Semi-analytical formulas for the Hertzsprung-Russell Diagram

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The absolute visual magnitude as function of the observed color (B-V) , also named Hertzsprung-Russell diagram can be described through five equations; that in presence of calibrated stars means eight constants. The developed framework allows to deduce the remaining physical parameters that are mass , radius and luminosity. This new technique is applied to the first 10 PC , the first 50 pc , the Hyades and to the determination of the distance of a cluster. The case of the white dwarfs is analyzed assuming the absence of calibrated data: our equation produces a smaller χ^2 in respect to the standard color-magnitude calibration when applied to the Villanova Catalog of Spectroscopically Identified White Dwarfs. The theoretical basis of the formulas for the colors and the bolometric correction of the stars are clarified through a Taylor expansion in the temperature of the Planck distribution. keywords

stars: formation ; stars: statistics ; methods: data analysis ; techniques: photometric

1 Introduction

The diagrams in absolute visual magnitude versus spectral type for the stars started with Hertzsprung (1905); Rosenberg (1911); Hertzsprung (1911). The original Russell version can be found in Russell (1914b,a,c), in the following H-R diagram. The common explanation is through the stellar evolution, see for example chapter VII in Chandrasekhar (1967). Actually the presence of uncertainties in the stellar evolution makes the comparison between theory and observations an open field of research, see Maeder & Renzini (1984); Madore (1985); Renzini & Fusi Pecci (1988); Chiosi et al. (1992); Bedding et al. (1998). Modern application of the H-R diagram can be found in de Bruijne et al. (2001) applied to the Hyades when the parallaxes are provided by Hipparcos, and in Al-Wardat (2007) applied to the binary systems COU1289 and COU1291.

The Vogt theorem, see Vogt (1926), states that

Theorem 1 The structure of a star is determined by it's mass and it's chemical composition.

Another approach is the parametrization of physical quantities such as absolute magnitude, mass, luminosity and radius as a function of the temperature, see for example Cox (2000) for Morgan and Keenan classification, in the following MK, see Morgan & Keenan (1973). The temperature is not an observable quantity and therefore the parametrization of the observable and not observable quantities of the stars as a function of the observable color is an open problem in astronomy.

Conjecture 1 The absolute visual magnitude M_V is a function, F, of the selected color

$$M_V = F(c_1,, c_8, (B - V))$$
.

The eight constants are different for each MK class.

In order to give an analytical expression to Conjecture 1 we first analyze the case in which we are in presence of calibrated physical parameters for stars of the various MK spectral types , see Section 2 and then the case of absence of calibration tables, see Section 3 . Different astrophysical environments such as the first 10 pc and 50 pc , the open clusters and distance determination of the open clusters are presented in Section 4. The theoretical dependence by the temperature for colors and bolometric corrections are analyzed in Section 5.

2 Presence of calibrated physical parameters

The M_V , visual magnitude, against (B-V) can be found starting from five equations, four of them were already described in Zaninetti (2005). When the numerical value of the symbols is omitted the interested reader is demanded to Zaninetti (2005). The luminosity of the star is

$$\log_{10}(\frac{L}{L_{\odot}}) = 0.4(M_{\text{bol},\odot} - M_{\text{bol}}) ,$$
 (1)

where $M_{\text{bol},\odot}$ is the bolometric luminosity of the sun that according to Cox (2000) is 4.74. The equation that regulates the total luminosity of a star with it's mass is

$$\log_{10}\left(\frac{L}{L_{\odot}}\right) = a_{\rm LM} + b_{\rm LM}\log_{10}\left(\frac{\mathcal{M}}{\mathcal{M}_{\odot}}\right) \quad , \tag{2}$$

here L is the total luminosity of a star , L_{\odot} the sun's luminosity, \mathcal{M} the star's mass , \mathcal{M}_{\odot} the sun's mass , $a_{\rm LM}$ and $b_{\rm LM}$ two coefficients that are reported in Table 1 for MAIN V , GIANTS III and SUPERGIANTS I ; more details can be found in Zaninetti (2005). We remember that the tables of calibration

Table 1: Table of coefficients derived from the calibrated data (see Table 15.7 in Cox (2000)) through the least square method

	MAIN, V	GIANTS, III,	SUPERGIANTS I
K_{BV}	-0.641 ± 0.01	-0.792 ± 0.06	-0.749 ± 0.01
$T_{\rm BV}[{ m K}]$	7360 ± 66	8527 ± 257	8261 ± 67
K_{BC}	42.74 ± 0.01	44.11 ± 0.06	42.87 ± 0.01
$T_{\rm BC}[{ m K}]$	31556 ± 66	36856 ± 257	31573 ± 67
$a_{\rm LM}$	0.062 ± 0.04	0.32 ± 0.14	1.29 ± 0.32
$b_{ m LM}$	3.43 ± 0.06	2.79 ± 0.23	2.43 ± 0.26

of MK spectral types unify SUPERGIANTS Ia and SUPERGIANTS Ib into SUPERGIANTS I, see Table 15.7 in Cox (2000) and Table 3.1 in Bowers & Deeming (1984). From the theoretical side Padmanabhan (2001) quotes $3 < b_{\rm LM} < 5$; the fit on the calibrated values gives $2.43 < b_{\rm LM} < 3.43$, see Table 1.

