A game theoretic model between two Emergency Departments and the Emergency Medical Services

Michalis Panayides

THIS.

Supervisors:
Dr. Vince Knight,
Prof. Paul Harper

Ambulance blockage problem in UK

Patients forced to wait for 24 hours in ambulances, data shows

Ambulance crews forced to wait outside A&Es for 24 hours, according to chiefs

Rebecca Thomas Health Correspondent • Tuesday 17 May 2022 08:26 • Comments

(AFP/Getty)

'Appalling' waits for ambulances in England leaving lives at risk

Exclusive: Royal College of Emergency Medicine president says NHS is breaking its agreement to treat sickest in a timely way

'For staff, this is heartbreaking': senior doctor's view on crisis
'I feel so let down': long waits for ambulances in south-west

Ambulance handover delays highest since start of winter

NHS 'on its knees' as ambulance response times for life-threatening calls rise to record high

Average response time to deal with Category 1 cases – such as cardiac arrests – is now nine minutes and 20 seconds with rises across all categories.

Queues - Custom network of queues

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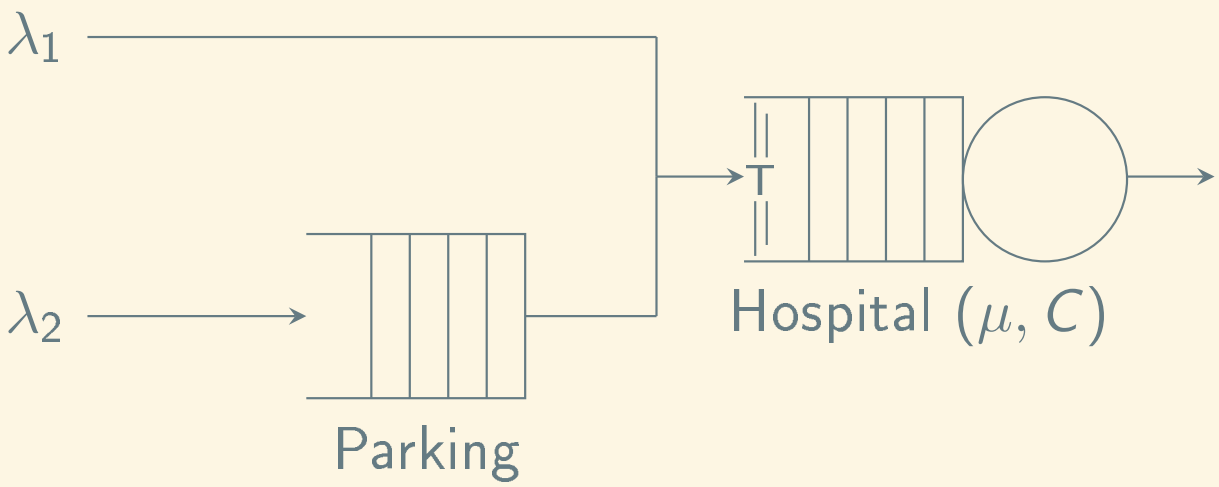
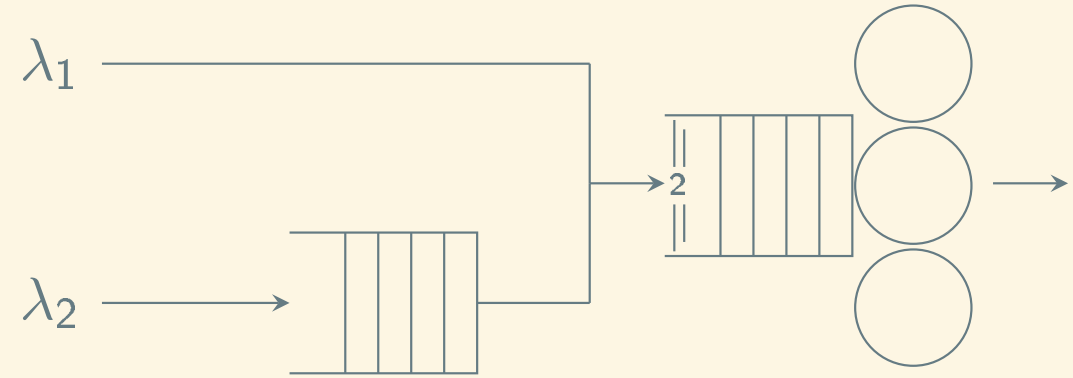
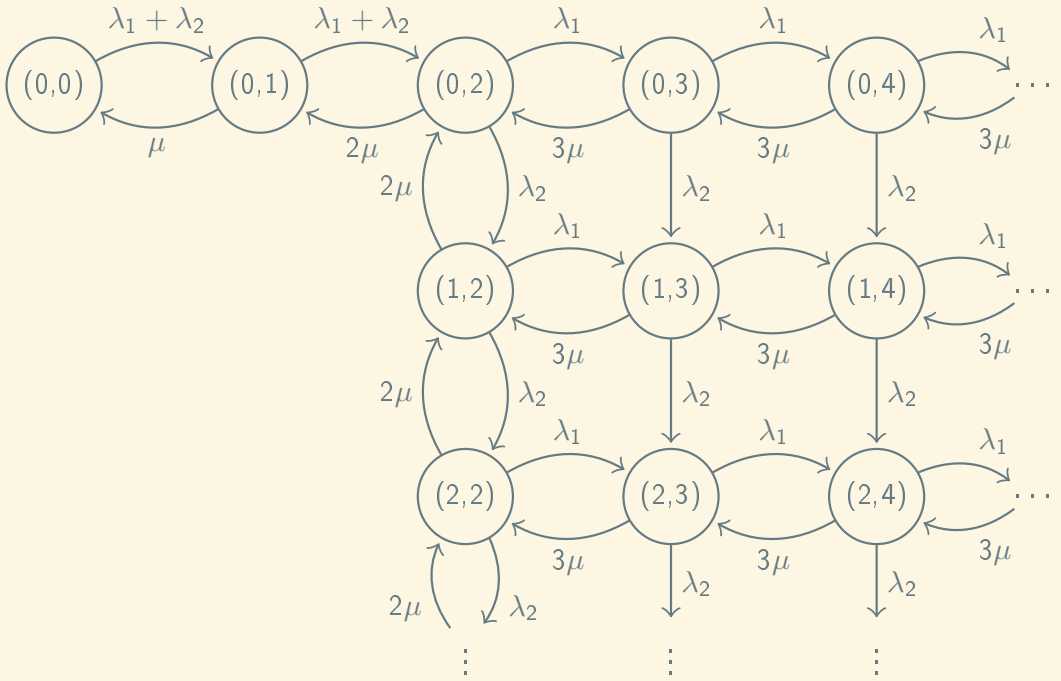


Parameters:

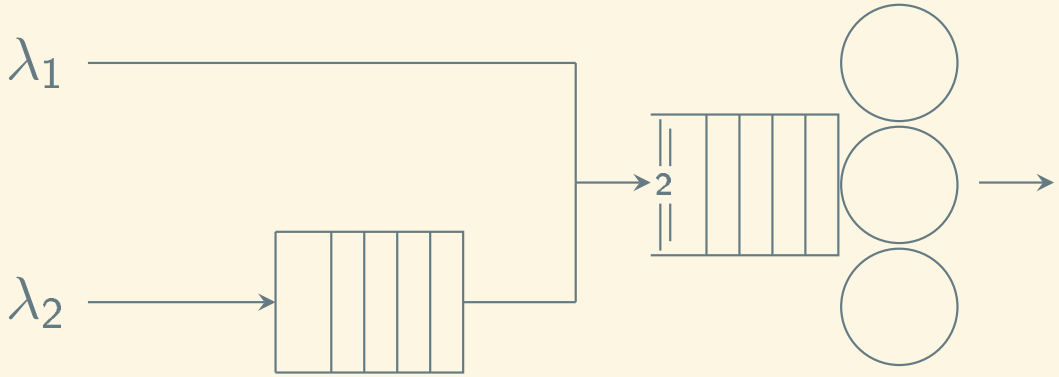
- λ_1: Arrival rate of type 1 individuals
- λ_2: Arrival rate of type 2 individuals
- μ: Service rate
- C: Number of servers
- T: Threshold

