Math 140 **Tutorial 4**
Jan 27 - 30

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1. By calculating the limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

find the derivatives of the following functions at $x = a$:

(a) $f(x) = \dfrac{1}{x}$ for $a \neq 0$.

(b) $f(x) = \sqrt{x}$ for $a \neq 0$.

(c) $f(x) = \dfrac{1}{\sqrt{x}}$ for $a \neq 0$.

(d) $f(x) = x^3 - 3x + 5$.

(e) $f(x) = x^{1/4}$.

(f) * $f(x) = \sin(x^2)$.

[Hint: For (f) you can use that as h approaches 0, $\sin h \approx h$ and $\cos h \approx 1 - h^2$]

GROUP WORK 1, SECTION 2.7

Follow that Car

The distance travelled by a car is given by $d(t) = 8\left(t^3 - 6t^2 + 12t\right)$, where d is in miles and t is in hours.

1. Draw a graph of $d(t)$ from $t = 0$ to $t = 3$.

2. Does the car ever stop?

3. What is the average velocity over $[1, 3]$? over $[1.5, 2.5]$? over $[1.9, 2.1]$?

4. Estimate the instantaneous velocity at $t = 2$. Give a physical interpretation of your answer.

GROUP WORK 3, SECTION 2.7

Connect the Dots

A company does a study on the effect of production value p of an advertisement on its consumer approval rating A. After interviewing eight focus groups, they come up with the following data:

Production Value	Consumer Approval
\$1000	32%
\$2000	33%
\$3000	46%
\$3500	55%
\$3600	61%
\$3800	65%
\$4000	69%
\$5000	70%

Assume that $A\,(p)$ gives the consumer approval percentage as a function of p.

1. Estimate A' (\$3500). Is this likely to be an overestimate or an underestimate?

2. Interpret your answer to Problem 1 in real terms. What does your estimate of A' (\$3500) tell you?

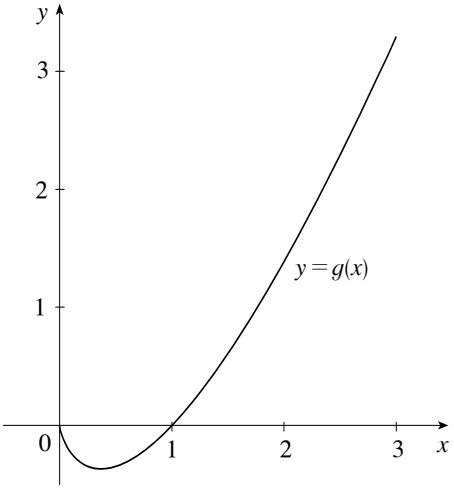
3. What are the units of $A'\,(p)$?

4. Estimate A' (\$3550). Is your estimate better or worse than your estimate of A' (\$3500)? Why?

GROUP WORK 1, SECTION 2.8

Tangent Lines and the Derivative Function

The following is a graph of $g(x) = x \ln x$.



It is a fact that the derivative of this function is $g'(x) = \ln x + 1$.

1. Sketch the line tangent to $g(x)$ at $x = 2$ on the graph above.

2. Find an equation of the tangent line at $x = 2$.

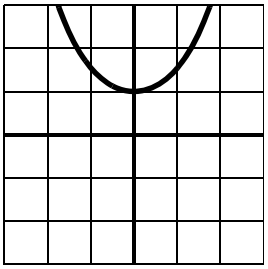
3. Now sketch the line tangent to $g(x)$ at $x = \dfrac{1}{e} \approx 0.368$.

4. Find an equation of the tangent line at $x = \dfrac{1}{e}$.

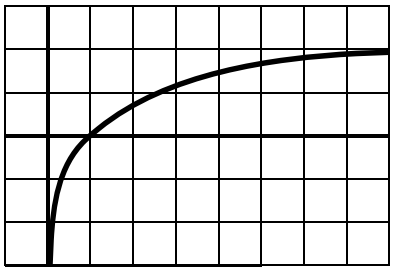
GROUP WORK 3, SECTION 2.8

The Derivative Function

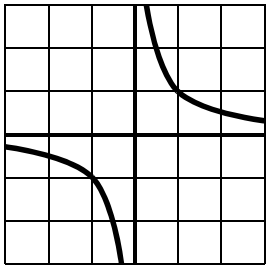
The graphs of several functions f are shown below. For each function, estimate the slope of the graph of f at various points. From your estimates, sketch graphs of f'.



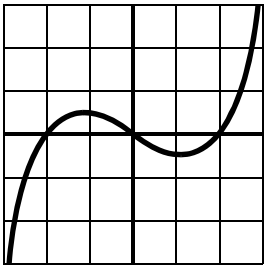
Graph 1



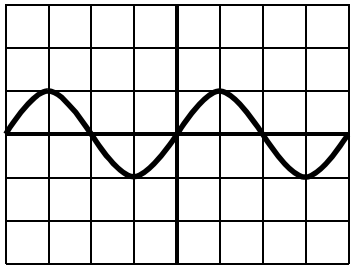
Graph 2



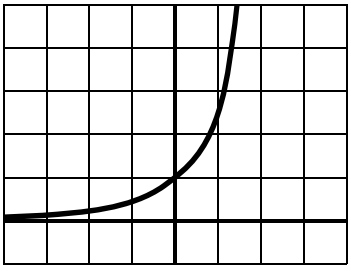
Graph 3



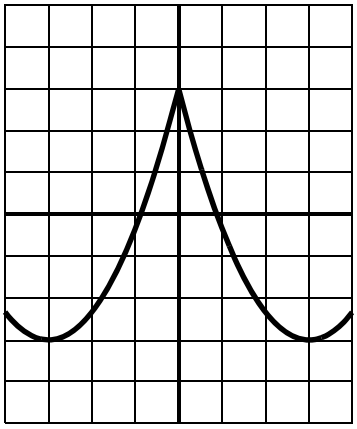
Graph 4



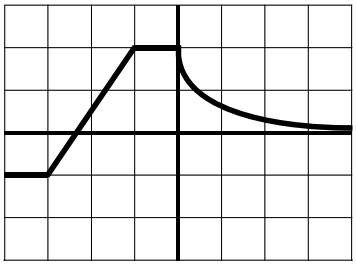
Graph 5



Graph 6



Graph 7



Graph 8