From a visual inspection of formula (2) is possible to conclude that a logarithmic expression for the mass as function of the temperature will allows us to continue with formulas easy to deal with. The following form of the mass-temperature relationship is therefore chosen

$$\log_{10}(\frac{\mathcal{M}}{\mathcal{M}_{\odot}}) = a_{\rm MT} + b_{\rm MT} \log_{10}(\frac{T}{T_{\odot}}) \quad , \tag{3}$$

where T is the star's temperature, T_{\odot} the sun's temperature, $a_{\rm MT}$ and $b_{\rm MT}$ two coefficients that are reported in Table 2 when the masses as function of the temperature (e.g. Table 3.1 in Bowers & Deeming (1984)) are processed. According to Cox (2000) $T_{\odot} = 5777~K$. Due to the fact that the masses of

Table 2: Table of $a_{\rm MT}$ and $b_{\rm MT}$ with masses given by Table 3.1 in Bowers & Deeming (1984).

	main,V	giants,III	supergiants,I	
			(B-V) > 0.76	(B-V) < 0.76
a_{MT}	-7.6569	5.8958	-3.0497	4.1993
$b_{ m MT}$	2.0102	-1.4563	-0.8491	1.0599
χ^2	28.67	3.41	20739	18.45

the SUPERGIANTS present a minimum at $(B-V)\approx 0.7$ or T $\approx 5700~K$ we have divided the analysis in two. Another useful formula is the bolometric correction BC

$$BC = M_{\text{bol}} - M_{\text{V}} = -\frac{T_{\text{BC}}}{T} - 10 \log_{10} T + K_{\text{BC}} ,$$
 (4)

where $M_{\rm bol}$ is the absolute bolometric magnitude, M_V is the absolute visual magnitude, $T_{\rm BC}$ and $K_{\rm BC}$ are two parameters that are derived in Table 1. The bolometric correction is always negative but in Allen (1973) the analytical formula was erroneously reported as always positive.

The fifth equation connects the physical variable T with the observed color (B-V), see for example Allen (1973),

$$(B - V) = K_{\rm BV} + \frac{T_{\rm BV}}{T} \quad , \tag{5}$$

where $K_{\rm BV}$ and $T_{\rm BV}$ are two parameters that are derived in Table 1 from the least square fitting procedure. Conversely in Section 5 we will explore

a series development for (B - V) as given by a Taylor series in the variable 1/T. Inserting formulas (4) and (3) in (2) we obtain

$$M_V = -2.5 a_{\rm LM} - 2.5 b_{\rm LM} a_{\rm MT} - 2.5 b_{\rm LM} b_{\rm MT} \log_{10} T$$
$$-K_{\rm BC} + 10 \log_{10} T + \frac{T_{\rm BC}}{T} + M_{\rm bol, \odot} . \tag{6}$$

Inserting equation (5) in (6) the following relationship that regulates M_V and (B-V) in the H-R diagram is obtained

$$M_{V} = -2.5 a_{\rm LM} - 2.5 b_{\rm LM} a_{\rm MT} - T_{\rm BV}$$

$$2.5 b_{\rm LM} b_{\rm MT} \log_{10} \left(\frac{T_{\rm BV}}{(B - V) - K_{\rm BV}} \right)$$

$$-K_{\rm BC} + 10 \log_{10} \left(\frac{T_{\rm BV}}{(B - V) - K_{\rm BV}} \right) + \frac{T_{\rm BC}}{T_{\rm BV}} \left[(B - V) - K_{\rm BV} \right] + M_{\rm bol, \odot}$$
(7)

Up to now the parameters $a_{\rm MT}$ and $b_{\rm MT}$ are deduced from Table 3.1 in Bowers & Deeming (1984) and Table 2 reports the merit function χ^2 computed as

$$\chi^2 = \sum_{j=1}^n (M_V - M_V^{cal})^2 \quad , \tag{8}$$

where M_V^{cal} represents the calibration value for the three MK classes as given by Table 15.7 in Cox (2000). From a visual inspection of the χ^2 reported in Table 2 we deduced that different coefficients of the mass-temperature relationship (3) may give better results. We therefore found by a numerical analysis the values $a_{\rm MT}$ and $b_{\rm MT}$ that minimizes equation (8) when (B-V) and $M_{\rm V}$ are given by the calibrated values of Table 15.7 in Cox (2000). This method to evaluate $a_{\rm MT}$ and $b_{\rm MT}$ is new and allows to compute them in absence of other ways to deduce the mass of a star. The absolute visual magnitude with the data of Table 3 is

$$M_{\rm V} = 31.34 3.365 \ln \left(7361.0 \left((B - V) + 0.6412 \right)^{-1} \right) +$$
 $4.287 \left(B - V \right)$ (9)

Table 3: Table of $a_{\rm MT}$ and $b_{\rm MT}$ when M_V^{cal} is given by calibrated data.

	main,V	giants, III,	supergiants, I	
			(B-V)>0.76	(B-V) < 0.76
a_{MT}	-7.76	3.41	3.73	0.20
$b_{ m MT}$	2.06	-2.68	-0.64	0.24
χ^2	11.86	0.152	0.068	0.567

$$MAIN\ SEQUENCE\ ,\ V\ when$$
 $-0.33 < (B-V) < 1.64,$

$$M_{\rm V} = -109.6 +$$

$$12.51 \ln \left(8528.0 \left((B - V) + 0.7920 \right)^{-1} \right) + 4.322 \left(B - V \right)$$

$$GIANTS, III \quad 0.86 < (B - V) < 1.33 \quad ,$$
(10)

$$M_{V} = -39.74 + 3.565 \ln \left(8261.0 \left((B - V) + 0.7491 \right)^{-1} \right) + 3.822 (B - V)$$

$$SUPERGIANTS, I \text{ when}$$

$$-0.27 < (B - V) < 0.76,$$

$$(11)$$

$$M_{\rm V} = -61.26 + 6.050 \ln \left(8261.0 \left((B - V) + 0.7491 \right)^{-1} \right) + 3.822 (B - V)$$

$$SUPERGIANTS , I when$$

$$0.76 < (B - V) < 1.80 .$$
(12)

Is now possible to build the calibrated and theoretical H-R diagram , see Figure 1.