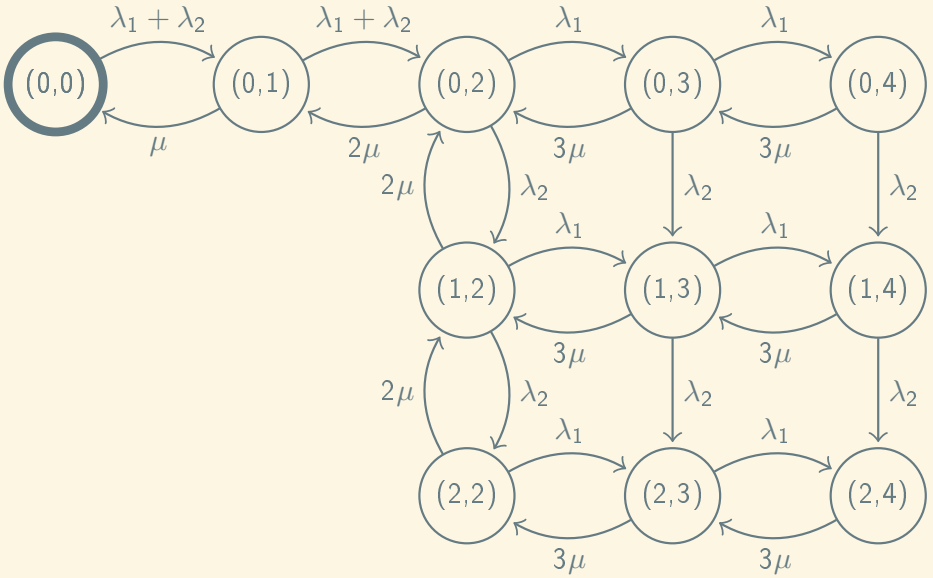
Markov Chain - Custom network





Markov Chain - Custom network - $N = 4, M = 2$





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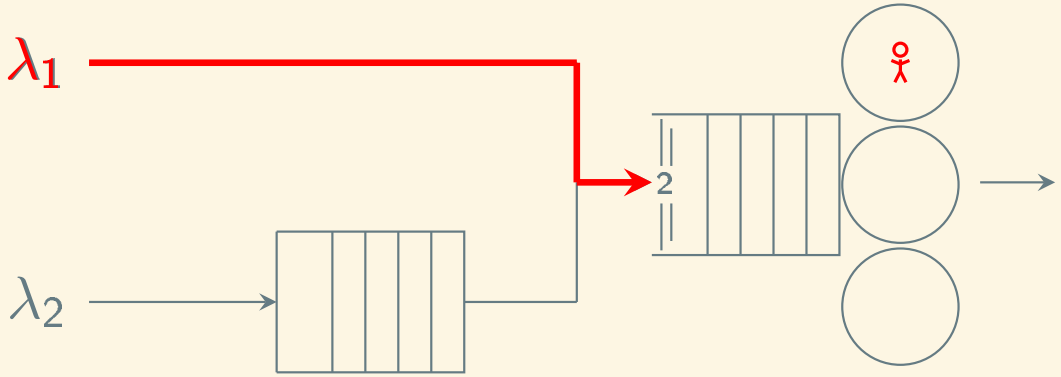

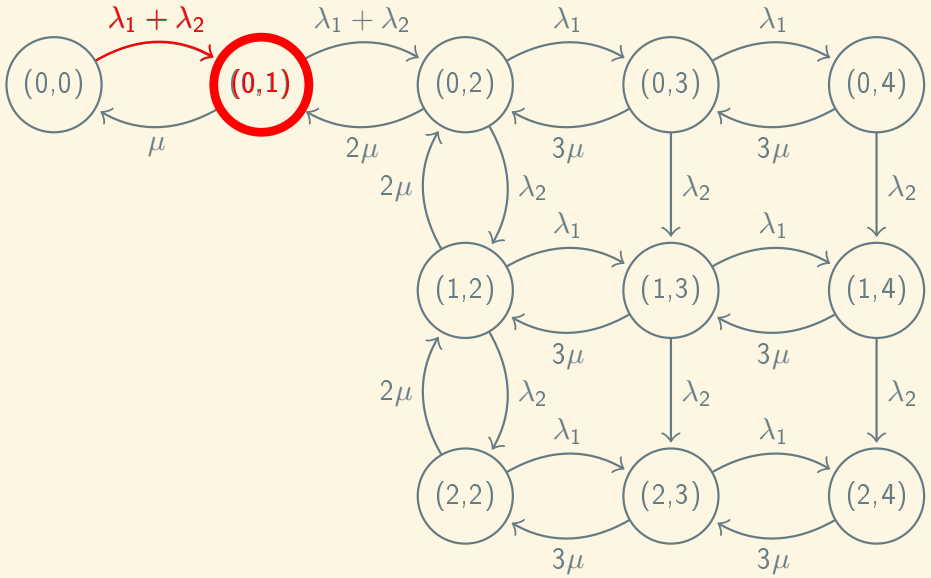

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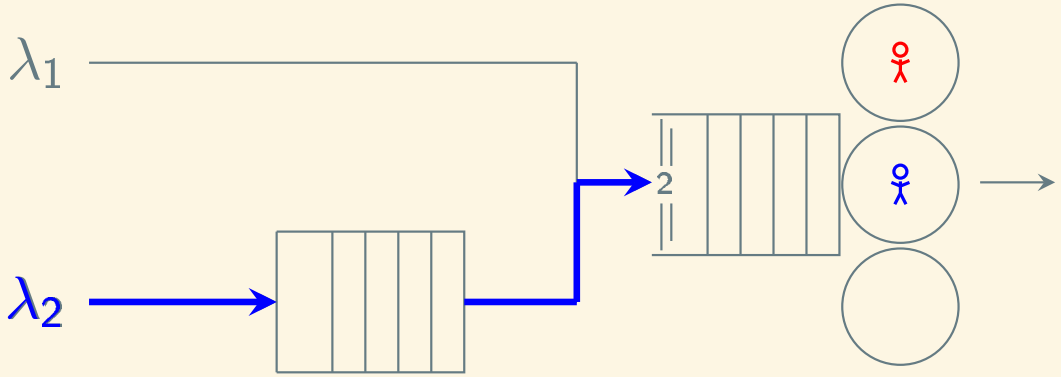

