Math 141 Tutorial 2

Main problems

1. Using the summation formulas seen in class, provide a closed form for the following summations in terms of n.

   (a) $\displaystyle\sum_{i=0}^{n}(i+1)$

   (b) $\displaystyle\sum_{i=2}^{n}(2i+n)$

   (c) $\displaystyle\sum_{i=-1}^{n}(i+2)^2$

   (d) $\displaystyle\sum_{i=0}^{n}(i-2)^2$

   (e) $\displaystyle\sum_{i=-n}^{0}-i^3$

   (f) $\displaystyle\sum_{i=-n}^{n}\left(i^3+3i^2n+3in^2+n^3\right)$

2. Evaluate the definite integrals by either taking the limit of (left or right) Riemann sums or interpreting the integral as an area.

   (a) $\int_0^2 2x \, \mathrm{d}x$

   (b) $\int_1^4 (3 - x) \, \mathrm{d}x$

   (c) $\int_0^2 (2x^2 + 1) \, \mathrm{d}x$

   (d) $\int_1^5 (x^2 + 2x) \, \mathrm{d}x$

   (e) $\int_0^3 x^3 \, \mathrm{d}x$

   (f) $\int_{-2}^2 |2x| \, \mathrm{d}x$

3. Suppose that $f, g : [a, b] \to \mathbb{R}$ are continuous functions and let $k \in \mathbb{R}$ be a constant. By using the corresponding properties for summations, prove the following

   (a) $\displaystyle\int_a^b (kf(x))\,\mathrm{d}x = k \int_a^b f(x)\,\mathrm{d}x$

   (b) $\displaystyle\int_a^b (f(x) + g(x))\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x + \int_a^b g(x)\,\mathrm{d}x$

Challenge problems

4. Using Riemann sums, show that

$$\int_{a(x)}^{b(x)} t \, \mathrm{d}t = \frac{(b(x))^2 - (a(x))^2}{2}$$

5. What do you expect the value of

$$\int_{-\pi/2}^{3\pi/2} \cos x \, \mathrm{d}x$$

to be? *Hint: use symmetry.*

6. Let $f(x) = x^2$.
   (a) Using Riemann sums, determine the function

$$F(x) = \int_0^x f(t) \, \mathrm{d}t.$$

   (b) Determine the derivative $F'(x)$ of $F(x)$. How does this function relate to f?