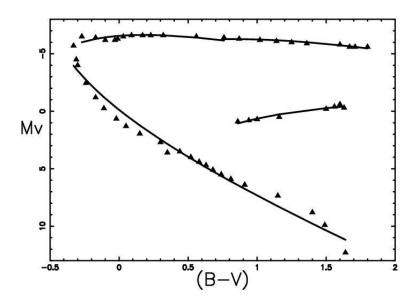


Figure 1: $M_{\rm V}$ against (B-V) for calibrated MK stars (triangles) and theoretical relationship with $a_{\rm MT}$ and $b_{\rm MT}$ as given by Table 3.

2.1 The mass (B-V) relationship

Is now possible to deduce the numerical relationship that connects the mass of the star \mathcal{M} , with the variable (B-V) and the constants a_{MT} and b_{MT}

$$\log_{10}\left(\frac{\mathcal{M}}{\mathcal{M}}_{\odot}\right) = a_{MT} + b_{MT} \ln\left(\frac{T_{BV}}{(B-V) - K_{BV}}\right) (\ln(10))^{-1} \quad . \tag{13}$$

When a_{MT} and b_{MT} are given by Table 3 and the other coefficients are as reported in Table 1 the following expression for the mass is obtained:

$$\log_{10}(\frac{\mathcal{M}}{\mathcal{M}_{\odot}}) = 7.769 +$$

$$+0.8972 \ln \left(7360.9 \left((B - V) + 0.6411 \right)^{-1} \right)$$

$$MAIN \ SEQUENCE \ , \ V \ when$$

$$-0.33 < (B - V) < 1.64 \ ,$$

$$(14)$$

$$\log_{10}\left(\frac{\mathcal{M}}{\mathcal{M}\odot}\right) = 10.41 +$$

$$-1.167 \ln\left(8527.5 \left((B - V) + 0.792\right)^{-1}\right)$$

$$GIANTS , III \quad 0.86 < (B - V) < 1.33 ,$$

$$(15)$$

$$\log_{10}(\frac{\mathcal{M}}{\mathcal{M}\odot}) = 0.2 +$$

$$+0.1276 \ln \left(8261 ((B - V) + 0.7491)^{-1}\right)$$

$$SUPERGIANTS, I \quad when$$

$$-0.27 < (B - V) < 0.76 ,$$

$$(16)$$

$$\log_{10}(\frac{\mathcal{M}}{\mathcal{M}_{\odot}}) = 3.73 +$$

$$-0.2801 \ln \left(8261 \left((B - V) + 0.7491 \right)^{-1} \right)$$

$$SUPERGIANTS , I when$$

$$0.76 < (B - V) < 1.80 .$$

$$(17)$$

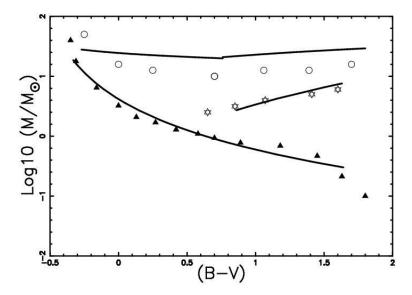


Figure 2: $\log_{10}(\frac{\mathcal{M}}{\mathcal{M}_{\odot}})$ against (B-V) for calibrated MK stars : MAIN SEQUENCE , V (triangles) , GIANTS , III (empty stars) and SUPERGIANTS , I (empty circles). The theoretical relationship as given by formulas (14-17) is reported as a full line.

Figure 2 reports the logarithm of the mass as function of (B-V) for the three classes here considered (points) as well the theoretical relationships given by equations (14-17) (full lines).

2.2 The radius (B-V) relationship

The radius of a star can be found from the Stefan-Boltzmann law, see for example formula (5.123) in Lang (1999). In our framework the radius is

$$\log_{10}\left(\frac{R}{R\odot}\right) = 1/2 a_{LM} + 1/2 b_{LM} a_{MT} + 2 \frac{\ln\left(T_{\odot}\right)}{\ln\left(10\right)} +$$

$$+1/2 b_{LM} b_{MT} \ln \left(\frac{T_{BV}}{(B-V)-K_{BV}} \right) (\ln (10))^{-1}$$
$$-2 \ln \left(\frac{T_{BV}}{(B-V)-K_{BV}} \right) (\ln (10))^{-1} . \tag{18}$$

When the coefficients are given by Table 3 and Table 1 the radius is

$$\log_{10}(\frac{R}{R\odot}) = -5.793 +$$

$$0.6729 \ln \left(7360 \left((B - V) + 0.6411 \right)^{-1} \right)$$

$$MAIN \ SEQUENCE \ , \ V \ when$$

$$-0.33 < (B - V) < 1.64 \ ,$$

$$(19)$$

$$\log_{10}(\frac{R}{R\odot}) = 22.25$$

$$-2.502 \ln \left(8527 ((B-V) + 0.7920)^{-1}\right)$$

$$GIANTS , III when$$

$$0.86 < (B-V) < 1.33 ,$$
(20)

$$\log_{10}(\frac{R}{R\odot}) = 8.417 - 0.7129 \ln \left(8261 ((B - V) + 0.7491)^{-1}\right)$$

$$SUPERGIANTS, I \quad when$$

$$-0.27 < (B - V) < 0.76 ,$$
(21)

$$\log_{10}(\frac{R}{R\odot}) = 12.71$$

$$-1.21 \ln \left(8261 ((B-V) + 0.7491)^{-1}\right)$$

$$SUPERGIANTS, I \text{ when}$$

$$0.76 < (B-V) < 1.80$$
(22)

Figure 3 reports the radius as function of (B-V) for the three classes (points) as well the theoretical relationship given by equations (20-23) (full lines).

