

# A game theoretic model between two Emergency Departments and the Emergency Medical Services

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# THIS.

**Supervisors:**

Dr. Vince Knight,  
Prof. Paul Harper

# Ambulance blockage problem in UK

## Patients forced to wait for 24 hours in ambulances, data shows

Ambulance crews forced to wait outside A&Es for 24 hours, according to chiefs

Rebecca Thomas Health Correspondent • Tuesday 17 May 2022 08:26 • Comments



(AFP/Getty)

'Appalling' waits for ambulances in England leaving lives at risk

Exclusive: Royal College of Emergency Medicine president says NHS is breaking its agreement to treat sickest in a timely way  
The staff, this is heartbreaking - senior doctor's view on crisis  
I feel so let down - long waits for ambulances on the south-west



Ambulance handover delays highest since start of winter  
© iStockphoto.com



**NHS 'on its knees' as ambulance response times for life-threatening calls rise to record high**

Average response time to deal with Category 1 cases – such as cardiac arrest – is now nine minutes and 20 seconds, with rises across all categories.

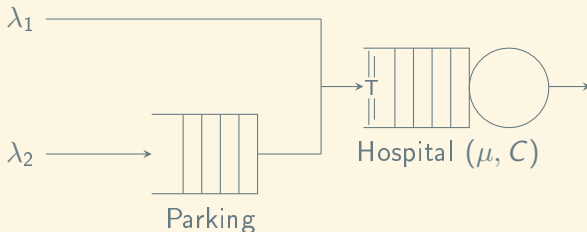


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## Queues - Custom network of queues



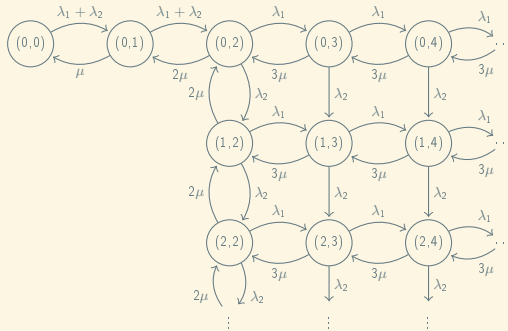
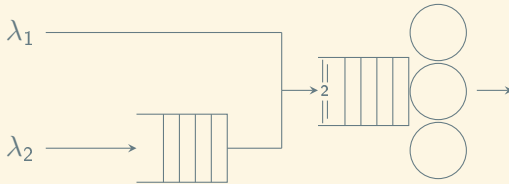
## Queues - Custom network of queues



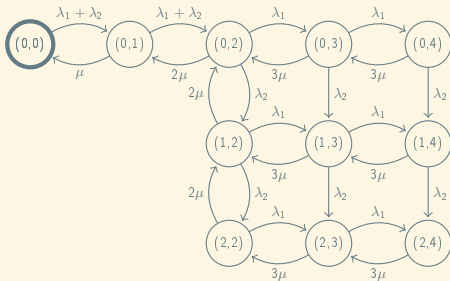
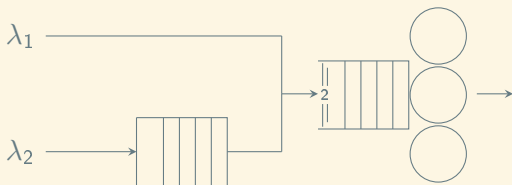
Parameters:

- ▶  $\lambda_1$ : Arrival rate of type 1 individuals
- ▶  $\lambda_2$ : Arrival rate of type 2 individuals
- ▶  $\mu$ : Service rate
- ▶  $C$ : Number of servers
- ▶  $T$ : Threshold

