5 Lecture hours 9-11

This Tutorial is a review for the midterm.

Definitions

Definition (Subspace - Span version)**.** A subspace of $\mathbb{R}^n$ is a set of vectors in $\mathbb{R}^n$ that can be described as a span of vectors.

Definition (Subspace - Standard version)**.** A subspace of $\mathbb{R}^n$ subset V of $\mathbb{R}^n$ with the following properties:

(i) V is a non-empty set.

(ii) If $\vec{u}$ is in V , $\vec{u}$ is also in V for any scalar $k \in \mathbb{R}$ (We say V is closed under scalar multiplication.)

(iii) If $\vec{u}$ and $\vec{w}$ are in V, their sum $\vec{u} + \vec{w}$ is also in V. (We say V is closed under addition.)

Definition (Basis)**.** The vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ are a basis of a subspace V if they span V and are linearly independent. In other words, a basis of a subspace V is the minimal set of vectors needed to span all of V.

Definition (Dimension of a subspace)**.** The dimension of the subspace V is the number of vectors in a basis of V.

Definition (Linear Transformations)**.** We define a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ as a function with two properties:

1. $T(\vec{x} + \vec{y}) = T(\vec{x}) + T(\vec{y})$ for all $\vec{x}, \vec{y} \in \mathbb{R}^n$ (we say T preserves vector addition)
2. $T(\vec{x}) = cT(\vec{x})$ for all $\vec{x} \in \mathbb{R}^n$ and $c \in \mathbb{R}$ (we say T preserves scalar multiplication)

Problem 21 (Subspace)**.** Suppose that V is the set of all solutions of the homogeneous system

$$\begin{cases} x_1 + 2x_2 - 2x_3 + 2x_4 - x_5 = 0, \\ x_1 + 2x_2 - x_3 + 3x_4 - 2x_5 = 0, \\ 2x_1 + 4x_2 - 7x_3 + x_4 + x_5 = 0. \end{cases} \tag{5.1}$$

Show that V is a subspace of $\mathbb{R}^5$.

Problem 22 (Basis of a subspace)**.** Let U be a subspace of $\mathbb{R}^5$ defined by $U = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 : x_1 = 3x_2, x_3 = 7x_4\}$. Find a basis for U.

Problem 23 (Basis of a subspace)**.** Suppose $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is a basis of $\mathbb{R}^4$. Prove that

$$\{\vec{v}_1 + \vec{v}_2, \vec{v}_2 + \vec{v}_3, \vec{v}_3 + \vec{v}_4, \vec{v}_4\}$$

is also a basis of $\mathbb{R}^4$.

Problem 24 (Linear Transformations). Find $a, b \in \mathbb{R}$ such that $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$T(x, y, z) = (2x - 4y + 3z + b, 6x + axyz)$$

is a linear transformation.

Problem 25 (Linear Transformations). Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation and let $\vec{v_1}, \ldots, \vec{v_k}$ be vectors in R^n. True or false? If false, give a counter-example. If true, explain why.

a) If the vectors $T(\vec{v_1}), \ldots, T(\vec{v_k})$ are linear independent, then $\vec{v_1}, \ldots, \vec{v_k}$ are also linear independent.

b) If the vectors $\vec{v_1}, \ldots, v_k$ are linear independent, then $T(\vec{v_1}), \ldots, T(\vec{v_k})$ are also linear independent.