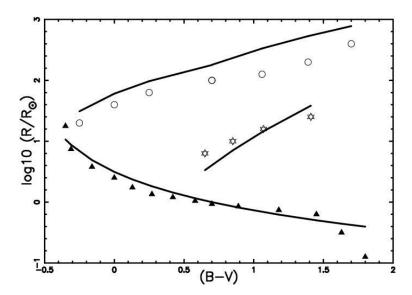


Figure 3: $\log_{10}(\frac{R}{R\odot})$) against (B-V) for calibrated MK stars : MAIN SEQUENCE , V (triangles) , GIANTS , III (empty stars) and SUPERGIANTS , I (empty circles). The theoretical relationships as given by formulas (20-23) are reported as a full line.

2.3 The luminosity (B-V) relationship

The luminosity of a star can be parametrized as

$$\log_{10}\left(\frac{L}{L\odot}\right) = a_{LM} + b_{LM} a_{MT} + b_{LM} \left(b_{MT} \ln\left(\frac{T_{BV}}{(B-V) - K_{BV}}\right) \frac{1}{\ln(10)}\right). \tag{23}$$

When the coefficients are given by Table 3 and Table 1 the luminosity is

$$\log_{10}(\frac{L}{L\odot}) = -26.63 +$$
+3.083 ln (7360.9 ((B - V) + 0.6411)⁻¹)
$$MAIN \ SEQUENCE \ , \ V \ when$$

$$-0.33 < (B - V) < 1.64 \ ,$$
(24)

$$\log_{10}(\frac{L}{L\odot}) = 29.469 +$$

$$-3.2676 \ln \left(8527.59 ((B-V) + 0.7920)^{-1}\right)$$

$$GIANTS, III \quad 0.86 < (B-V) < 1.33 \quad ,$$

$$(25)$$

$$\log_{10}(\frac{L}{L\odot}) = 1.7881 +$$

$$+0.3112 \ln \left(8261.19 ((B-V) + 0.7491)^{-1}\right)$$

$$SUPERGIANTS, I when$$

$$-0.27 < (B-V) < 0.76 ,$$
(26)

$$\log_{10}(\frac{L}{L\odot}) = 10.392 +$$

$$-0.682 \ln \left(8261.19 ((B - V) + 0.749)^{-1}\right)$$

$$SUPERGIANTS, I \quad when$$

$$0.76 < (B - V) < 1.80 \quad .$$
(27)

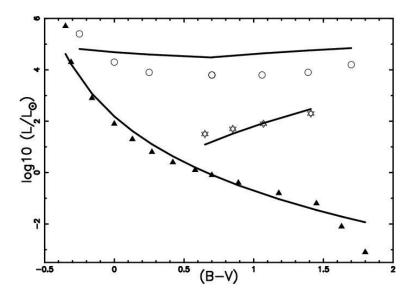


Figure 4: $\log_{10}(\frac{L}{L\odot})$ against (B-V) for calibrated MK stars : MAIN SEQUENCE , V (triangles) , GIANTS , III (empty stars) and SUPER-GIANTS , I (empty circles). The theoretical relationship as given by formulas (25-27) is reported as a full line.

Figure 4 reports the luminosity as function of (B-V) for the three classes (points) as well as the theoretical relationship given by equations (25-27) (full lines).

3 Absence of calibrated physical parameters

The already derived framework can be applied to a class of stars in which the calibration data of BC, (B-V), \mathcal{M} , L versus the temperature are absent, for example the white dwarfs. The presence of the fourth edition of the Villanova Catalog of Spectroscopically Identified White Dwarfs, see McCook & Sion (1999), makes possible to build the H-R diagram of 568 white dwarfs that have trigonometric parallax. Once the observed absolute

magnitude is derived the merit function χ^2 is computed as

$$\chi^2 = \sum_{j=1}^n (M_V - M_V^{obs})^2 \quad , \tag{28}$$

where M_V^{obs} represents the observed value of the absolute magnitude and the theoretical absolute magnitude, M_V , is given by equation (7). The four parameters $K_{\rm BV}$, $T_{\rm BV}$, $T_{\rm BC}$ and $K_{\rm BC}$ are supplied by theoretical arguments, i.e. numerical integration of the fluxes as given by the Planck distribution ,see Planck (1901). The remaining four unknown parameters $a_{\rm MT}$, $b_{\rm MT}$, $a_{\rm LM}$ and $b_{\rm LM}$ are supplied by the minimization of equation (28). Table 4 reports the eight parameters that allow to build the H-R diagram.

Table 4: Table of the adopted coefficients for white dwarfs.

Coefficient	value	method
$K_{ m BV}$	-0.4693058	from the Planck law
$T_{\mathrm{BV}}[\mathrm{K}]$	6466.229	from the Planck law
$K_{ m BC}$	42.61225	from the Planck law
$T_{\mathrm{BC}}[\mathrm{K}]$	29154.75	from the Planck law
$a_{\rm LM}$	0.28	minimum χ^2 on real data
$b_{ m LM}$	2.29	minimum χ^2 on real data
$a_{ m MT}$	-7.80	minimum χ^2 on real data
$b_{ m MT}$	1.61	minimum χ^2 on real data

The numerical expressions for the absolute magnitude (equation (7)), radius (equation (18)), mass (equation (13)) and luminosity (equation (23)) are:

$$M_{V} = 8.199$$

$$+0.3399 \ln \left(6466.0 \left((B - V) + 0.4693 \right)^{-1} \right) + 4.509 \left(B - V \right)$$

$$white dwarf, \quad -0.25 < (B - V) < 1.88 \quad ,$$
(29)

$$\log_{10}(\frac{R}{R_{\odot}}) = -1.267$$

$$-0.0679 \ln \left(6466 ((B - V) + 0.4693)^{-1}\right)$$
white $dwarf$, $-0.25 < (B - V) < 1.88$, (30)

