
- Lecture 2: Barycentric Coordinates for Closed Curves by Scott Schaefer
- Lecture 3: Hermite Mean Value Interpolation by Michael S. Floater
- Lecture 4: A General, Geometric Construction of Coordinates in any Dimensions by Tao Ju
- Lecture 5: Transfinite Mean Value Interpolation over Volumetric Domains by Solveig Bruvoll
- Lecture 6: Barycentric Finite Element Methods by N. Sukumar

Minisymposium on **Theory and Applications of Barycentric Coordinates**

- Lecture 1: Constructing Barycentric Coordinates on Surfaces by Raif Rustamov
- Lecture 2: Injective Barycentric Mappings on Convex Domains by Jiří Kosinka
- Lecture 3: Geometric Criteria for Generalized Barycentric Finite Elements by Andrew Gillette
- Lecture 4: Variational Space Deformations with Barycentric Coordinates by Mirela Ben-Chen

Keynote Lecture on Generalized Barycentric Coordinates by Kai Hormann

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Papers

before 1900

Der barycentrische Calcul
August F. Möbius, 1827

before 2000

Continuity and convexity of projections and barycentric coordinates in convex polyhedra
John A. Kalman, 1961

A rational basis for function approximation
Eugene L. Wachspress, 1971

A rational basis for function approximation. II: Curved sides
Eugene L. Wachspress, 1973

Pseudo-harmonic interpolation on convex domains
William J. Gordon and James A. Wixom, 1974

A Rational Finite Element Basis
Eugene L. Wachspress, 1975

Rational basis functions for curved elements
Eugene L. Wachspress, 1979

High-order curved finite elements
Eugene L. Wachspress, 1981

A pentagonal surface patch for computer aided geometric design
Peter Charrot and John A. Gregory, 1984

A multisided generalization of Bézier surfaces
Charles T. Loop and Tony D. DeRose, 1989

Computing discrete minimal surfaces and their conjugates
Ulrich Pinkall and Konrad Polthier, 1993

Multiresolution analysis of arbitrary meshes
Matthias Eck, Tony DeRose, Tom Duchamp, Hugues Hoppe, Michael Lounsbery, and Werner Stuetzle, 1995

Barycentric coordinates for convex polytopes
Joe Warren, 1996

Parametrization and smooth approximation of surface triangulations
Michael S. Floater, 1997

Reflection space image based rendering
Brian Cabral, Marc Olano, and Philip Nemec, 1999

2002

Generalized barycentric coordinates on irregular polygons
Mark Meyer, Haeyoung Lee, Alan Barr, and Mathieu Desbrun

2003

Interpolants within convex polygons: Wachspress' shape functions
Gautam Dasgupta

Integration within polygonal finite elements
Gautam Dasgupta

Mean value coordinates
Michael S. Floater

On the uniqueness of barycentric coordinates
Joe Warren

2004

A quadrilateral rendering primitive
Kai Hormann and Marco Tarini

Interpolations for temperature distributions: A method for all non-concave polygons
Elisabeth A. Malsch and Gautam Dasgupta

Shape functions for polygonal domains with interior nodes
Elisabeth A. Malsch and Gautam Dasgupta

Construction of polygonal interpolants: A maximum entropy approach
N. Sukumar

Conforming polygonal finite elements
N. Sukumar and Alireza Tabarraei

2005

Mean value coordinates in 3D

Michael S. Floater, Géza Kós, and Martin Reimers

Sharp error estimates for interpolatory approximation on convex polytopes
Allal Guessab and Gerhard Schmeisser

Algebraic construction of smooth interpolants on polygonal domains
Elisabeth A. Malsch and Gautam Dasgupta

Smooth two dimensional interpolants: A recipe for all polygons
Elisabeth A. Malsch, John J. Lin, and Gautam Dasgupta

Mean value coordinates for closed triangular meshes
Tao Ju, Scott Schaefer, and Joe Warren

Geometric construction of coordinates for convex polyhedra using polar duals
Tao Ju, Scott Schaefer, Joe Warren, and Mathieu Desbrun

2006

Local *maximum-entropy* approximation schemes: a seamless bridge between finite elements and meshfree methods.
Marino Arroyo and Michael Ortiz

On transfinite barycentric coordinates
Alexander Belyaev

A general construction of barycentric coordinates over convex polygons
Michael S. Floater, Kai Hormann, and Géza Kós