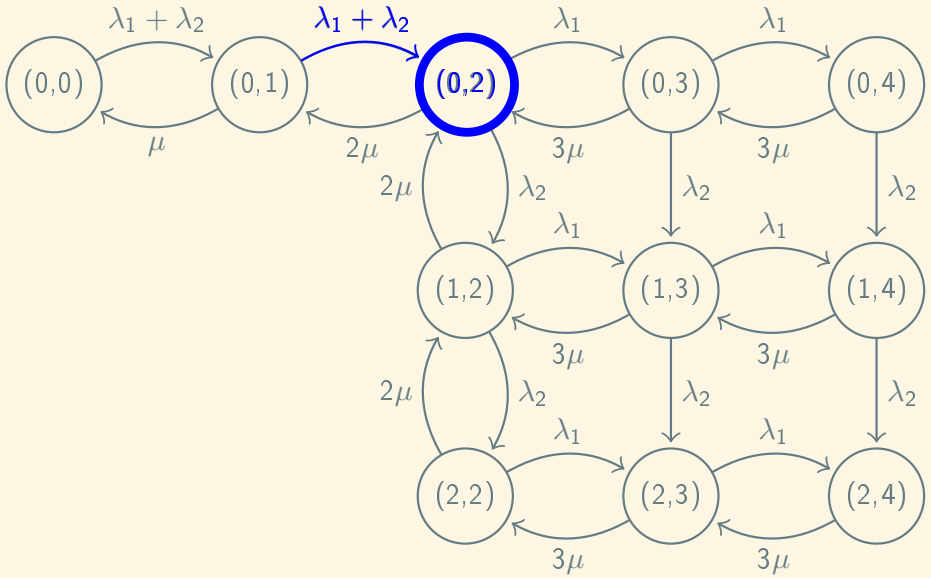

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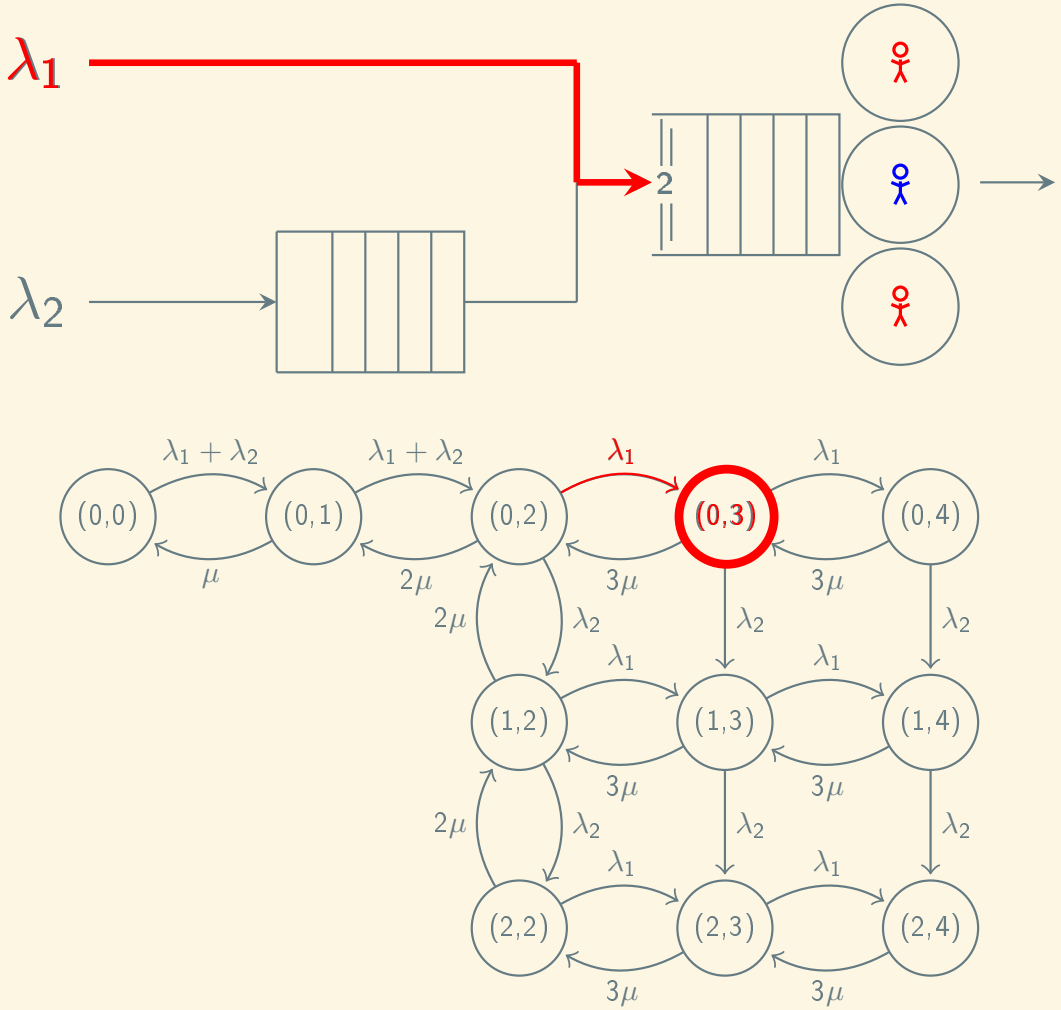

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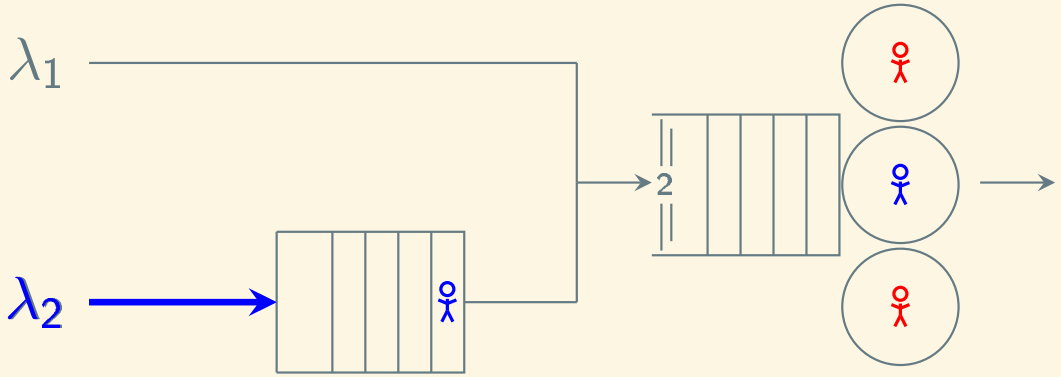