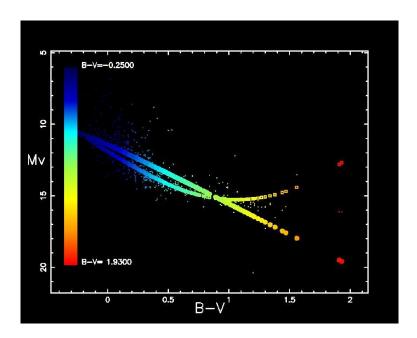


Figure 5: $M_{\rm V}$ against (B-V) (H-R diagram) of the fourth edition of the Villanova Catalog of Spectroscopically Identified White Dwarfs. The observed stars are represented through small points, the theoretical relationship of white dwarfs through big full points and the reference relationship given by formula (33) is represented as a little square points.

$$\log_{10}(\frac{\mathcal{M}}{\mathcal{M}_{\odot}}) = -7.799$$

$$+0.6992 \ln \left(6466 \left((B - V) + 0.4693 \right)^{-1} \right)$$
white $dwarf$, $-0.25 < (B - V) < 1.88$, (31)

$$\log_{10}(\frac{L}{L_{\odot}}) = -17.58 +$$

$$1.601 \ln \left(6466 \left((B - V) + 0.4693 \right)^{-1} \right)$$

$$white \ dwarf, \quad -0.25 < (B - V) < 1.88 \quad .$$

$$(32)$$

Figure 5 reports the observed absolute visual magnitude of the white dwarfs as well as the fitting curve; Table 5 reports the minimum , the average and the maximum of the three derived physical quantities.

Table 5: Table of derived physical parameters of the Villanova Catalog of Spectroscopically Identified White Dwarfs

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parameter min		average	maximum			
$\mathcal{M}/\mathcal{M}_{\odot}$	$5.28 \ 10^{-3}$	$4.23 \ 10^{-2}$	0.24			
R/R_{\odot}	$1.07 \ 10^{-2}$	$1.31 \ 10^{-2}$	$1.56 \ 10^{-2}$			
L/L_{\odot}	$1.16 \ 10^{-5}$	$2.6 \ 10^{-3}$	$7.88 \ 10^{-2}$			

Our results can be compared with the color-magnitude relation as suggested by McCook & Sion (1999), where the color-magnitude calibration due to Dahn et al. (1982) is adopted,

$$M_{V} = 11.916 ((B - V) + 1)^{0.44} - 0.011$$

$$when(B - V) < 0.4$$

$$M_{V} = 11.43 + 7.25 (B - V) - 3.42 (B - V)^{2}$$

$$when 0.4 < (B - V)$$
(33)

The previous formula (33) is reported in Figure 5 as a line made by little squares and Table 6 reports the χ^2 computed as in formula (28) for our

formula (30) and for the reference formula (33). From a visual inspection of Table 6 is possible to conclude that our relationship represents a better fit of the data in respect to the reference formula.

Table 6: Table of χ^2 when the observed data are those of the fourth edition of the Villanova Catalog of Spectroscopically Identified White Dwarfs.

equation	χ^2
our formula (30)	710
reference formula (33)	745

The three classic white dwarfs that are Procyon B, Sirius B, and 40 Eridani-B can also be analyzed when (B-V) is given by Wikipedia (http://www.wikipedia.org/). The results are reported in Table 7, 8 and 9 where is also possible to visualize the data suggested by Wikipedia.

Table 7: Table of derived physical parameters of 40 Eridani-B where (B-V)=0.04

parameter	here	suggested in Wikipedia
M_V	11.59	11.01
$\mathcal{M}/\mathcal{M}_{\odot}$	$6.41 \ 10^{-2}$	0.5
R/R_{\odot}	$1.23 \ 10^{-2}$	$2 \ 10^{-2}$
L/L_{\odot}	$3.53 \ 10^{-3}$	$3.3 \ 10^{-3}$

Table 8: Table of derived physical parameters of Procyon B where (B - V)=0.0

parameter	here	suggested in Wikipedia
M_V	11.43	13.04
$\mathcal{M}/\mathcal{M}_{\odot}$	$7.31 \ 10^{-2}$	0.6
R/R_{\odot}	$1.21 \ 10^{-2}$	$2 \ 10^{-2}$
L/L_{\odot}	$4.77 \ 10^{-3}$	$5.5 \ 10^{-4}$

Table 9: Table of derived physical parameters of Sirius B where (B - V)=-0.03

parameter	here	suggested in Wikipedia
M_V	11.32	11.35
$\mathcal{M}/\mathcal{M}_{\odot}$	$8.13 \ 10^{-2}$	0.98
R/R_{\odot}	$1.21 \ 10^{-2}$	$0.8 10^{-2}$
L/L_{\odot}	$6.08 \ 10^{-3}$	$2.4 \ 10^{-3}$

4 Application to the astronomical environment

The stars in the first 10 pc, as observed by Hipparcos (ESA (1997)), belong to the MAIN V and Figure 6 reports the observed stars, the calibration stars and the theoretical relationship given by equation (10) as a continuous line.

Different is the situation in the first 50 pc where both the MAIN V and the GIANTS III are present, see Figure 7 where the theoretical relationship for GIANTS III is given by equation (11).

Another astrophysical environment is that of the Hyades cluster, with (B-V) and m_v as given in Stern et al. (1995) and available on the VizieR Online Data Catalog. The H-R diagram is build in absolute magnitude adopting a distance of 45 pc for the Hyades, see Figure 8, where also the theoretical relationship of MAIN V is reported.

Another interesting open cluster is that of the Pleiades with the data of Micela et al. (1999) and available on the VizieR Online Data Catalog.

Concerning the distance of the Pleiades we adopted 135 pc, according to Bouy et al. (2006); other authors suggest 116 pc as deduced from the Hipparcos data, see Mermilliod et al. (1997). The H-R diagram of the Pleiades is reported in Figure 9.

