Lecture hours 22 - 23

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This tutorial is also a review for the second midterm.

Problem 41. Let T be the linear transformation induced by the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 2 \end{bmatrix},$$

and S be the linear transformation induced by the matrix

$$\begin{bmatrix} -2 & 0 \\ 0 & 1 \\ 1 & 3 \end{bmatrix}.$$

a) Find the matrix that induces the linear transformation $S \circ T$.

b) Find the rank and nullity of $S \circ T$.

c) Find a basis for the image of $S \circ T$ and a basis for the kernel of $S \circ T$.

d) Is the linear transformation $S \circ T$ invertible? If so, find the inverse. If not, explain why.

e) If $\mathcal{B}$ is the standard basis of $\mathbb{R}^3$ find the matrix for $S \circ T$ in the $\mathcal{B}$ coordinates.

f) Let $\mathcal{D}$ the basis of $\mathbb{R}^3$ given by

$$\mathfrak{D} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\}.$$

Find the matrix for $S \circ T$ in the $\mathcal{D}$ coordinates.

Problem 42 (Distances)**.** Let P be the plane spanned by the vectors

$$\vec{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{z} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}.$$

1. Assume S is the projection onto P

   a) Find the distance from the vector $\vec{y} \times \vec{z}$ to the vector $S(\vec{y} \times \vec{z})$.

   b) Find the distance from the vector $\vec{z}$ to the vector $S(\vec{z})$.

2. Assume T is the reflection across P.

   c) Find the distance from the vector $\vec{y} \times \vec{z}$ to the vector $T(\vec{y} \times \vec{z})$.

   d) Find the distance from the vector $\vec{y}$ to the vector $T(\vec{y})$.