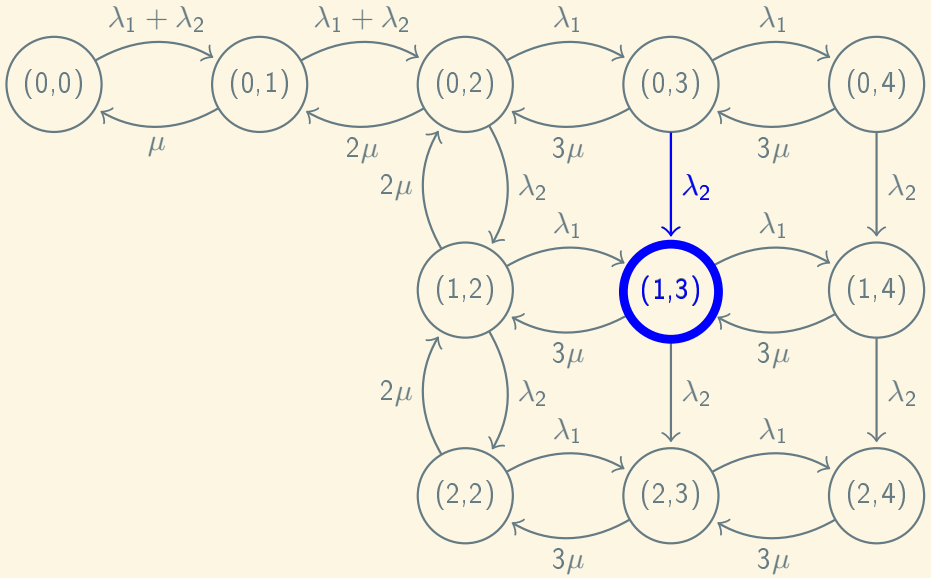

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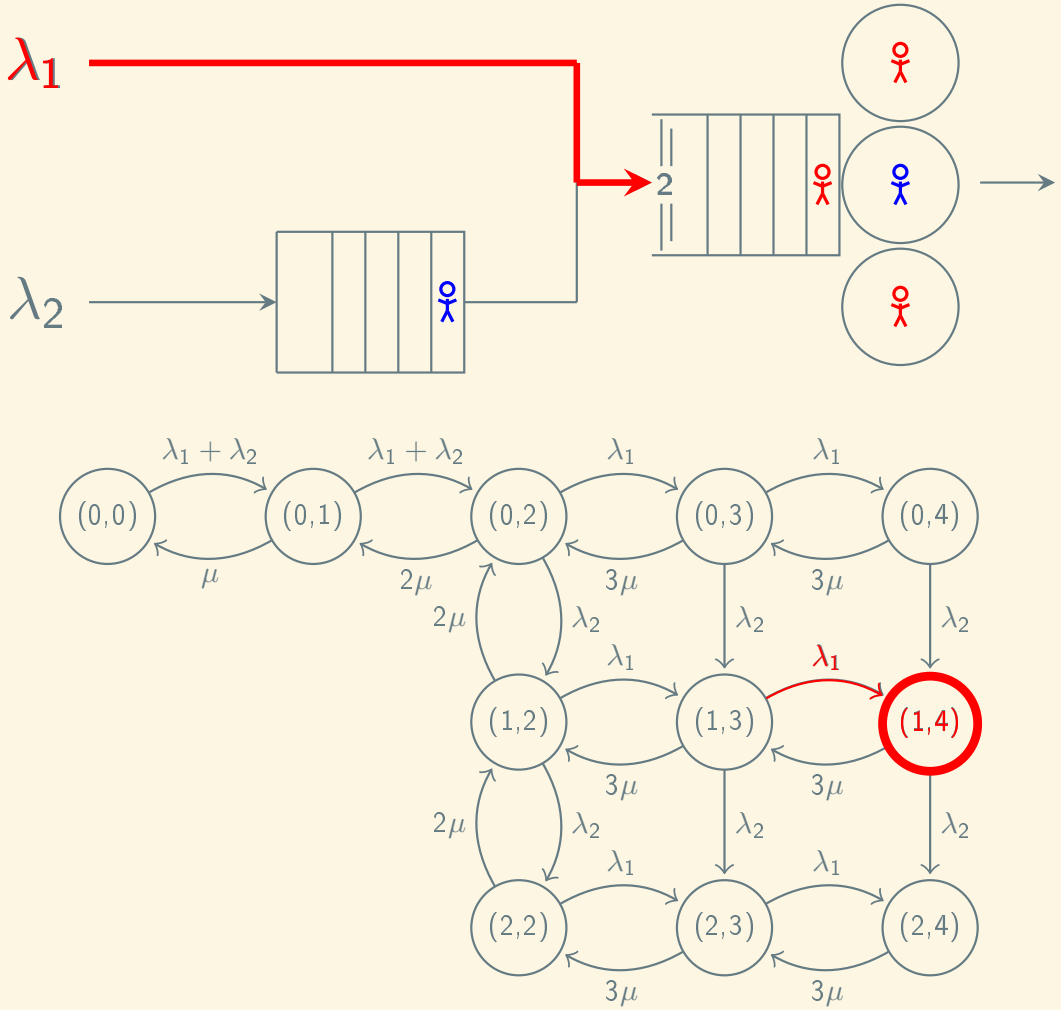

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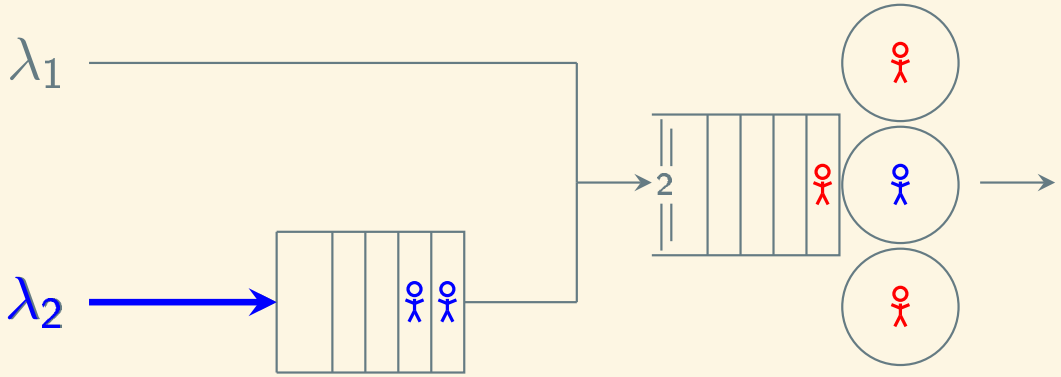

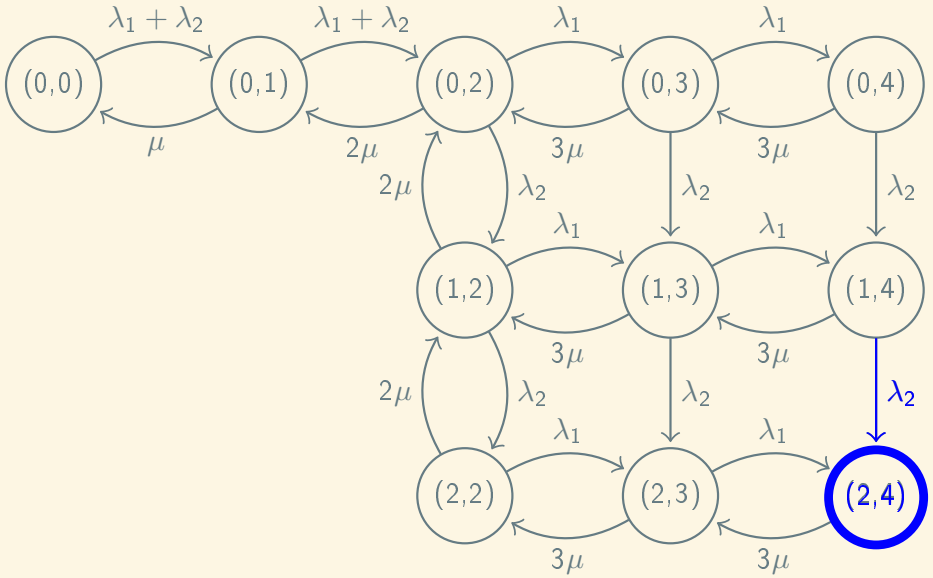

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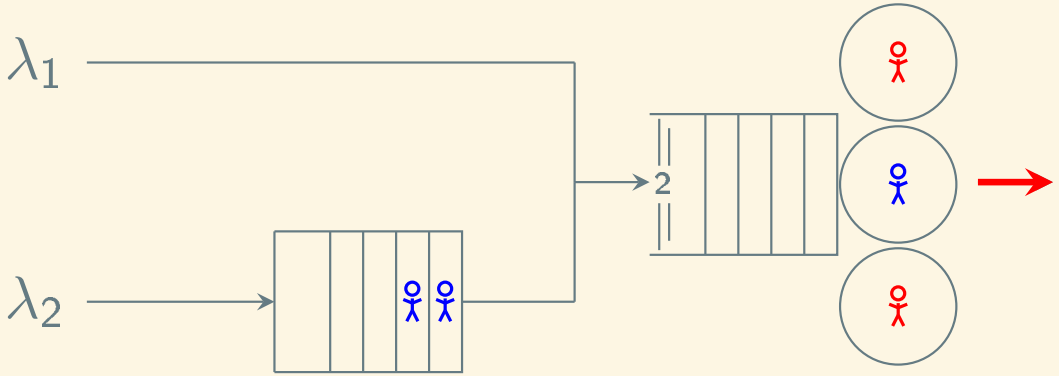

