Math 141 Tutorial 4

Derivatives of Trigonometric and Inverse Trigonometric Functions

Trigonometric Functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}\sec(x) = \sec(x)\tan(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos(x) = -\sin(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan(x) = \sec^2(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}\cot(x) = \cot^2(x)$$

Inverse Trigonometric Functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\arcsin(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{arcsec}(x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\arccos(x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{arccsc}(x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\arctan(x) = \frac{1}{1+x^2} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{arccot}(x) = \frac{-1}{1+x^2}$$

Main problems

1. In this exercise, you will practice using the Fundamental Theorem of Calculus (Form 2). For every integral below, do the following:

   - find an antiderivative (i.e. primitive) of the integrand,
   - evaluate the given integral by applying the FTC if possible. If the FTC does not apply, explain why.

   (a) $\displaystyle\int_0^1 \left(3x^2 + \sqrt{x} - 2\right) \mathrm{d}x$

   (b) $\displaystyle\int_0^{3\pi/2} (\sin(x) + \cos(x))\, \mathrm{d}x$

   (c) $\displaystyle\int_0^{\pi} \sec^2(x)\, \mathrm{d}x$

   (d) $\displaystyle\int_0^{\frac{1}{\pi}} \frac{1}{1+x^2}\, \mathrm{d}x$

   (e) $\displaystyle\int_{-1}^{2} \left((x+1)^2 + \frac{1}{x}\right) \mathrm{d}x$

   (f) $\displaystyle\int_1^4 (3^x + 1)\, \mathrm{d}x.$

2. Consider a particle moving along a line such that, at any time t, the instantaneous velocity of this particle is given by

$$v(t) = t^2 - 2t - 3, \quad (m/s).$$

   (a) Express the displacement of the particle from times $t = 2$ to $t = 4$ using an integral and evaluate this integral.

   (b) Express the distance traveled by the particle from times $t = 2$ to $t = 4$ using an integral and evaluate this integral.

   Briefly explain the difference between the integrals obtained in (a)-(b). How does this relate to the total area under the curve of a sign-changing function?

3. Using the Fundamental Theorem of Calculus (FTC), evaluate the following:

   (a) $\dfrac{\mathrm{d}}{\mathrm{d}x} \displaystyle\int_1^{x+4} \left(\frac{1}{t} + 2\right) \mathrm{d}t$

   (b) $\displaystyle\int_1^{x+4} \left(\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{t} + 2\right]\right) \mathrm{d}t$

   (c) $\dfrac{\mathrm{d}}{\mathrm{d}x} \displaystyle\int_{\sin(x)}^{2\pi} \cos(t)\, dt$

   (d) $\dfrac{\mathrm{d}}{\mathrm{d}x} \displaystyle\int_{\sin(x)}^{x^2+1} e^{\sqrt[3]{t}}\, \mathrm{d}t.$

4. Compute the following integrals using substitution:

(a) $\displaystyle\int_0^2 2e^{3s+5}\,\mathrm{d}s$

(b) $\displaystyle\int 2x\cos\left((x-1)(x+1)\right)\,\mathrm{d}x$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{1+\sin(x)}\,\mathrm{d}x$

(d) $\displaystyle\int \frac{2x^3+1}{(x^4+2x)^3}\,\mathrm{d}x$

(e) $\displaystyle\int_0^1 \sqrt{2t+1}\,dt$

(f) $\displaystyle\int_{\frac{\pi}{2}}^{-2\pi} e^{\cos(x)}\sin(x)\,dx$

(g) $\displaystyle\int_0^1 t^4(1+t^5)^{10}\,dt$

5. Suppose that f is a continuous function on $\mathbb{R}$. Prove that $\displaystyle\int_a^b f(x+c)\,\mathrm{d}x = \int_{a+c}^{b+c} f(x)\,\mathrm{d}x$ for any $c \in \mathbb{R}$.

Challenge Problem

6. Compute

$$I = \int_0^{\pi} \frac{x\sin(x)}{1+\cos^2(x)}\,dx$$

Hint: make the substitution $u = \pi - x$ and with trig identities write down a simple definite integral for $2I$