Math 140 **Tutorial 10**

March 17 - 20

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1. Prove the following identities

   (a)
   $$\frac{1+\tanh x}{1-\tanh x} = e^{2x}$$

   (b)
   $$\cosh 2x = \cosh^2 x + \sinh^2 x$$

   (c)
   $$(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$$

   For any real number n.

2. Find the derivative of the function.

   (a) $y = \tan^{-1}(\sinh x)$

   (b) $y = \tanh^{-1}\left(x^3\right)$

   (c) $y = \operatorname{sech}(\tanh x)$

3. Verify that $f(x) = \sqrt{x} - \frac{1}{3}x$ satisfies the hypotheses of Rolle's Theorem on the interval $[0, 9]$. Find all numbers c satisfying the conclusion of the theorem.

4. Explain why, if the graph of a polynomial function has three x-intercepts, then it must have at least two points at which its tangent line is horizontal. Is this true for any function having three x-intercepts?

5. Show that the equation $x^4 + 4x + c = 0$ has at most two solutions which are real numbers.

6. Verify that the function satisfies the hypotheses of the MVT on the given interval. Then, find all values of c that satisfy the conclusion of the MVT.

   (a) $f(x) = \dfrac{x}{x+2}$ on $[1, 4]$

   (b) $f(x) = e^{-2x}$ on $[0, 3]$

7. A number a is called a **fixed point** of a function f if $f(a) = a$. Prove that if f satisfies the hypothesis of the MVT and $f'(x) \neq 1$ for all x, then f has at most one fixed point. *Hint: Suppose there are at least two fixed points, call them a and b, $a \neq b$. Then, use the MVT.*