Math 141 Tutorial 7

Main problems

1. For the following problem compute the area enclosed by the given functions in the specified region:
   (a) between $y = x - 1$ and $y = x^2 - x - 2$
   (b) between $y = \cos(2x)$ and $y = \sin(x)$ for $\frac{-\pi}{2} \leq x \leq \frac{3\pi}{2}$
   (c) between $x = y^2 - 4y$ and $x = 2y - y^2$
   (d) between $y = e^x$ and $y = e^{5x}$ for $-2 \leq x \leq 1$

2. Find the area enclosed by the curves $y = \ln x$, $y = \ln 2 + \ln(x - 1)$ and $y = 2$. Which approach is easier, integrating with respect to x or y?

3. Consider an object S whose base is a circle of radius r. Suppose that the cross-sections along one of the diameters of this circle are isosceles right triangles such that the hypotenuse does *not* rest on the base.

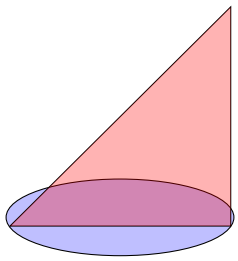

Figure 1: A cross section of S depicted in red

   Show that the volume of S is
$$V = \frac{8}{3}r^3.$$

4. Consider the following problems on volumes of revolution using disks/washers.
   (a) Find the volume of the object created by rotating the region trapped between $f(x) = x - x^3$ and $y = 0$ about the x-axis.
   (b) Find the volume of the object created by rotating the region trapped between $y = x$, $x = 0$ and $y = 3$ about the y-axis.

(c) Find the volume of the object created by rotating the region trapped between $y = (x - 2)^2$, $y = 0$ and $x = 1$ about the the line $x = 1$.

(d) Find the volume of the object created by rotating the region trapped between $y = \cos(x)$, $y = -\cos(x)$ *which contains the origin* about the the line $y = -1$.