Math 141 Tutorial 1

Main problems

1. Compute the following sums and simplify your answer to a single number

   (a) $\displaystyle\sum_{i=-2}^{2}(2i+1)$

   (b) $\displaystyle\sum_{i=-2}^{2} 2i+1$

   (c) $\displaystyle\sum_{i=1}^{4}\frac{1}{i}+1$

   (d) $\displaystyle\sum_{i=2}^{5}\frac{i^2-2i+1}{i-1}$

   (e) $\displaystyle\sum_{i=-1}^{1} 2^i$

   (f) $\displaystyle\sum_{i=1}^{4}\log_{24} i$

2. Using Riemann sums with n subintervals, approximate the area under the following curves. Draw a picture in order to visualize the rectangles whose areas are being summed.

   (a) With $n = 4$ and either left or right Riemann sums, approximate the area under the curve of $f(x) = 2x - 4$ from $x = 2$ to $x = 4$. What is the true area?

   (b) With $n = 4$ and both left and right Riemann sums, approximate the area under $f(x) = x^3$ from $x = 0$ to $x = 2$. By using both left and right Riemann sums, one obtains upper and lower bounds for the true area. Which method gives a lower bound on the true area? How do you explain this?

   (c) What do you expect to happen if we repeat the process in part (b) with $n = 6$? Should we be closer or further from the "true area"?

3. For each function $f(x)$ and interval, write the Riemann sum that approximates the area under the curve for any $n \geq 1$.

   (a) The area under $f(x) = 2x$ between $x = 0$ and $x = 2$

   (b) The area between $f(x) = -x + 3$, the x-axis and the lines $x = 1$ and $x = 4$

   (c) The area under $f(x) = 2x^2 + 1$ between $x = 0$ and $x = 2$

4. We tackle now the inverse process: for each of the following Riemann sums find a function $f(x)$ and values a and b such that the limit expresses the area above/below $f(x)$ between $x = a$ and $x = b$.

Note: There may be several valid answers for each problem.
Hint: Every sum appearing in this problem can be realized as a right Riemann sum.

(a) $\displaystyle\sum_{i=1}^{n} \left(\frac{3i}{n} - 3\right) \frac{3}{n}$

(b) $\displaystyle\sum_{i=1}^{n} \frac{\left(2 + \frac{i}{n}\right)^2 + \left(2 + \frac{i}{n}\right)}{n}$

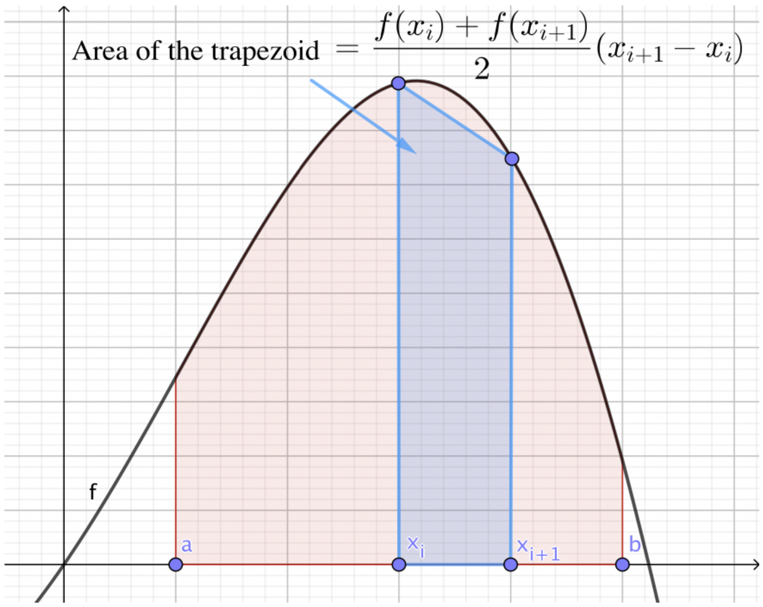
(c) $\displaystyle\sum_{i=1}^{n} e^{\frac{6i}{n} - 2} \frac{6}{n}$

(d) $\displaystyle\sum_{i=1}^{n} \left(\frac{3i}{2n} + \frac{1}{2}\right) \tan\left(\frac{3i}{2n} - \frac{3}{2}\right) \frac{3}{2n}$

Challenge problems

5. Using a combination of left and right Riemann sums with $n = 4$, find both an upper and lower bound for the area under $f(x) = 2x - x^2$ from $x = 0$ to $x = 2$.

6. Another method for approximating the area under a curve is known as the Trapezoidal Rule. Here, instead of using rectangles, trapezoids are used. The area of the trapezoid below $f(x)$ and between the nodes x_i and x_{i+1} is given by



(a) Write down an equation for the Trapezoidal Rule when using a fixed number n of trapezoids to approximate the area under $f(x)$ between $x = a$ and $x = b$.

(b) Can you write down the formula for the Trapezoidal Rule in terms of left and right Riemann sums?