Mean value coordinates for arbitrary planar polygons
Kai Hormann and Michael S. Floater

Spherical barycentric coordinates
Torsten Langer, Alexander Belyaev, and Hans-Peter Seidel

Recent advances in the construction of polygonal finite element interpolants
N. Sukumar and Elisabeth A. Malsch

2007

Barycentric rational interpolation with no poles and high rates of approximation
Michael S. Floater and Kai Hormann

Harmonic coordinates for character articulation
Pushkar Joshi, Mark Meyer, Tony DeRose, Brian Green, and Tom Sanocki

A general geometric construction of coordinates in a convex simplicial polytope
Tao Ju, Peter Liepa, and Joe Warren

Mean value coordinates for arbitrary spherical polygons and polyhedra in R^3
Torsten Langer, Alexander Belyaev, and Hans-Peter Seidel

Mean value Bézier surfaces
Torsten Langer and Hans-Peter Seidel

GPU-assisted positive mean value coordinates for mesh deformations
Yaron Lipman, Johannes Kopf, Daniel Cohen-Or, and David Levin

A unified, integral construction for coordinates over closed curves
Scott Schaefer, Tao Ju, and Joe Warren

Deriving the continuity of maximum-entropy basis functions via variational analysis
N. Sukumar and Roger J-B Wets

Overview and construction of meshfree basis functions: from moving least squares to entropy approximants
N. Sukumar and Roy W. Wright

Discrete Laplace operators: No free lunch
Max Wardetzky, Saurabh Mathur, Felix Kälberer, and Eitan Grinspun

Barycentric coordinates for convex sets
Joe Warren, Scott Schaefer, Anil N. Hirani, and Mathieu Desbrun

2008

The adjoint for an algebraic finite element
Gautam Dasgupta and Eugene L. Wachspress

Basis functions for concave polygons
Gautam Dasgupta and Eugene L. Wachspress

Pointwise radial minimization: Hermite interpolation on arbitrary domains
Michael S. Floater and Christian Schulz

Maximum entropy coordinates for arbitrary polytopes
Kai Hormann and N. Sukumar

Reusable skinning templates using cage-based deformations
Tao Ju, Qian-Yi Zhou, Michiel van de Panne, Daniel Cohen-Or, and Ulrich Neumann

Higher order barycentric coordinates
Torsten Langer and Hans-Peter Seidel

Mean value Bézier maps
Torsten Langer and Hans-Peter Seidel

Green coordinates
Yaron Lipman, David Levin, and Daniel Cohen-Or

Polyhedral finite elements using harmonic basis functions
Sebastian Martin, Peter Kaufmann, Mario Botsch, Martin Wicke, and Markus Gross

Boundary element formulation of harmonic coordinates
Raif M. Rustamov

Barycentric coordinates computation in homogeneous coordinates
Vaclav Skala

Extended finite element method on polygonal and quadtree meshes
Alireza Tabarraei and N. Sukumar

Voronoi-based interpolants for fracture modelling
N. Sukumar and John E. Bolander

2009

Variational harmonic maps for space deformation
Mirela Ben-Chen, Ofir Weber, and Craig Gotsman

Spatial deformation transfer
Mirela Ben-Chen, Ofir Weber, and Craig Gotsman

Transfinite mean value interpolation in general dimension
Solveig Bruvoll and Michael S. Floater

Transfinite mean value interpolation
Christopher Dyken and Michael S. Floater

Coordinates for instant image cloning
Zeev Farbman, Gil Hoffer, Yaron Lipman, Daniel Cohen-Or, and Dani Lischinski

On the injectivity of Wachspress and mean value mappings between convex polygons
Michael S. Floater and Jiří Kosinka

Poisson-based weight reduction of animated meshes
Eric Landreneau and Scott Schaefer

Interior distance using barycentric coordinates
Raif Rustamov, Yaron Lipman, and Thomas Funkhouser

Complex barycentric coordinates with applications to planar shape deformation
Ofir Weber, Mirela Ben-Chen, and Craig Gotsman

2010

Barycentric interpolation and mappings on smooth convex domains
Michael S. Floater and Jiří Kosinka