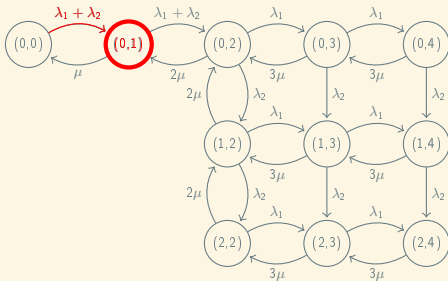
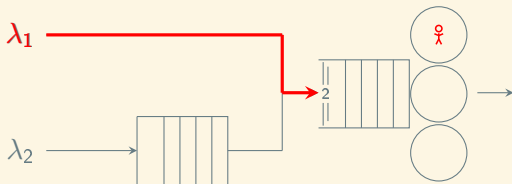
# Markov Chain - Custom network



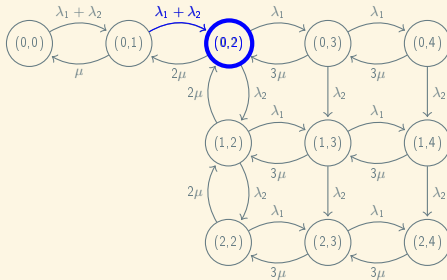
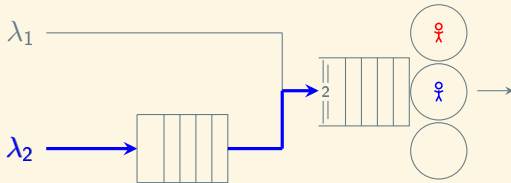
# Markov Chain - Custom network - $N = 4, M = 2$



# Markov Chain - Custom network - $N = 4, M = 2$

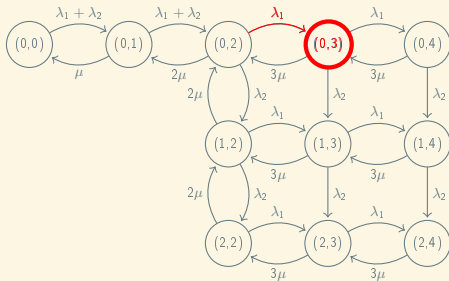
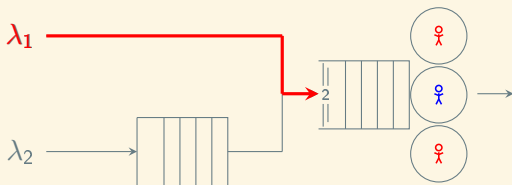


# Markov Chain - Custom network - $N = 4, M = 2$

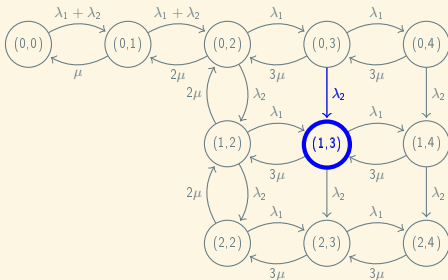
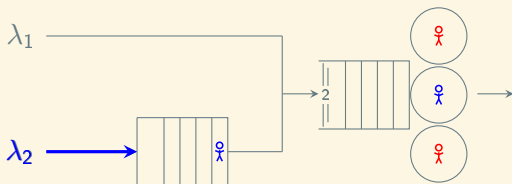




