Math 140 **Tutorial 2**
January 13 - 16

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1. Sketch the graph of a function f that satisfies all of the following conditions

   (a) $\lim_{x\to 0} f(x) = +\infty$
   (b) $\lim_{x\to 2^+} f(x) = -\infty$
   (c) $\lim_{x\to 2^-} f(x) = 3$
   (d) $\lim_{x\to 1} f(x) = 0$
   (e) $\lim_{x\to -1} f(x)$ does not exist.

2. Investigate the following limits. Back up your answers by sketching (or looking up) the graphs of the associated functions, and by plugging in individual values.

   (a) $\lim_{x\to -1} x^2 + 1$
   (b) $\lim_{x\to -1} \frac{x+1}{x^3-1}$
   (c) $\lim_{x\to -1} \frac{x+1}{x^3+1}$
   (d) $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$
   (e) $\lim_{x\to 1^+} \frac{x+1}{x^3-1}$
   (f) $\lim_{x\to 1^-} \frac{x+1}{x^3-1}$
   (g) $\lim_{x\to 2} \frac{2-x}{\sqrt{x+2}-2}$

3. Given that

$$\lim_{x\to 2} f(x) = 4,\ \lim_{x\to 2} g(x) = -2,\ \lim_{x\to 2} h(x) = 0$$

   find the value of the following limits if they exist, or explain why the limits don't exist.

   (a) $\lim_{x\to 2}(f(x) + 5g(x))$
   (b) $\lim_{x\to 2} g(x)^3$
   (c) $\lim_{x\to 2} \frac{f(x)g(x)}{h(x)}$
   (d) $\lim_{x\to 2} \cos(h(x))$

4. If the limit $\lim_{x\to a} f(x)$ does not exist and the limit $\lim_{x\to a} g(x)$ does not exist, does it follow that the limit

$$\lim_{x\to a} (f(x) + g(x))$$

   does not exist as well?

5. In this exercise we will be using the $\epsilon - \delta$ definition of limits to calculate some limits.

   (a) For $f(x) = x + 1$, for each value of ϵ find a value of δ such that

   $$\text{if } |x - 1| < \delta \text{ then } |f(x) - f(1)| < \epsilon$$

   i. $\epsilon = 0.1$
   ii. $\epsilon = 0.01$
   iii. $\epsilon = 0.001$

   Can you write a formula for δ in terms of ϵ that will work for any value of ϵ? Write the limit statement for $f(x)$ that we are trying to justify.

   (b) For $f(x) = \dfrac{x}{5}$, for each value of ϵ find a value of δ such that

   $$\text{if } |x - 3| < \delta \text{ then } |f(x) - f(3)| < \epsilon$$

   i. $\epsilon = 0.1$
   ii. $\epsilon = 0.01$
   iii. $\epsilon = 0.001$

   Can you write a formula for δ in terms of ϵ that will work for any value of ϵ? Write the limit statement for $f(x)$ that we are trying to justify.

   (c) For $f(x) = x^3$, for each value of ϵ find a value of δ such that

   $$\text{if } |x - 0| < \delta \text{ then } |f(x) - f(0)| < \epsilon$$

   i. $\epsilon = 0.1$
   ii. $\epsilon = 0.01$
   iii. $\epsilon = 0.001$

   Can you write a formula for δ in terms of ϵ that will work for any value of ϵ? Write the limit statement for $f(x)$ that we are trying to justify.

   (d) For $f(x) = \dfrac{1}{x^2}$, for each value of M find a value of δ such that

   $$\text{if } |x - 0| < \delta \text{ then } f(x) > M$$

   i. $M = 10$
   ii. $M = 100$
   iii. $M = 1000$

   Can you write a formula for δ in terms of M that will work for any value of M? Write the limit statement for $f(x)$ that we are trying to justify.