Math 141 Tutorial 8

Main problems

1. Consider the following problems on volumes of revolution using **cylindrical shells**.

   (a) Find the volume of the object created by rotating the region trapped between $y = \ln(x^2)$, the x-axis, $x = 1$ and $x = \sqrt{e}$ about the y-axis.

   (b) Find the volume of the object created by rotating the region trapped between $y = x$ and $y = x^2$ about the line $y = 1$

2. Using disks or cylindrical shells, find the volume of each object.

   (a) The object created by rotating the region in the first quadrant trapped between $y = x^2\sqrt{1-x}$ and $y = 0$ about the line $x = -1$.

   (b) The object created by rotating the region in the first quadrant trapped between $y = |x-2|$, $y = |x-2| + 1$, $x = 1$ and $x = 3$ about the line $y = -3$.

3. The integral below represents the volume of a solid object obtained by rotating a region R about the x-axis.

$$\int_0^{\sqrt{\pi}} \pi \sin^2(x^2)\, dx.$$

   Find the volume obtained by rotating R about the y-axis. Include a sketch of R and the new solid in your solution.

4. Find an interval $[a, b]$ of length 6 such that the average value of x^2 over $[a, b]$ is 4.

5. Determine whether each improper integral is convergent or divergent and justify your statement. When convergent, is it always possible to compute the value of the improper integral? If so, compute it.

   (a) $\displaystyle\int_1^2 \frac{1}{(x-2)^2}\, dx$ (b) $\displaystyle\int_0^\infty \sin\theta e^{\cos\theta}\, d\theta$ (c) $\displaystyle\int_0^{10} \frac{1}{x^2 + 6x - 55}\, dx$

Extra Practice problems:

1. Consider the object created by rotating the region trapped between $y = x$ and $y = x^2$ about the y-axis.

   (a) Find the volume using disks.

   (b) Find the volume using cylindrical shells.

2. Find the volume of the object created by rotating the region in the first quadrant trapped between $f(x) = |x|$, $y = 1 - |x|$ and $x = 0$ about the y-axis.

3. Find the volume of the object created by rotating the region in the first quadrant trapped between $y = x$ and $y = \sqrt{x}$ about the x-axis.

4. Find the volume of the object created by rotating the region bounded by the curves $x = 0$ and $x = 9 - y^2$ about $x = -1$.

5. Find the volume of the object created by rotating the region bounded by the curves $y = x^2 + 1$ and $y = 9 - x^2$ about $y = -1$.

6. A bathtub which can hold a maximum of 300 litres is currently filled with 290 liters of water. At time $t = 0$, water begins to pour into the tub at a (varying) rate of $10\sqrt{t}$ litres per minute, while at the same time water begins to drain out of the tub at a rate of $2t^2$ litres per minute (where t is measured in minutes). Assuming these rates continue indefinitely, does the bathtub overflow?

7. Compute the average value of the function $f(x) = \dfrac{e^{1/x}}{x^2}$ on $[1, 4]$.