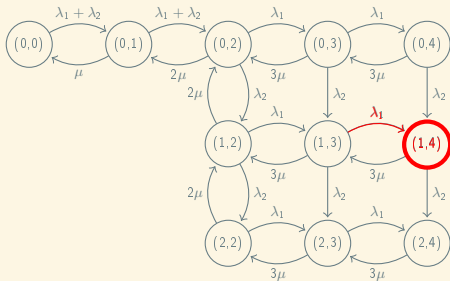
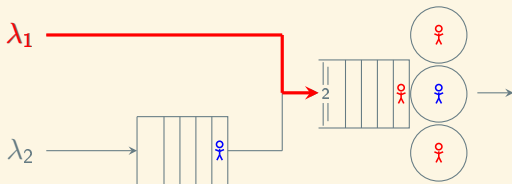
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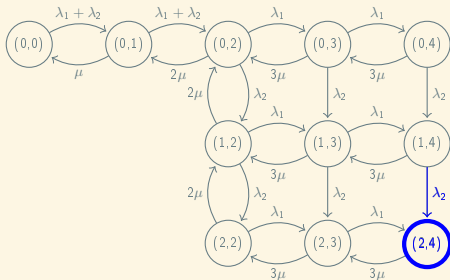
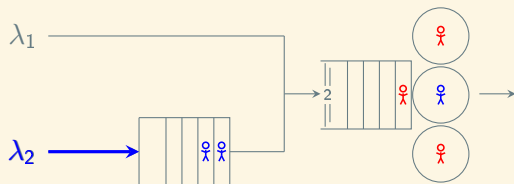
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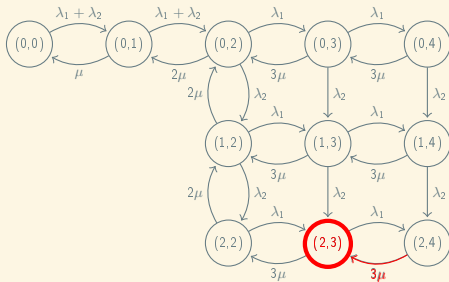
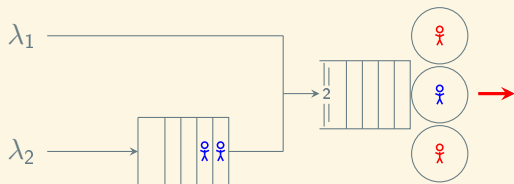
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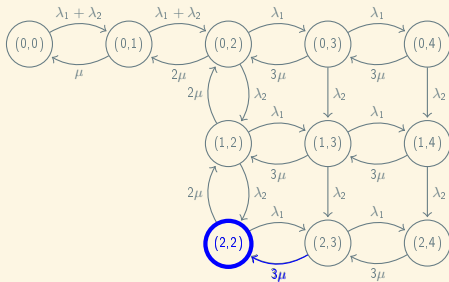
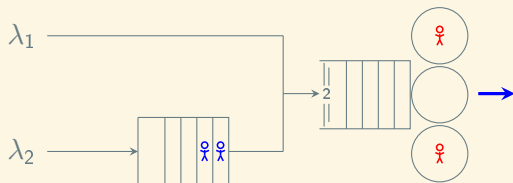
# Markov Chain - Custom network - $N = 4, M = 2$



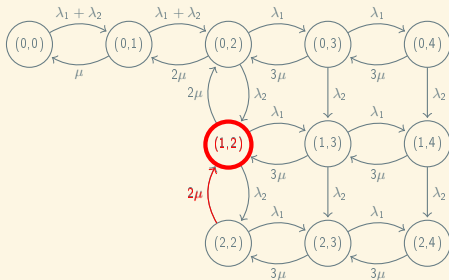
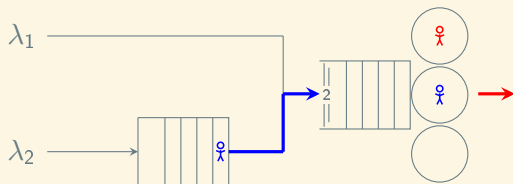
# Markov Chain - Custom network - $N = 4, M = 2$



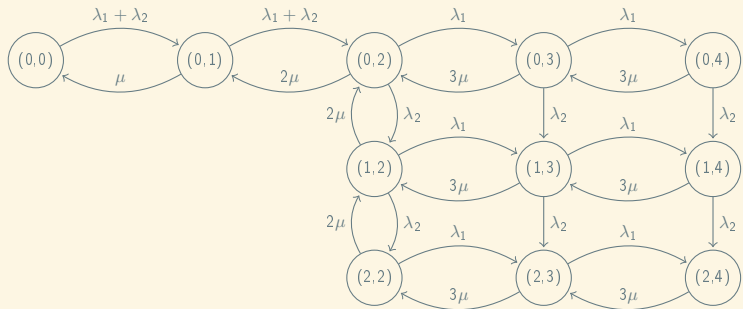
# Markov Chain - Custom network - $N = 4, M = 2$



