

1. For each of the following functions find the first, second and third derivatives ($\frac{df}{dx}$, $\frac{d^2f}{dx^2}$, $\frac{d^3f}{dx^3}$):

(a) $f(x) = \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1$

(b) $f(x) = \sin x$

(c) $f(x) = \cos x$

2. Evaluate the following derivatives:

(a) $\frac{d}{dx} \left(ae^x + \frac{b}{x} + \frac{c}{x^2} \right)$

(f) $\frac{d^2}{dx^2} (x^4 e^x)$

(b) $\frac{d}{dx} \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}} \right)^2$

(g) $\frac{d}{dx} (x \sec x \tan x)$

(c) $\frac{d}{dx} \left(\frac{x - x^2}{\sqrt{x}} \right)$

(h) $\frac{d}{dx} \left(\frac{ax + b}{cx + d} \right)$

(d) $\frac{d}{dx} ((3x^3 + 2)(7x + 2))$

(i) $\frac{d}{dx} \left(\frac{1 - \sec x}{\cot x} \right)$

(e) $\frac{d}{dx} (\sec x e^x)$

(j) $\frac{d}{dx} \left(\frac{x^3 e^x + 1}{2x + e^x} \right)$

3. Find the equations of the tangent line of the curves at the given point:

(a) $y = xe^x + x^3$ at $(0, 0)$

(b) $y = \sin x + \cos x$ at $(0, 1)$

(c) $y = e^x \cos x + \sin x$ at $(0, 1)$

4. (a) Where does $f(x) = e^x \cos x$ have a horizontal tangent line?
(b) Where does $f(x) = \frac{x^2+1}{x+1}$ have a tangent line parallel to the line $2y = x - 3$?

5. Let

$$f(x) = \begin{cases} x^2, & x \leq 2 \\ mx + b, & x > 2. \end{cases}$$

Find the values of m and b that make f differentiable everywhere.

6. Suppose that f, g, h are differentiable functions. Prove that

$$(fgh)' = f'gh + fg'h + fgh'$$

What can you say about the derivative of a product of n differentiable functions?

7. Find the derivative of the function. Do not simplify.

(a) $y = (x^2 + 7x + 2)^{20}$

(e) $f(x) = \ln(e^{-x} + xe^{-x})$

(b) $y = x(4x + 1)^{100}$

(f) $h(t) = t \ln\left(\frac{1}{t}\right)$

(c) $y = \cos\left(\sqrt{\sin x}\right)$

(g) $f(x) = x^x$

(d) $y = \tan^2(\sin(3x + \ln x))$

(h) $y = e^{e^x}$