4.1 Distance determination

The distance of an open cluster can be found through the following algorithm :

1. The absolute magnitude is computed introducing a guess value of the distance.

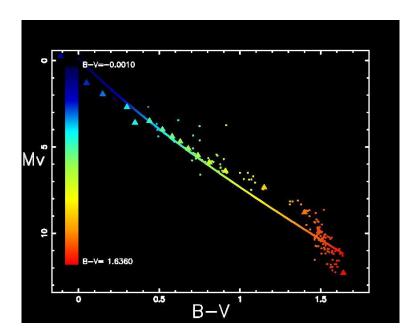


Figure 6: $M_{\rm V}$ against (B-V) (H-R diagram) in the first 10 pc. The observed stars are represented through points, the calibrated data of MAIN V with great triangles and the theoretical relationship of MAIN V with a full line.

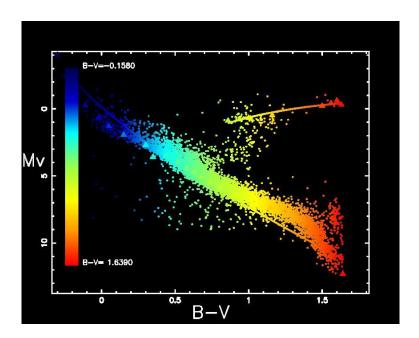


Figure 7: $M_{\rm V}$ against (B-V) (H-R diagram) in the first 50 pc. The observed stars are represented through points, the calibrated data of MAIN V and GIANTS III with great triangles , the theoretical relationship of MAIN V and GIANTS III with a full line.

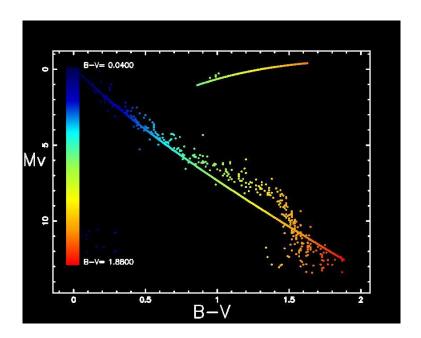


Figure 8: $M_{\rm V}$ against (B-V) (H-R diagram) for the Hyades . The observed stars are represented through points, the theoretical relationship of MAIN V with a full line.

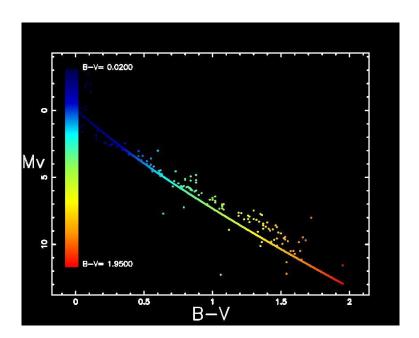


Figure 9: $M_{\rm V}$ against (B-V) (H-R diagram) for the Pleiades. The observed stars are represented through points, the theoretical relationship of MAIN V with a full line.

- 2. Only the stars belonging to MAIN V are selected.
- 3. The χ^2 between observed an theoretical absolute magnitude (see formula (10)) is computed for different distances.
- 4. The distance of the open cluster is that connected with the value that minimize the χ^2 .

In the case of the Hyades this method gives a distance of 37.6~pc with an accuracy of 16% in respect to the guess value or 19% in respect to 46.3~pc of Wallerstein (2000).

5 Theoretical relationships

In order to confirm or not the physical basis of formulas (4) and (5) we performed a Taylor-series expansion to the second order of the exact equations as given by the Planck distribution for the colors, see Section 5.1, and for the bolometric correction, see Section 5.2. A careful analysis on the numerical results applied to the Sun is reported in Section 5.3.

5.1 Colors versus Temperature

The brightness of the radiation from a blackbody is

$$B_{\lambda}(T) = \left(\frac{2hc^2}{\lambda^5}\right) \frac{1}{\exp(\frac{hc}{\lambda kT}) - 1} \quad , \tag{34}$$

where c is the light velocity , k the Boltzmann constant , T the equivalent brightness temperature and λ the considered wavelength , see formula (13) in Planck (1901),or formula (275) in Planck (1959), or formula (1.52) in Rybicki & Lightman (1985) , or formula (3.52) in Kraus (1986).

The color-difference, $(C_1 - C_2)$, can be expressed as

$$(C_1 - C_2) = m_1 - m_2 = K - 2.5 \log_{10} \frac{\int S_1 I_{\lambda} d\lambda}{\int S_2 I_{\lambda} d\lambda}$$
, (35)

where S_{λ} is the sensitivity function in the region specified by the index λ , K is a constant and I_{λ} is the energy flux reaching the earth. We now define a sensitivity function for a pseudo-monochromatic system

$$S_{\lambda} = \delta(\lambda - \lambda_i) \quad i = U, B, V, R, I \quad , \tag{36}$$

where δ denotes the Dirac delta function. In this pseudo-monochromatic color system the color-difference is

$$(C_1 - C_2) = K - 2.5 \log_{10} \frac{\lambda_2^5}{\lambda_1^5} \frac{(\exp(\frac{hc}{\lambda_2 kT}) - 1)}{(\exp(\frac{hc}{\lambda_1 kT}) - 1)} . \tag{37}$$

Table 10: Johnson system

symbol	wavelength (Å)
U	3600
В	4400
V	5500
R	7100
I	9700

The previous expression for the color can be expanded through a Taylor series about the point $T=\infty$ or making the change of variable $x=\frac{1}{T}$, about the point x=0. When the expansion order is 2 we have

$$(C_1 - C_2)_{app} = -\frac{5}{2} \ln \left(\frac{\lambda_2^4}{\lambda_1^4} \right) \frac{1}{\ln(10)}$$
$$-\frac{5}{4} \frac{hc(\lambda_1 - \lambda_2)}{\lambda_2 \lambda_1 k \ln(10) T} - \frac{5}{48} \frac{h^2 c^2 (\lambda_1^2 - \lambda_2^2)}{\lambda_2^2 \lambda_1^2 k^2 \ln(10) T^2} ,$$
 (38)

where the index app means approximated. We now continue inserting the value of the physical constants as given by CODATA Mohr & Taylor (2005) and wavelength of the color as given by Table 15.6 in Cox (2000) and visible in Table 10. The wavelength of U,B and V are exactly the same of the multicolor photometric system defined by Johnson (1966), conversely R(7000 \mathring{A}) and I(9000 \mathring{A}) as given by Johnson (1966) are slightly different from the values here used. We now continue parameterizing the color as

$$(C_1 - C_2)_{app} = a + \frac{b}{T} + \frac{d}{T^2} \quad . \tag{39}$$

Another important step is the calibration of the color on the maximum temperature T_{cal} of the reference tables. For example for MAIN SEQUENCE