# Markov Chain - Custom network - $N = 4, M = 2$



# Steady state probabilities - Generator matrix

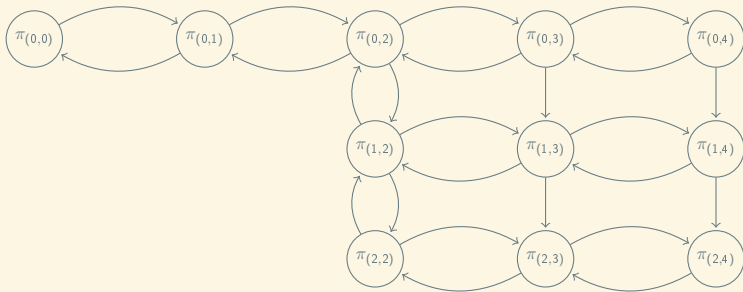


From \ To	(0,0)	(0,1)	(0,2)	(2,3)	(2,4)
(0,0)	$-\lambda_1 - \lambda_2$	$\lambda_1 + \lambda_2$	0	...	0
(0,1)	$\mu$	$-\mu - \lambda_1 - \lambda_2$	$\lambda_1 + \lambda_2$	...	0
(0,2)	0	$2\mu$	$-2\mu - \lambda_1 - \lambda_2$	...	0
...	...	...	...	...	...
(2,3)	0	0	0	...	$-\lambda_1 - 3\mu$
(2,4)	0	0	0	...	$3\mu$



# Steady state probabilities - Generator matrix ( $Q$ )

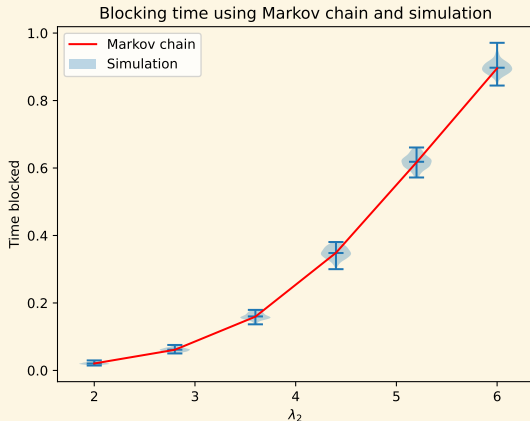
$$\pi = [\pi_{(0,0)} \quad \pi_{(0,1)} \quad \pi_{(0,2)} \quad \dots \quad \pi_{(2,3)} \quad \pi_{(2,4)}], \quad \sum \pi_{(u,v)} = 1$$



$$\frac{d\pi}{dt} = \pi Q = 0$$

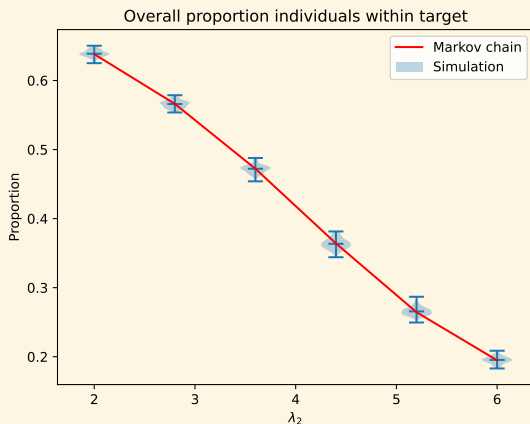
# Performance Measures - Blocking time

$$B = \frac{\sum_{(u,v) \in S_A^{(2)}} \pi(u,v) b(u,v)}{\sum_{(u,v) \in S_A^{(2)}} \pi(u,v)}$$



