Math 140 **Tutorial 3**

January 20 - 23

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1. Evaluate the following limits if the exist:

   (a) $\lim_{x \to \pi} \sin(x + \sin x)$

   (b) $\lim_{x \to 0} x \sin\left(\frac{\pi}{x^2}\right) + \cos x$

   (c) $\lim_{t \to 0} \frac{1}{t\sqrt{t+1}} - \frac{1}{t}$

   (d) $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$

   (e) $\lim_{x \to -1} \frac{x^2 - 4x}{x^2 - 3x - 4}$

   (f) $\lim_{x \to 2} \arctan\left(\frac{x^2 - 4}{3x^2 - 6x}\right)$

2. Which of the following functions is continuous on the given intervals?

   (a) $g(t) = \frac{t^2 + 5t}{2t + 1}$, $[0, \infty)$

   (b) $f(x) = \begin{cases} \frac{x-3}{x^2-9} & x \neq 3 \\ 0 & x = 3 \end{cases}$, $[0, \infty)$

   (c) $f(x) = \begin{cases} \frac{x-3}{x^2-9} & x \neq 3 \\ \frac{1}{6} & x = 3 \end{cases}$, $[0, \infty)$

   (d) $f(x) = \begin{cases} \sin(x) & x < \pi/4 \\ \cos(x) & x \geq \pi/4 \end{cases}$, $[0, \infty)$

3. Determine if the statements below are true or false. If true, justify your answer; if not, provide a counterexample.

   (a) If $f(x)$ is continuous on the interval $(0, 1)$, then $f(x)^2$ is also continuous on the same interval.

   (b) If $f(x)^2$ is continuous on the interval $(0, 1)$, then $f(x)$ is also continuous on the same interval.

   (c) If $f(x)$ and $g(x)$ are both continuous on the interval $(0, 1)$, then $f(x) + g(x)$ is continuous on the same interval.

   (d) If $f(x) + g(x)$ is continuous on the interval $(0, 1)$, then so are $f(x)$ and $g(x)$ on the same interval.

(e) If both $f(x)+g(x)$ and $f(x)-g(x)$ are continuous on $(0,1)$, then so are $f(x)$ and $g(x)$ on the same interval.

4. Use the Intermediate Value Theorem to show that the following equations have a solution on the given intervals

   (a) $\cos x = x$ on $(0,1)$

   (b) $\ln x = e^{-x}$ on $(1,2)$

5. In this question we will use the Intermediate Value Theorem to approximate a solution to the equation
$$x = e^{x-2}$$

   (a) First, show that this equation has a solution in the interval $(0,1)$.

   (b) Now split this interval into two halves $(0,\frac{1}{2})$ and $(\frac{1}{2},1)$. Show that one of these two intervals has a solution to our equation. Which one is it?

   (c) Split the interval you got in part (b) into two halves again to show that one of the four intervals $(0,\frac{1}{4}),(\frac{1}{4},\frac{1}{2}),(\frac{1}{2},\frac{3}{4}),(\frac{3}{4},1)$ has a solution to our equation. Which one is it?

   (d) Repeat this process of splitting the intervals in halves until you have narrowed down the solution to 2 decimal places.

6. A mountain climber starts climbing a mountain at 10am and they reach the top at 2pm where they stay for the night. The next day the climber begins their descent at 10am and reaches the bottom at 2pm, they take the exact same path they took the day before. Is there necessarily a time between 10am and 2pm at which the climber was at the exact same spot on both days? (Note the the climber doesn't necessarily ascend and descend at the same speed)

7. Evaluate the following limits if they exist

(a) $\lim_{x\to+\infty} \cos x$

(b) $\lim_{x\to+\infty} \dfrac{x+1}{x}$

(c) $\lim_{x\to+\infty} \dfrac{x}{x+1}$

(d) $\lim_{x\to-\infty} \dfrac{x}{x^2+1}$

(e) $\lim_{x\to+\infty} \dfrac{x^3-3x^2+3x-1}{3x^3+27x^2+9x+1}$

(f) $\lim_{x\to1^+} \arctan\left(\dfrac{1}{x-1}\right)$

8. Find a rational function $f(x)$ such that $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x)$. Find another rational function $g(x)$ such that $\lim_{x\to\infty} g(x) \neq \lim_{x\to-\infty} g(x)$.

Reminder: a rational function is a polynomial divided by a polynomial.