Lecture hours 30-32

14

This tutorial is a review for Unit 3.

Problem 56.

a) Find a 2×2 matrix such that $A^2 = A$ and compute its eigenvalues.

b) If A is an $n \times n$ matrix such that $A^2 = A$. What can you say about the eigenvalues of A?

65

Problem 57. Find:

a) A diagonalizable matrix that is not invertible.

b) An invertible matrix that is not diogonalizable.

c) A non invertible matrix that is not diagonalizable.

Problem 58. Let A be the 3×3 matrix given by

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}.$$

a) Find the eigenvalues of A.

b) Find a basis for each one of the eigenspaces of A.

c) Find a diagonal matrix D and an invertible matrix S such that A = SDS^{-1}.

d) For any $n \times n$ matrices X and Y it holds that

$$det(XY) = det(X)det(Y) \quad trace(XY) = trace(YX)$$

Can you use this fact to find det(A) and trance(A)? How?