Math 141 Tutorial 5

Main problems

1. Compute the following integrals using integration by parts (IBP)

   (a) $\displaystyle\int_0^{\ln(2)} se^s \, \mathrm{d}s$

   (b) $\displaystyle\int x\cosh(x)\, \mathrm{d}x$

   (c) $\displaystyle\int_0^1 \arctan(x)\, \mathrm{d}x$

   (d) $\displaystyle\int_1^e \ln(x^8)\, \mathrm{d}x$

   (e) $\displaystyle\int x\sec^2 x \, \mathrm{d}x$

   (f) $\displaystyle\int \arcsin(x)\, \mathrm{d}x$

   (g) $\displaystyle\int \frac{\ln x}{x^2}\, \mathrm{d}x$

2. (a) Using integration by parts, prove the following reduction formula:

   $$\int (\ln x)^n \ \mathrm{d}x = x\,(\ln x)^n - n\int (\ln x)^{n-1} \ \mathrm{d}x.$$

   (b) Using your result from (a), determine

   $$\int_1^e (\ln x)^3 \ \mathrm{d}x.$$

3. Compute the following trigonometric integrals.

   (a) $\displaystyle\int_0^{\pi/2} \sin^8(x)\cos^5(x)\, \mathrm{d}x$

   (b) $\displaystyle\int \sin^5(x)\, \mathrm{d}x$

   (c) $\displaystyle\int_{-\pi/4}^0 \tan^3(x)\sec^4(x)\, \mathrm{d}x$

   (d) $\displaystyle\int \sin^2(x)\cos^4(x)\, \mathrm{d}x$

   (e) $\displaystyle\int \tan^3(x)\sec(x)\, \mathrm{d}x$

   (f) $\displaystyle\int_0^{\pi/10} \cos^4(5x)\, \mathrm{d}x$

4. Compute the integrals below using Trigonometric Substitution

(a) $\displaystyle\int \frac{\sqrt{2-x^2}}{x^2}\,\mathrm{d}x$

(b) $\displaystyle\int_{\sqrt{3}}^{2} \frac{\sqrt{x^2-3}}{x}\,\mathrm{d}x$

(c) $\displaystyle\int \sqrt{7+6x-x^2}\,\mathrm{d}x$

(d) $\displaystyle\int_0^a x^2\sqrt{a^2-x^2}\,\mathrm{d}x$

Practice Problems

5. Evaluate the following integrals using a method of your choice.

(a) $\displaystyle\int x\sec^2(x)\,\mathrm{d}x$

(b) $\displaystyle\int_0^{\sqrt{\pi}} x^3\cos(x^2)\,\mathrm{d}x$

(c) $\displaystyle\int x\sin^3(x)\cos^3(x)\,\mathrm{d}x$

(d) $\displaystyle\int \sin(ax)\cos(bx)\,\mathrm{d}x, (a, b \neq 0, a \neq \pm b)$

(e) $\displaystyle\int_0^1 \frac{x}{x^4+1}\,\mathrm{d}x$

(f) $\displaystyle\int_1^e \frac{\ln x}{x}\,\mathrm{d}x$

(g) $\displaystyle\int \frac{1}{\sqrt{1-4x^2}}\,\mathrm{d}x$

(h) $\displaystyle\int \frac{-3x}{\sqrt{x^2-16}}\,\mathrm{d}x$

(i) $\displaystyle\int_0^{3/10} \frac{x^2}{\sqrt{9-25x^2}}\,\mathrm{d}x$

(j) $\displaystyle\int \frac{4x^5}{(2x^2-3)^{\frac{3}{2}}}\,\mathrm{d}x$

(k) $\displaystyle\int_0^{\pi/3} \frac{\sin(t)\cos(t)}{\sqrt{1+\cos^2(t)}}\,\mathrm{d}t$

(l) $\displaystyle\int \tan^2(x)\,\mathrm{d}x$

(m) $\displaystyle\int \frac{\sin^2\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x$

(n) $\displaystyle\int \left(\frac{\ln x}{x}\right)^2\,\mathrm{d}x$

Challenge Problems

6. Prove that the following equation is correct for any continuously differentiable functions $f(x), g(x)$ and $h(x)$:

$$\int_a^b f'(x)g(x)h(x)\,dx = f(x)g(x)h(x)\Big|_a^b - \int_a^b f(x)g'(x)h(x)\,dx - \int_a^b f(x)g(x)h'(x)\,dx$$

7. Evaluate

$$\int_{-\pi}^{\pi} \arctan\left(\pi^x\right)\,\mathrm{d}x.$$

Hint: consider using the substitution $u := -x$*. You might need the identity*

$$\arctan(1/s) = \operatorname{arccot}(s) = \frac{\pi}{2} - \arctan(s),$$

where the first equality is valid for $s > 0$.

8. Compute the following integral with the appropriate method(s)

$$\int \frac{x\ln(x)}{\sqrt{x^2-1}}\,\mathrm{d}x$$

Hint: start with integration by parts.

9. (a) Using trigonometric substitution show that

$$\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right) + C.$$

(b) Use the hyperbolic substitution $x = a \cdot \sinh(t)$ to show that

$$\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \operatorname{arcsinh}\left(\frac{x}{a}\right) + C$$

where $a > 0$ is a constant.

(c) Using part (a), provide an expression for $\operatorname{arcsinh}\left(\frac{x}{a}\right)$ in terms of the logarithm function.

Recall: $\cosh^2(\phi) = 1 + \sinh^2(\phi)$ and $\cosh(x) > 0$ for all $x \in \mathbb{R}$.