Derivation and analysis of Green coordinates
Yaron Lipman and David Levin

Moving least squares coordinates
Josiah Manson and Scott Schaefer

Barycentric coordinates on surfaces
Raif Rustamov

A versatile framework for shape description
Raif Rustamov

Conversion of performance mesh animation into cage-based animation
Yann Savoye and Jean-Sébastien Franco

Cage-based tracking for performance animation
Yann Savoye and Jean-Sébastien Franco

CageIK: dual-Laplacian cage-based inverse kinematics
Yann Savoye and Jean-Sébastien Franco

Volumetric modeling with diffusion surfaces
Kenshi Takayama, Olga Sorkine, Andrew Nealen, and Takeo Igarashi

Controllable conformal maps for shape deformation and interpolation
Ofir Weber and Craig Gotsman

2011

Discrete Laplacians on general polygonal meshes
Marc Alexa and Max Wardetzky

Error estimates for generalized barycentric interpolation
Andrew Gillette, Alexander Rand, and Chandrajit Bajaj

Bounded biharmonic weights for real-time deformation
Alec Jacobson, Ilya Baran, Jovan Popović, and Olga Sorkine

Positive Gordon–Wixom coordinates
Josiah Manson, Kuiyu Li, and Scott Schaefer

HOT: Hodge-optimized triangulations
Patrick Mullen, Pooran Memari, Fernando de Goes, and Mathieu Desbrun

Quadratic serendipity finite elements on polygons using generalized barycentric coordinates
Alexander Rand, Andrew Gillette, and Chandrajit Bajaj

Interpolated eigenfunctions for volumetric shape processing
Raif Rustamov

Video-based toon character from surface performance capture
Yann Savoye

Geometric computation, duality and projective space
Vaclav Skala

Jacobians and Hessians of mean value coordinates for closed triangular meshes
Jean-Marc Thiery, Julien Tierny, and Tamy Boubekeur

Affine generalised barycentric coordinates
Shayne Waldron

A complex view of barycentric mappings
Ofir Weber, Mirela Ben-Chen, Craig Gotsman, and Kai Hormann

2012

Direct and converse results for generalized multivariate Jensen-type inequalities
Allal Guessab

A multivariate extension of an inequality of Brenner–Alzer
Allal Guessab, Otheman Nouisser, and Josip Pečarić

PolyMesher: a general-purpose mesh generator for polygonal elements written in Matlab
Cameron Talischi, Glaucio H. Paulino, Anderson Pereira, and Ivan F. M. Menezes

PolyTop: a Matlab implementation of a general topology optimization framework using unstructured polygonal finite element meshes
Cameron Talischi, Glaucio H. Paulino, Anderson Pereira, and Ivan F. M. Menezes

2013

Generalized barycentric coordinates and Jensen type inequalities on convex polytopes
Allal Guessab

Generalized barycentric coordinates and approximations of convex functions on arbitrary convex polytopes
Allal Guessab

Poisson coordinates
Xian-Ying Li and Shi-Min Hu

Cubic mean value coordinates
Xian-Ying Li, Tao Ju, and Shi-Min Hu

Interpolation error estimates for mean value coordinates
Alexander Rand, Andrew Gillette, and Chandrajit Bajaj

Bijective composite mean value mappings
Teseo Schneider, Kai Hormann, and Michael S. Floater

2014

Wachspress and mean value coordinates
Michael S. Floater

Gradient bounds for Wachspress coordinates on polytopes
Michael S. Floater, Andrew Gillette, and N. Sukumar

Local barycentric coordinates
Juyong Zhang, Balin Deng, Zishun Liu, Giuseppe Patanè, Sofien Bouaziz, Kai Hormann, and Ligang Liu

Geodesic based conformal mesh parameterization
Qiang Zou, Jibin Zhao, Yanguo Zhao, Guangbao Liu, and Haiyang Jin

2015

On transfinite Gordon–Wixom interpolation schemes and their extensions
Alexander G. Belyaev and Pierre-Alain Fayolle

Generalized barycentric coordinates and applications
Michael S. Floater

Optimality of gradient bound for polyhedral Wachspress coordinates
Michael S. Floater

Links basic facts on barycentric coordinates

Barycentric coordinates from Wolfram MathWorld

Barycentric coordinates from Wikipedia

Barycentric coordinates from Cut-the-Knot

Barycentric coordinates from Ken Joy's On-Line Computer Graphics Notes

other useful online resources

Ceva's Theorem

Routh's Theorem

Van Obel's Theorem