

Math 141 Tutorial 6

Main problems

1. Compute the following using trigonometric substitution.

(a) $\int t^3(3t^2 - 4)^{5/2} dt$

(c) $\int \frac{1}{\sqrt{9x^2 - 36x + 37}} dx$

(b) $\int \frac{\sqrt{x^2 + 16}}{x^4} dx$

(d) $\int \frac{(x+3)^5}{(40 - 6x - x^2)^{3/2}} dx$

2. Use long division to express each of the following functions $f(x)$ as a proper fraction. That is, find polynomials $S(x)$, $R(x)$, $Q(x)$ such that

$$f(x) = S(x) + \frac{R(x)}{Q(x)}$$

and $\deg R < \deg Q$.

(a) $f(x) = \frac{x^2 + 1}{x + 1}$

(c) $f(x) = \frac{x^3 + x^2 - 4x + 6}{x^2 - 2x + 2}$

(b) $f(x) = \frac{2x^3 - x}{x + 3}$

(d) $f(x) = \frac{x^4 + x + 1}{(x^2 + 1)(x - 1)}$

3. Write out the partial fraction decomposition for each of the following rational functions.

(a) $f(x) = \frac{1}{(x+a)(x+b)}$ when $a \neq b$

(c) $f(x) = \frac{x^2 + x + 1}{(x+1)^2(x+2)}$

(b) $f(z) = \frac{3z^2 - z + 8}{z^3 + 4z}$

(d) $f(t) = \frac{t^2 + t + 1}{t^4 + 2t^2 + 1}$

4. Integrate the following rational functions.

(a) $\int_0^{1/2} \frac{1}{1-x^2} dx$

(c) $\int \frac{1}{x^3 + x^2 - x - 1} dx$

(b) $\int \frac{1}{x^2(x+1)^2} dx$

(d) $\int \frac{x^4 + x + 1}{(x^2 + 1)(x - 1)} dx$

$$(e) \int_0^4 \frac{y-1}{y^2+4y+3} dy$$

$$(f) \int_2^4 \frac{t+1}{t^3-t^2} dt$$

5. Integrate the following rational functions by using partial fractions. Use the rational root theorem (stated below) and long division.

$$(a) \int_0^1 \frac{x}{x^4+2x^3+2x^2+2x+1} dx$$

$$(b) \int \frac{x-1}{2x^4+x^3-6x^2+x+2} dx$$

Theorem (Rational Root Theorem). *Consider a polynomial*

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$$

where the coefficients a_0, a_1, \dots, a_n are integers and $a_0 \neq 0$. If r is a rational root of $f(x)$, i.e. if $r \in \mathbb{Q}$ and $f(r) = 0$, then writing r in it's lowest terms

$$r = \pm p/q$$

we have that p is a factor of a_0 and q is a factor of a_n .

Practice Problems

6. Compute the following using any method available to you

$$(a) \int \frac{1}{(x^2-1)^2} dx$$

$$(h) \int_1^3 \frac{1}{\sqrt{x} + x\sqrt{x}} dx$$

$$(b) \int_1^2 \frac{3x^2+6x+2}{x^2+3x+2} dx$$

$$(i) \int \frac{x^2+x}{x^2+2x} dx$$

$$(c) \int \arcsin(x) dx$$

$$(j) \int_{-3}^{-1} (x+2)^{99} dx$$

$$(d) \int \sqrt{x^2+1} dx$$

$$(k) \int \frac{x}{\sqrt{x^2+2}} dx$$

$$(e) \int_{-10^{10}}^{10^{10}} x^{100} \sin(x^5) dx$$

$$(l) \int \frac{x^3 e^{x^2}}{(x^2+1)^2} dx$$

$$(f) \int \frac{e^{1/x}}{x^2} dx$$

$$(m) \int \tan^7(x) \sec^4(x) dx$$

$$(g) \int_1^e \frac{\ln(x)}{x^2} dx$$

$$(n) \int \frac{\sqrt{25-x^2}}{x^2} dx$$

Hint: for problem (e), use symmetry (i.e. is the function even or odd?).

Challenge Problems

7. Solve the following integrals

(a) $\int (1 + \ln(x)) \ln(\ln(x)) dx$

(b) $\int \sqrt{1 - \sqrt{x}} dx$

(c) $\int \frac{1}{1 + \cos^2(x)} dx$