# Performance Measures - Proportion within target

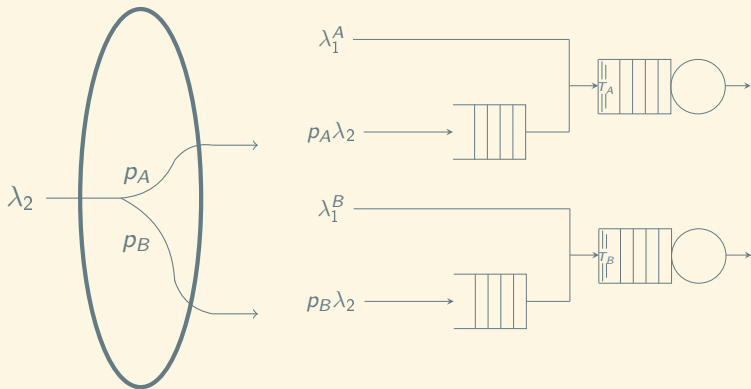
$$P(W < t) = \frac{\lambda_1 P_{L'_1}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(W^{(1)} < t) + \frac{\lambda_2 P_{L'_2}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(W^{(2)} < t)$$



# Game - Definition

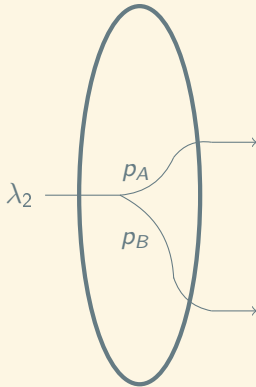


# Game - Diagrammatic representation

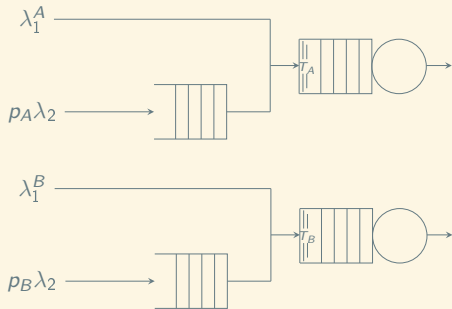


# Game - Diagrammatic representation

Blocking time



Proportion of individuals within target

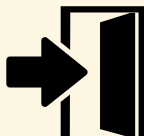


# Game - Players, Strategies and Objectives



$$p_A, p_B \in [0, 1]$$
$$p_A + p_B = 1$$

$\min B$



$$T_A \in [1, N_A]$$

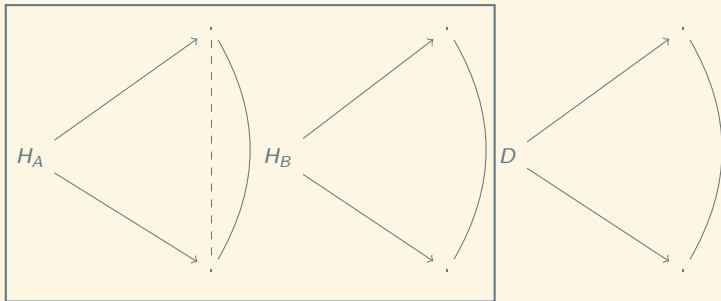
$$P(W^{(A)} < t) > 0.95$$



$$T_B \in [1, N_B]$$

$$P(W^{(B)} < t) > 0.95$$

# Game - Formulation





## Hospital's utility

$$U_{T_A, T_B}^{(i)} = 1 - \left[ (P(W^{(i)} < t) - 0.95)^2 \right]$$

## Game - Payoff matrices

$$A = \begin{pmatrix} U_{1,1}^A & U_{1,2}^A & \cdots & U_{1,N_B}^A \\ U_{2,1}^A & U_{2,2}^A & \cdots & U_{2,N_B}^A \\ \vdots & \vdots & \ddots & \vdots \\ U_{N_A,1}^A & U_{N_A,2}^A & \cdots & U_{N_A,N_B}^A \end{pmatrix}, \quad B = \begin{pmatrix} U_{1,1}^B & U_{1,2}^B & \cdots & U_{1,N_B}^B \\ U_{2,1}^B & U_{2,2}^B & \cdots & U_{2,N_B}^B \\ \vdots & \vdots & \ddots & \vdots \\ U_{N_A,1}^B & U_{N_A,2}^B & \cdots & U_{N_A,N_B}^B \end{pmatrix}$$

