11 Lecture hours 24-26

Definitions and Theorems

Definition (Transpose of a matrix Matrix).

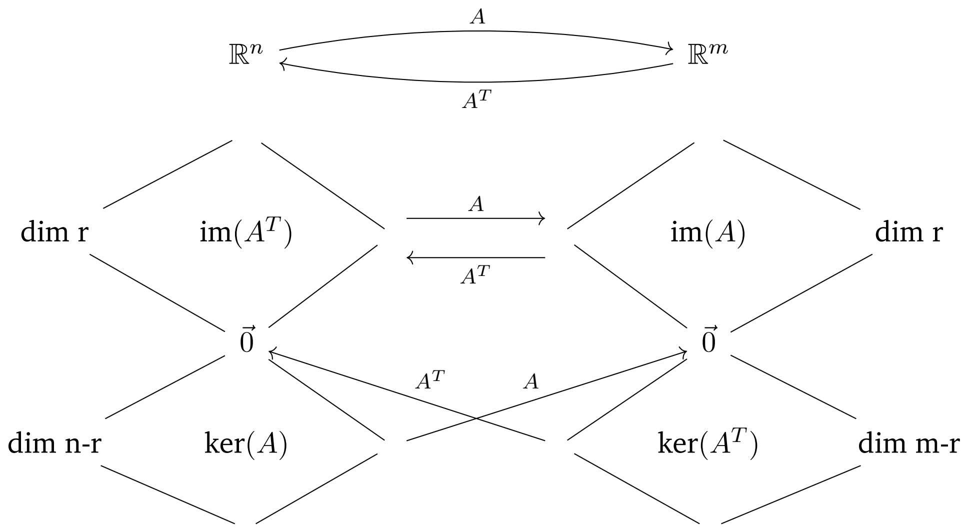
The transpose of a matrix A is A^T, and it has columns the rows of A (same order).

Definition (Perpendicular complement).

Let V be a subspace of $\mathbb{R}^n$, then W is called the "perpendicular complement" of V and denoted $V^\perp$ (pronounced "V perp", symbol $\perp$ is a superscript) if W contains all vector in $\mathbb{R}^n$ that are perpendicular to all vectors in V.

Definition (Fundamental subspaces of linear algebra).

For any m by n matrix A we have



$$(\ker A)^\perp = \mathrm{im}(A^T), \qquad (\mathrm{im} A)^\perp = \ker(A^T).$$

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Problem 43 (Fundamental subspaces of linear algebra)**.** Consider the matrix

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Find $\ker(A)$, $\operatorname{im}(A)$, $\ker(A^T)$, and $\operatorname{im}(A^T)$. For each of these subspaces, determine the value of n for which they are a subspace of $\mathbb{R}^n$.

Problem 44 (Transpose of a matrix). Let A be an invertible $n \times n$ matrix.

a) Explain why A^T is invertible.

b) Explain why $(A^T)^{-1} = (A^{-1})^T$. (Hint: $I^T = I$.)

Problem 45 (Least squares - Normal Equations). You are given data points $(x, y) = (1, 1), (2, 3), (-1, 3)$. Use a least squares line of best fit to predict the y-value when $x = 7$.