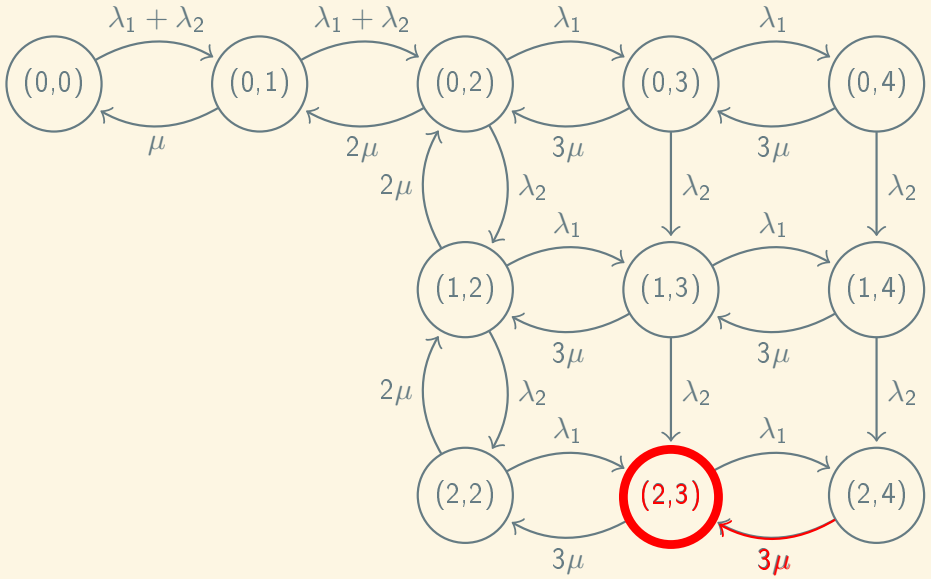

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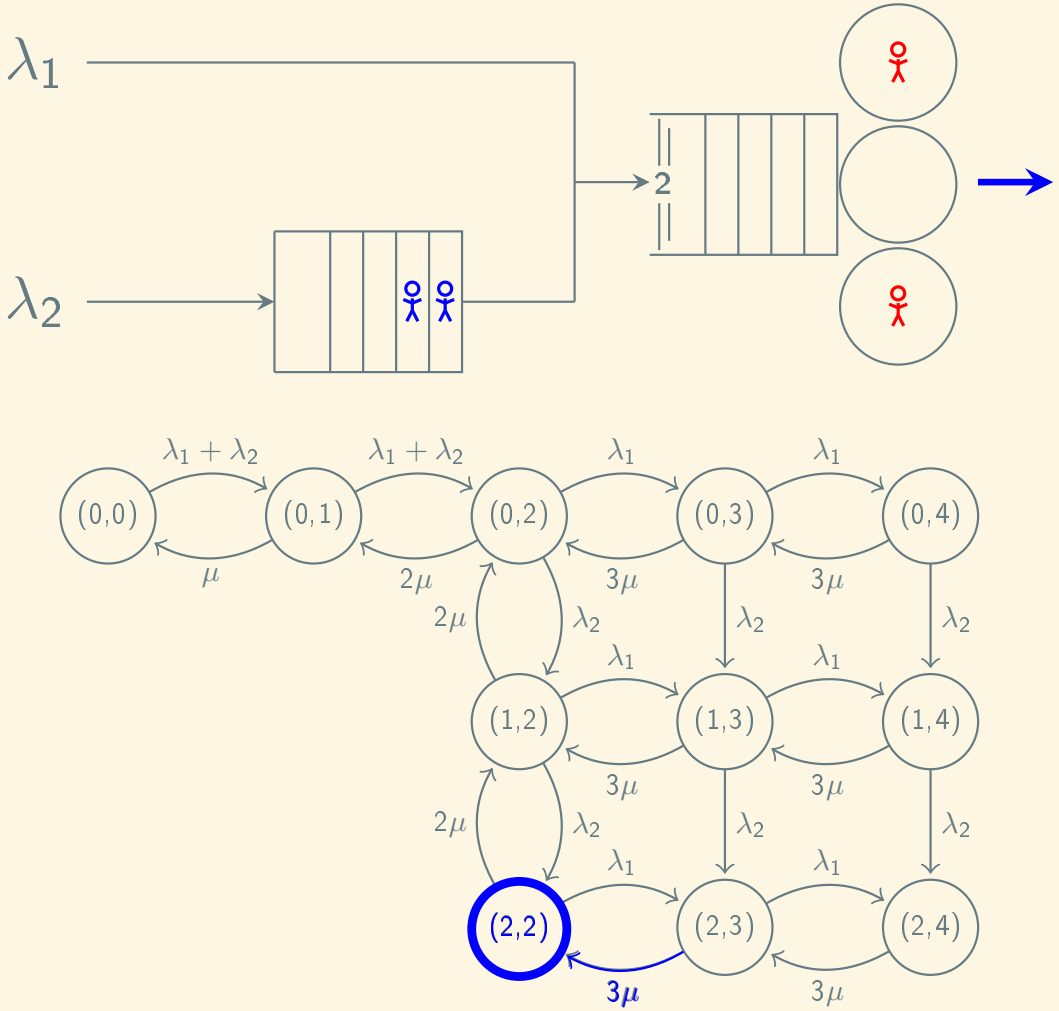

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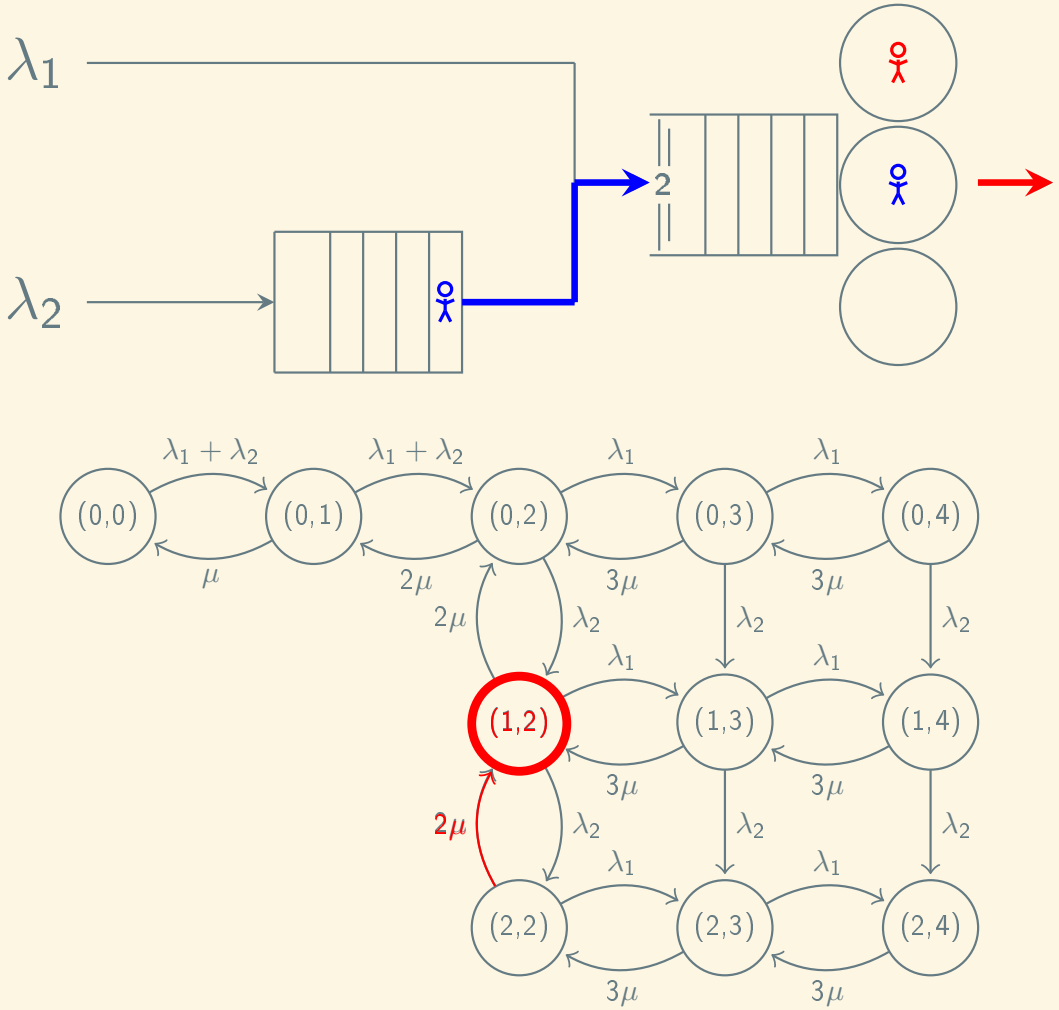
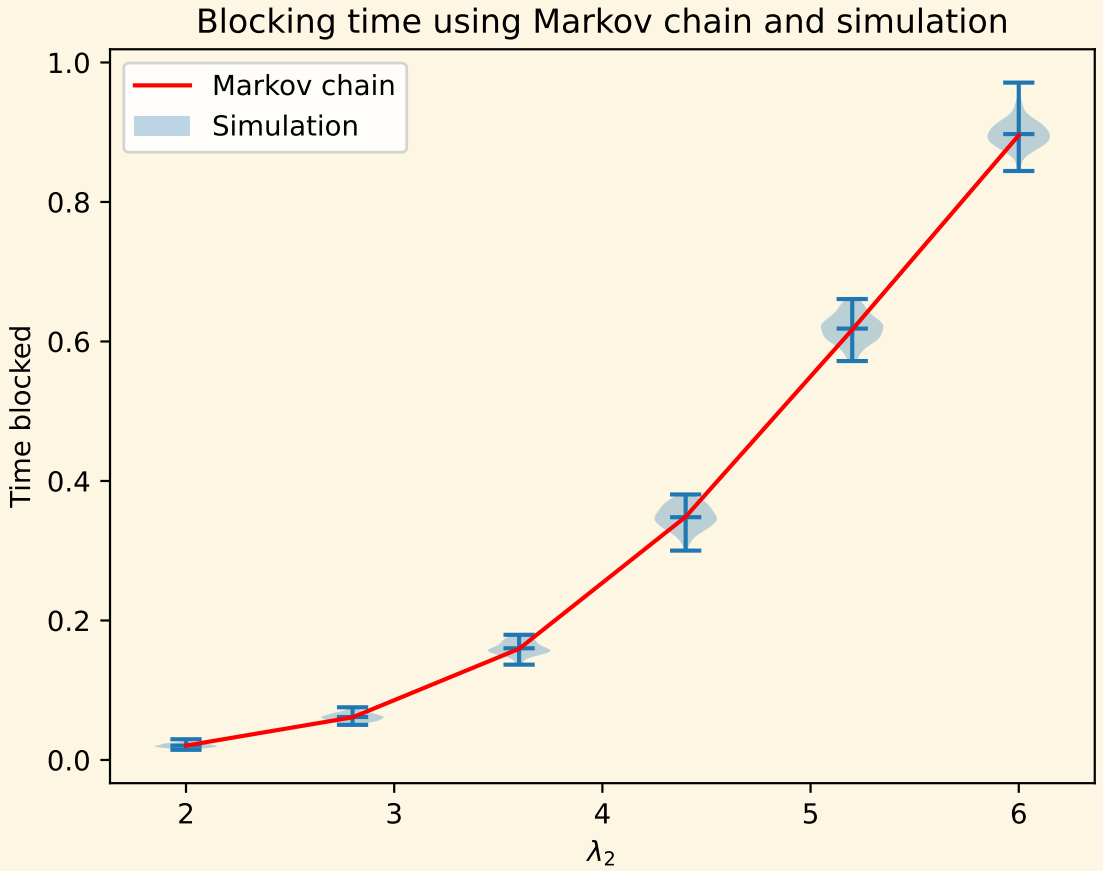