V at $T_{cal}=42000$, see Table 15.7 in Cox (2000), (B-V)=-0.3 and therefore a constant should be added to formula (38) in order to obtain such a value. With these recipes we obtain, for example

$$(B-V) = -0.4243 + \frac{3543}{T} + \frac{17480000}{T^2}$$
 (40)
MAIN SEQUENCE, V when $-0.33 < (B-V) < 1.64$.

The basic parameters b and d for the four colors here considered are reported in Table 10. The parameter a when WHITE DWARF, MAIN SEQUENCE, V, GIANTS, III and SUPERGIANTS, I are considered is reported in Table 11, conversely the Table 12 reports the coefficient b and d that are in common to the classes of stars here considered. The WHITE DWARF

Table 11: Coefficient a

	(B-V)	(U-B)	(V-R)	(R-I)
WHITE DWARF, $T_{cal}[K]=25200$	- 0.1981	- 1.234		
MAIN SEQUENCE, V, $T_{cal}[K]=42000$	- 0.4243	- 1.297	- 0.233	- 0.395
GIANTS III, $T_{cal}[K]=5050$	- 0.5271	- 1.156	- 0.4294	- 0.4417
SUPERGIANTS I , $T_{cal}[K]=32000$	- 0.3978	- 1.276	- 0.2621	- 0.420

calibration is made on the values of (U - B) and (B - V) for Sirius B, see Wikipedia (http://www.wikipedia.org/).

Table 12: Coefficients b and d

	(B-V)	(U-B)	(V-R)	(R-I)
b	3543	3936	3201	2944
d	17480000	23880000	12380000	8636000

The Taylor expansion agrees very well with the original function and Figure 10 reports the difference between the exact function as given by the ratio of two exponential and the Taylor expansion in the (B-V) case.

In order to establish a range of reliability of the polynomial expansion we solve the nonlinear equation

$$(C_1 - C_2) - (C_1 - C_2)_{app} = f(T) = -0.4$$
 , (41)

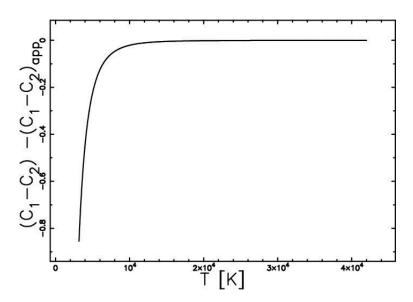


Figure 10: Difference between (B-V), the exact value from the Planck distribution, and $(B-V)_{app}$, approximate value as deduced from the Taylor expansion for MAIN SEQUENCE, V

Table 13: Range of existence of the Taylor expansion for MAIN SE-QUENCE, V

	(B-V)	(U-B)	(V-R)	(R-I)
T_{min} [K]	4137	4927	3413	2755
T_{max} [K]		42000	42000	42000

for T. The solutions of the nonlinear equation are reported in Table 13 for the four colors here considered. For the critical difference we have chosen the value -0.4 that approximately corresponds to 1/10 of range of existence in (B-V). Figure 11 reports the exact and the approximate value of (B-V) as well as the calibrated data.

5.2 Bolometric Correction versus Temperature

The bolometric correction BC , defined as always negative, is

$$BC = M_{\text{bol}} - M_{\text{V}} \quad , \tag{42}$$

where M_{bol} is the absolute bolometric magnitude and M_V is the absolute visual magnitude. It can be expressed as

$$BC = \frac{5}{2} \frac{\ln\left(15(\frac{hc}{kT\pi})^4(\frac{1}{\lambda_V})^5 \frac{1}{\exp(\frac{hc}{kT\lambda_V}) - 1}\right)}{\ln(10)} + K_{BC} \quad , \tag{43}$$

where λ_V is the visual wavelength and K_{BC} a constant. We now expand with a Taylor series about the point $T=\infty$

$$BC_{app} = -\frac{15}{2} \frac{\ln(T)}{\ln(10)} - \frac{5}{4} \frac{hc}{k\lambda_V \ln(10)T} - \frac{5}{48} \frac{h^2c^2}{k^2\lambda_V^2 \ln(10)T^2} + K_{BC} \quad . \tag{44}$$

The constant K_{BC} can be found with the following procedure. The maximum of BC_{app} is at T_{max} , where the index max stands for maximum

$$T_{max} = \frac{1}{6} \frac{\left(\frac{\sqrt{5}}{2} + \frac{1}{2}\right)ch}{k\lambda_V} \quad . \tag{45}$$

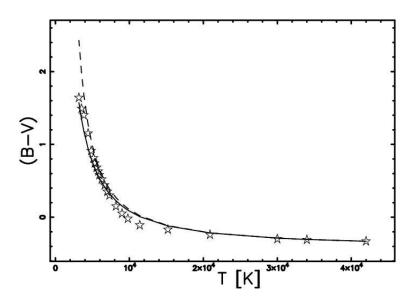


Figure 11: Exact (B-V) as deduced from the Planck distribution , or equation (37), traced with a full line. Approximate (B-V) as deduced from the Taylor expansion, or equation (38), traced with a dashed line. The calibrated data for MAIN SEQUENCE V are extracted from Table 15.7 in Cox (2000) and are represented through empty stars.

