Math 140 **Tutorial 7**

February 17 - 20

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1. Find $\frac{dy}{dx}$ by using implicit differentiation

   (a) $y^2 + x^2 = 1$

   (b) $\sin x + \cos y = x^3 - 3y^2$

   (c) $2(x^2 + y^2)^2 = 9(x^2 - y^2)$

   (d) $\sin x \cos y = \sin^2(x + y)$. (Check out the graph of this equation)

2. Use implicit differentiation to find an equation for the tangent line to the curve at the given point.

   (a) $x^2 + 2xy + 4y^2 = 12$ at $(2, 1)$

   (b) $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at $(0, \frac{1}{2})$

   (c) $y^2(y^2 - 4) = x^2(x^2 - 5)$ at $(0, -2)$

3. Use implicit differentiation to prove that

$$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2 - 1}}$$

   Hint: let $y(x) = \sec^{-1}(x)$, so that $x = \sec(y(x))$. To simplify the expression you get in the end, it might be helpful to rewrite things using trig identities.

4. In this problem, we'll evaluate the derivative of $f(x) = (\sin x)^{\ln x}$ in two 'different' ways.

   (a) Use the fact that $\ln x$ is the inverse of e^x to write $f(x) = \exp(\ln(f(x)))$ and then use the Chain Rule to evaluate $f'(x)$.

   (b) Take logarithms on both sides of $f(x) = (\sin x)^{\ln x}$ and then use implicit differentiation to evaluate $f'(x)$. Note: this technique is known as *logarithmic differentiation.*

5. For the following functions $y(x)$. Write down the domain and range of the function y and find the derivative of the function.

   (a) $y = \sin^{-1}\left(\sqrt{\sin x}\right)$

   (b) $y = \sqrt{\tan^{-1}(x)}$

   (c) $y = \ln(\sec x + \tan x)$

   (d) $y = \ln(xe^{x^2})$