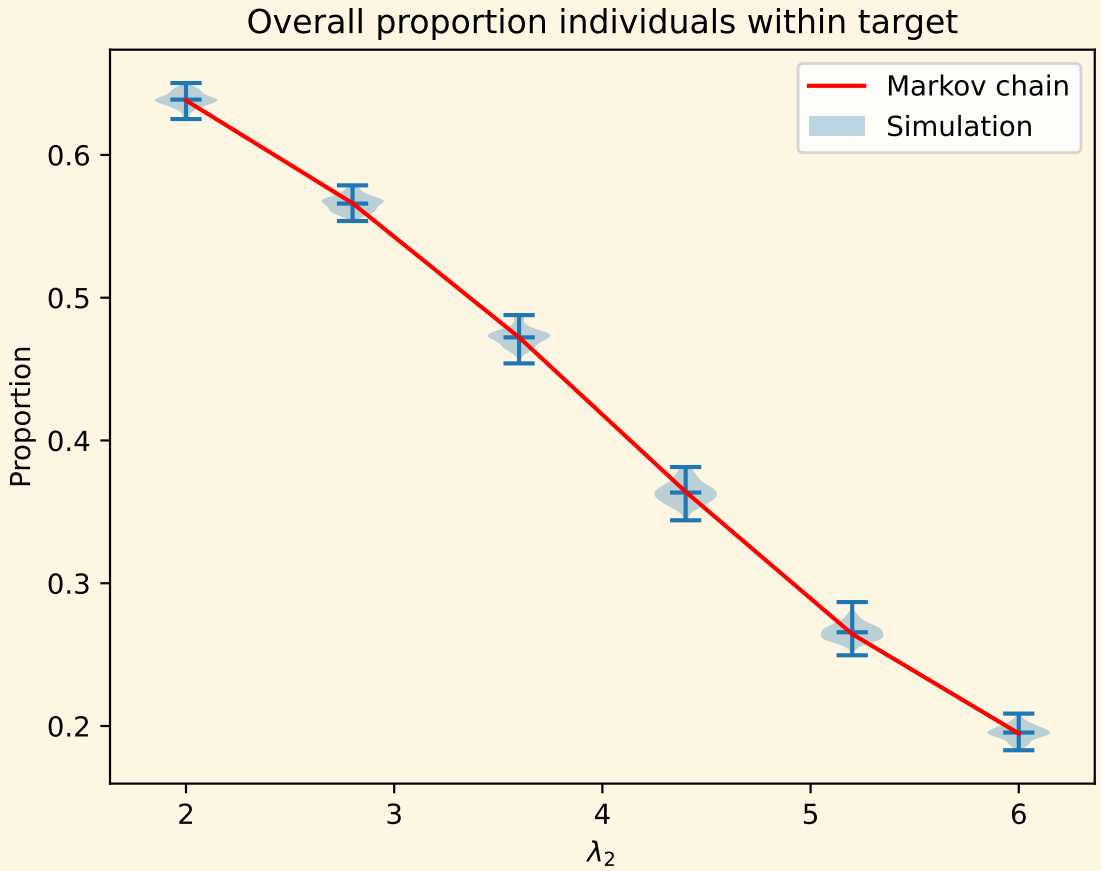
Performance Measures - Blocking time

$$B = \frac{\sum_{(u,v)\in S_A^{(2)}} \pi_{(u,v)}\ b(u,v)}{\sum_{(u,v)\in S_A^{(2)}} \pi_{(u,v)}}$$



