Math 140 **Tutorial 12**
March 31 - April 4

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1. For the following functions f, g and real number a verify the formula
$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$
without using l'Hospital's Rule!
   (a) $f(x) = 3x - 6,\ g(x) = 7x - 14,\ a = 2$
   (b) $f(x) = 3x^2 + 5x,\ g(x) = 10x,\ a = 0$
   (c) $f(x) = x^3 + 6x^2 + 11x + 6,\ g(x) = x^3 - 4x^2 + x + 6,\ a = -1$
   (d) $f(x) = (x - a)p(x),\ g(x) = (x - a)q(x)$, where p, q are differentiable functions with continuous derivatives, such that $q(a) \neq 0$.

2. Evaluate the following limits using l'Hospital's Rule, if it applies.
   (a) $\lim\limits_{x\to\pi/4}\dfrac{\sin x - \cos x}{\tan x - 1}$
   (b) $\lim\limits_{x\to\infty}\dfrac{\ln\ln x}{x}$
   (c) $\lim\limits_{x\to 0^+}(\sin x)(\ln x)$
   (d) $\lim\limits_{x\to 0^+}(\tan(2x))^x$
   (e) $\lim\limits_{\theta\to\pi/2}\dfrac{1 - \sin\theta}{\csc\theta}$
   (f) $\lim\limits_{x\to\infty}\left(1 + \dfrac{a}{x}\right)^{bx}$

3. Show that l'Hospital's Rule fails to yield a solution for $\lim\limits_{x\to\infty}\dfrac{x}{\sqrt{x^2+1}}$. Evaluate the limit by other means.

4. For the following functions

- Find the intervals on which f is increasing or decreasing,
- Find the local maximum and minimum values of f,
- Find the interval of concavity and the inflection points,
- Use this information to sketch a graph of f.

(a) $\dfrac{e^x}{x^2}$

(b) $\dfrac{1}{1+e^{-x}}$

(c) $\dfrac{\ln x}{x^2}$

(d) $\dfrac{1}{x} + \ln x$