Math 141 Tutorial 11

Main problems

1. Find the values of p for which these series are convergent.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n \ln n[\ln(\ln n)]^p}$ (c) $\displaystyle\sum_{n=1}^{\infty} n(1+n^2)^p$

2. Using the direct comparison test, determine whether the following series are convergent or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{4+2^n}$

3. Using the limit comparison test, determine whether the following series are convergent or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^2 e^{-n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}e^n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

4. Determine whether the following series are convergent or divergent.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{n^3}{n^4-1}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n}$

Practice Problems

1. Determine whether the following series are convergent or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(\ln n)^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{5^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{3^n + 4^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{n \sin n}{n^2 + 1}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{(2n+7)^3}$