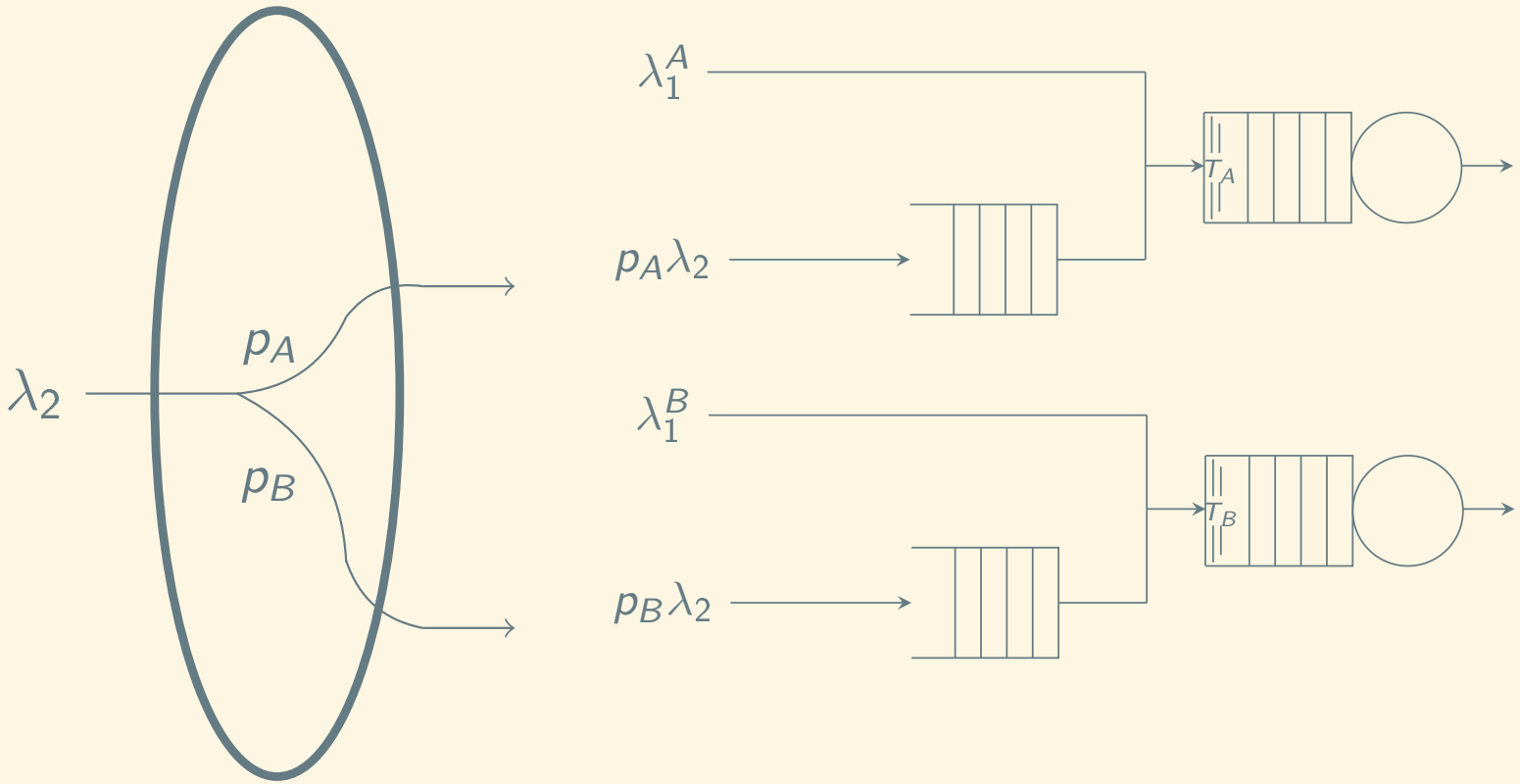
Performance Measures - Proportion within target

$$P(W < t) = \frac{\lambda_1 P_{L_1'}}{\lambda_2 P_{L_2'} + \lambda_1 P_{L_1'}} P(W^{(1)} < t) + \frac{\lambda_2 P_{L_2'}}{\lambda_2 P_{L_2'} + \lambda_1 P_{L_1'}} P(W^{(2)} < t)$$



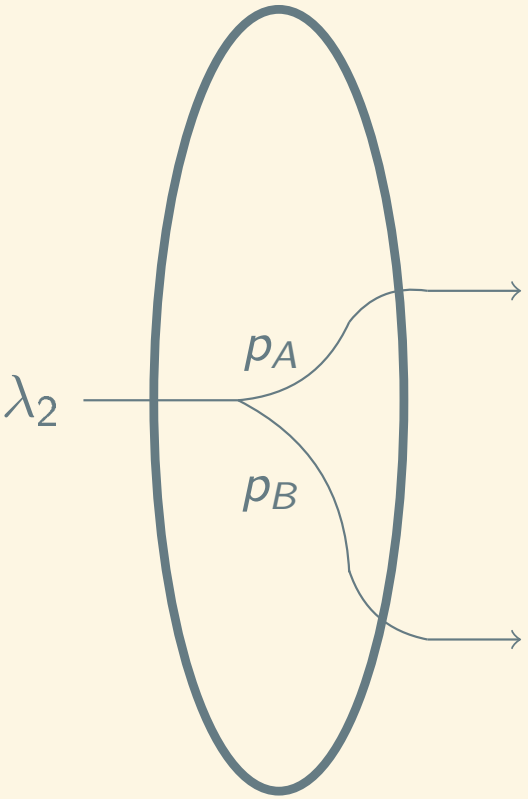
Game - Definition

Game - Diagrammatic representation

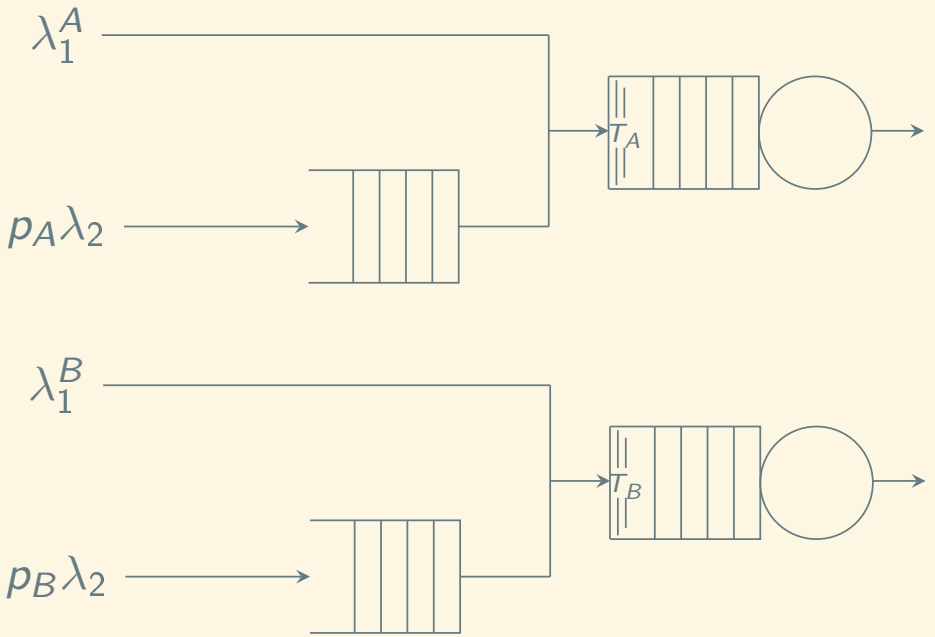


Game - Diagrammatic representation

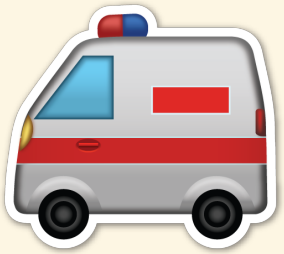
Blocking time

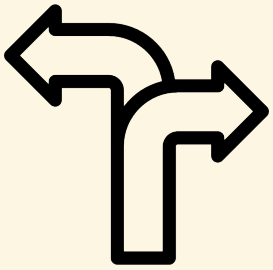


Proportion of individuals within target



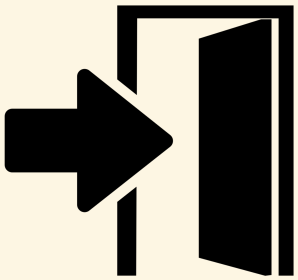
Game - Players, Strategies and Objectives