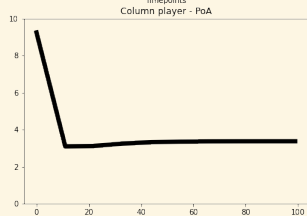
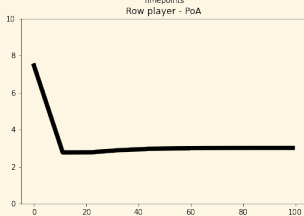
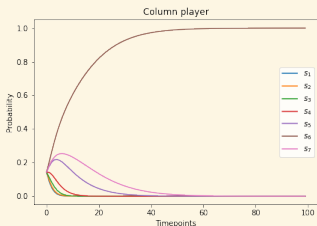
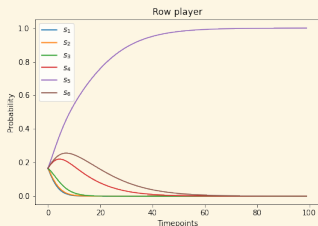
$$R = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N_B} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N_A,1} & p_{N_A,2} & \cdots & p_{N_A,N_B} \end{pmatrix}$$

# Asymmetric Replicator Dynamics

$$\frac{dx}{dt}_i = x_i((f_x)_i - \phi_x), \quad \text{for all } i$$

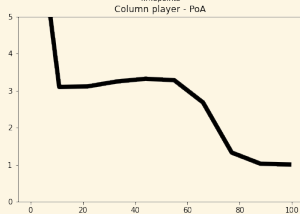
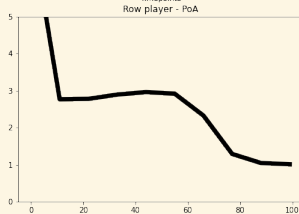
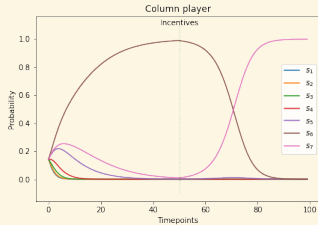
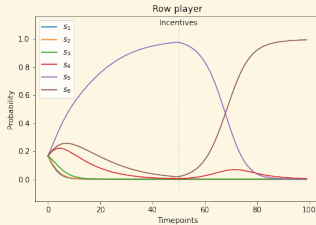
$$\frac{dy}{dt}_i = y_i((f_y)_i - \phi_y), \quad \text{for all } i$$

# Learning algorithms - Asymmetric replicator dynamics



Inefficiencies can be learned and  
emerge naturally

# Learning algorithms - Asymmetric replicator dynamics



Targeted incentivisation of  
behaviours can help escape  
learned inefficiencies

# Reinforcement Learning - Server's behaviour



1



2



2



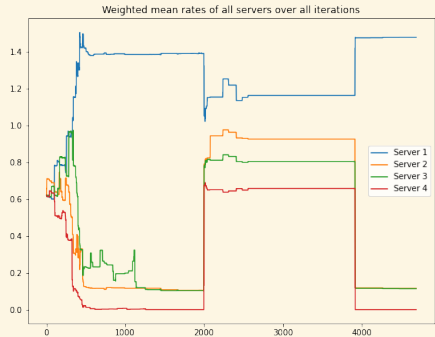
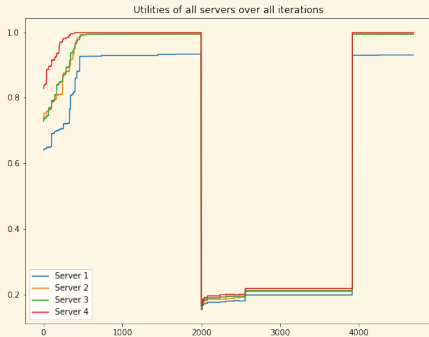
3

Utility

Idle time + Lost inds



# Reinforcement Learning - Server's behaviour



# Thank you!

```
$ pip install ambulance_game  
https://github.com/11michalis11/AmbulanceDecisionGame
```

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