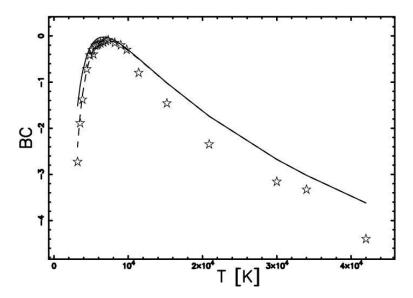


Figure 12: Exact BC as deduced from the Planck distribution , or equation (43), traced with a full line. Approximate BC as deduced from the Taylor expansion, or equation (46), traced with a dashed line. The calibrated data for MAIN SEQUENCE V are extracted from Table 15.7 in Cox (2000) and are represented through empty stars.

Given the fact that the observed maximum in the BC is -0.09 at 7300 K in the case of MAIN SEQUENCE V we easily compute K_{BC} and the following approximate result is obtained

$$BC_{app} = 31.41 - 3.257 \ln (T) - \frac{14200}{T} - \frac{3.096 \cdot 10^7}{T^2}$$
 (46)

Figure 12 reports the exact and the approximate value of BC as well as the calibrated data.

The Taylor expansion agrees very well with the original function and Figure 13 reports the difference between exact function as given by equation (43) and the Taylor expansion as given by equation (46).

In order to establish a range of reliability of the polynomial expansion we solve the nonlinear equation for T

$$BC - BC_{app} = -0.4 \quad . \tag{47}$$

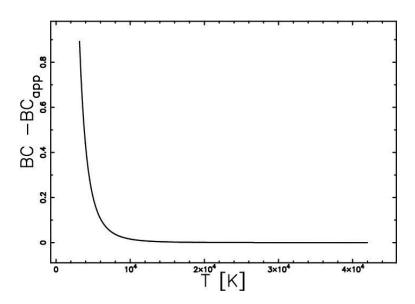


Figure 13: Difference between (B-V), the exact value from the Planck distribution, and $(B-V)_{app}$, approximate value as deduced from the Taylor expansion for MAIN SEQUENCE, V

Table 14: (B-V) of the Sun , T=5777 K

meaning	(B-V)
calibration, Cox (2000)	0.65
here, Taylor expansion	0.711
here , Planck formula	0.57
least square method, Zaninetti (2005)	0.633
Allen (1973)	0.66
Sekiguchi & Fukugita (2000)	0.627
Johnson (1966)	0.63

Table 15: (R-I) of the Sun, T=5777 K

meaning	(R-I)
calibration, Cox (2000)	0.34
here, Taylor expansion here, Planck formula	0.37 0.34

The solution of the previous nonlinear equation allows to state that the bolometric correction as derived from a Taylor expansion for MAIN SE-QUENCE, V is reliable in the range 4074~K < T < 42000~K.

5.3 The Sun as a blackbody radiator

The framework previously derived allows to compare our formulas with one specific star of spectral type G2V with T=5777 K named Sun. In order to make such a comparison we reported in Table 14 the various value of (B-V) as reported by different methods as well as in Table 15 the value of the infrared color (R-I). From a careful examination of the two tables we conclude that our model works more properly in the far-infrared window in respect to the optical one.

6 Conclusions

New formulas A new analytical approach based on five basic equations allows to connect the color (B-V) of the stars with the absolute visual magnitude, the mass, the radius and the luminosity. The suggested method is based on eight parameters that can be precisely derived from the calibration tables; this is the case of MAIN V, GIANTS III and SUPERGIANTS I. In absence of calibration tables the eight parameters can be derived mixing four theoretical parameters extracted from Planck distribution with four parameters that can be found minimizing the χ^2 connected with the observed visual magnitude; this is the case of white dwarfs. In the case of white dwarfs the mass-luminosity relationship, see Table 4, is

$$\log_{10}(\frac{L}{L_{\odot}}) = 0.28 + 2.29 \log_{10}(\frac{\mathcal{M}}{\mathcal{M}_{\odot}}) , \qquad (48)$$
white $dwarf$, $0.005\mathcal{M}_{\odot} < \mathcal{M} < \prime. \in \triangle \mathcal{M}_{\odot}$.

Applications The applications of the new formulas to the open clusters such as Hyades and Pleiades allows to speak of universal laws for the star's main parameters. In absence of accurate methods to deduce the distance of an open cluster an approximate evaluation can be done.

Theoretical bases The reliability of an expansion at the second order of the colors and bolometric correction for stars as derived from the Planck distribution is carefully explored and the range of existence in temperature of the expansion is determined.

Inverse function In this paper we have chosen a simple hyperbolic behavior for (B-V) as function of the temperature as given by formula (5). This function can be easily inverted in order to obtain T as function of (B-V) (MAIN SEQUENCE, V)

$$T = \frac{7360}{(B-V) + 0.641} K$$
 (49)
MAIN SEQUENCE, V when $4137 < T[K] < 42000$
or when $-0.33 < (B-V) < 1.45$.

When conversely a more complex behavior is chosen, for example a two degree polynomial expansion in $\frac{1}{T}$ as given by formula (38), the inverse

formula that gives T as function of (B-V) (MAIN SEQUENCE, V) is more complicated than formula (49),

$$T = 5000 \frac{0.355 \cdot 10^{8} + \sqrt{4.217 \times 10^{15} + 6.966 \times 10^{15} (B - V)}}{10^{8} (B - V) + 0.4244 \cdot 10^{8}} K$$

$$MAIN SEQUENCE, V \text{ when } 4137 < T[K] < 42000$$

$$or \text{ when } -0.33 < (B - V) < 1.45 .$$

The mathematical treatment that allows to deduce the coefficients of the series reversion can be found in Morse & Feshbach (1953); Dwight (1961); Abramowitz & Stegun (1965).

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