Math 140 **Tutorial 11**

March 24 - 27

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1. Sketch the graphs of a continuous functions on the interval $[1, 5]$ and which satisfies the following properties

   (a) Absolute maximum at 1, absolute minimum at 3, local minimum at 2 and local maximum at 4.

   (b) Absolute maximum at 2, absolute minimum at 5, 4 is a critical number but there is no local maximum or minimum there.

2. Find the absolute maximum and absolute minimum values of f on the given interval.

   (a) $f(x) = x^3 - 6x + 5,\ [-2, 5]$

   (b) $f(x) = x + \dfrac{1}{x},\ [0.2, 4]$

   (c) $f(x) = x^a(1 - x)^b,\ [0, 1]$ (a and b are positive, real numbers)

   (d) $f(t) = 2\cos t + \sin 2t,\ [0, \pi/2]$

3. Prove that the function
$$f(x) = x^{101} + x^{51} + x + 1$$
has neither a local maximum or a local minimum.

4. Find two numbers whose difference is 10 and whose product is minimal.

5. Which point on the line
$$y = 2x + 1$$
is closest to the origin?

6. Find the points on the ellipse $4x^2 + y^2 = 4$ which are farthest from the point $(0, 1)$.

7. A cylindrical can is made from material such that the top and bottom cost twice as much as the sides. What dimensions will minimize the cost of a 4L can?

8. A person is standing at coordinates $(1, 4)$. They are making their way back to their house which has coordinates $(6, 8)$, but first they need to pass by a river, which goes down the x-axis. What is the fastest path for this trip? At what x-coordinate should this person reach the river?