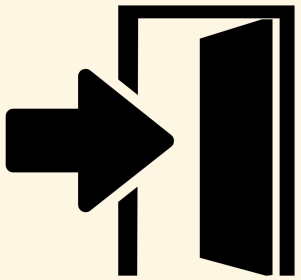


$p_A, p_B \in [0, 1]$

$p_A + p_B = 1$



$T_A \in [1, N_A]$



$T_B \in [1, N_B]$

$\min B$

$P(W^{(A)} < t) > 0.95$

$P(W^{(B)} < t) > 0.95$

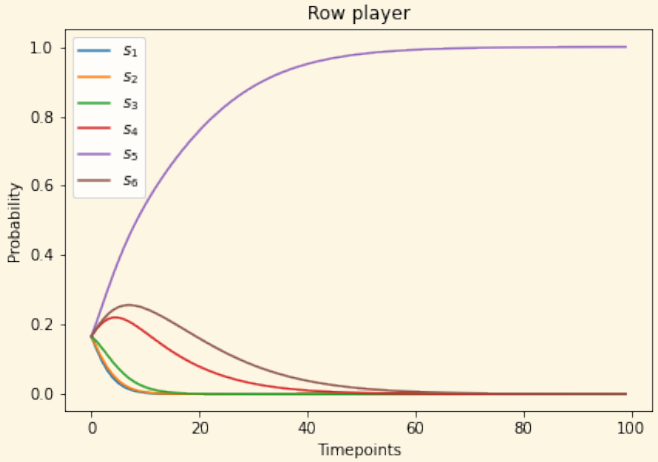
Hospital's utility

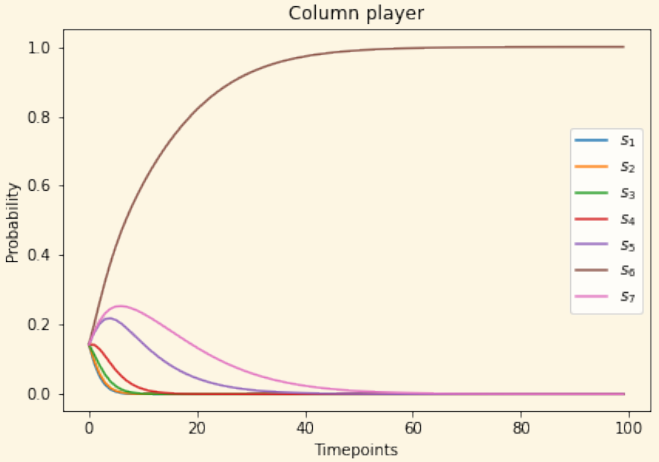
$$U^{(i)}_{T_A,T_B} = 1 - \left[(P(W^{(i)} < t) - 0.95)^2\right]$$

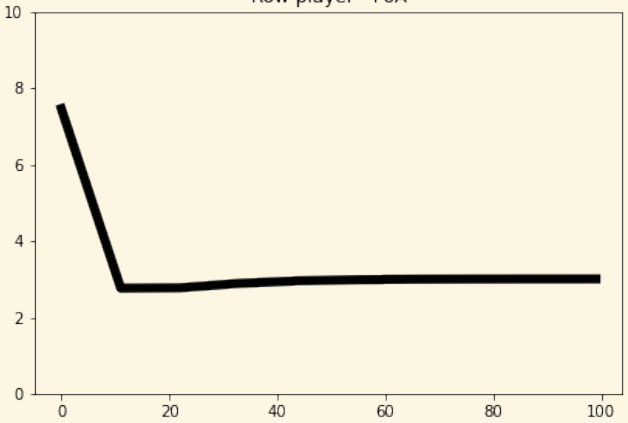
$$A = \begin{pmatrix} U^A_{1,1} & U^A_{1,2} & \cdots & U^A_{1,N_B} \\ U^A_{2,1} & U^A_{2,2} & \cdots & U^A_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^A_{N_A,1} & U^A_{N_A,2} & \cdots & U^A_{N_A,N_B} \end{pmatrix}, \quad B = \begin{pmatrix} U^B_{1,1} & U^B_{1,2} & \cdots & U^B_{1,N_B} \\ U^B_{2,1} & U^B_{2,2} & \cdots & U^B_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^B_{N_A,1} & U^B_{N_A,2} & \cdots & U^B_{N_A,N_B} \end{pmatrix}$$

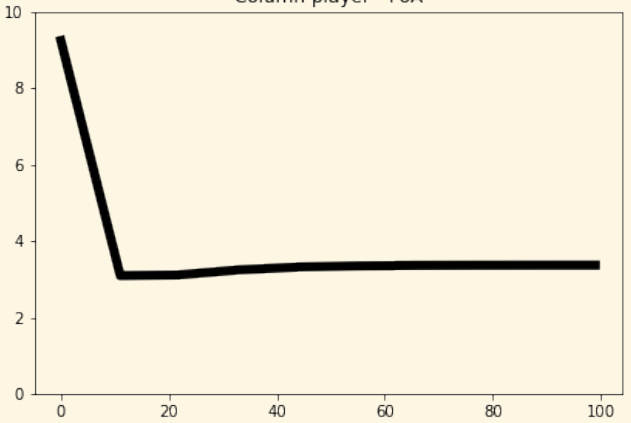
$$R = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N_B} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N_A,1} & p_{N_A,2} & \cdots & p_{N_A,N_B} \end{pmatrix}$$

Learning algorithms - Asymmetric replicator dynamics



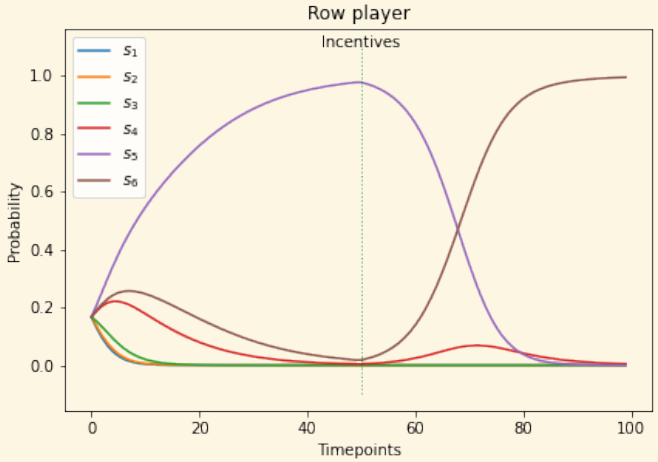


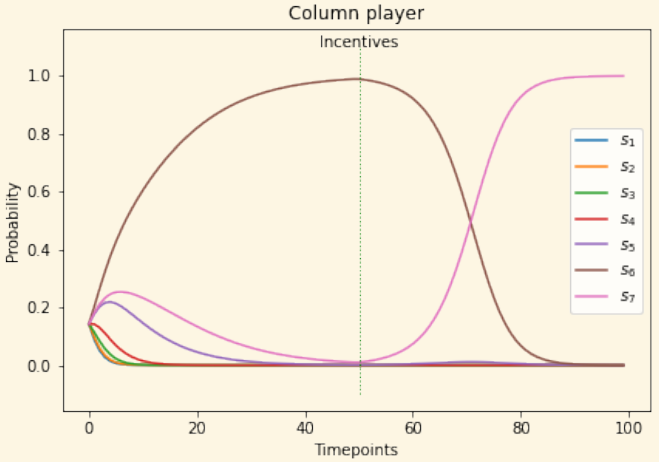


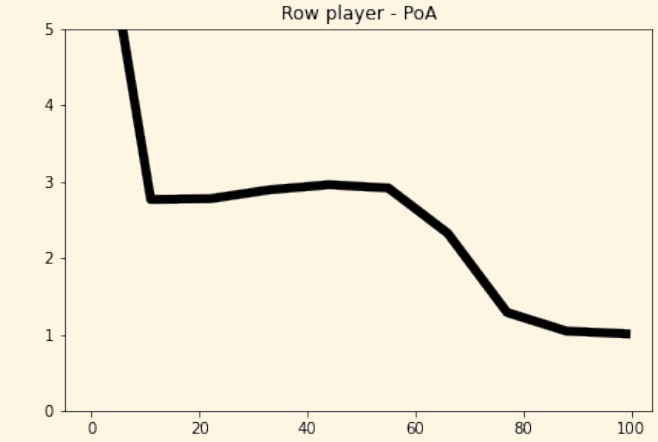


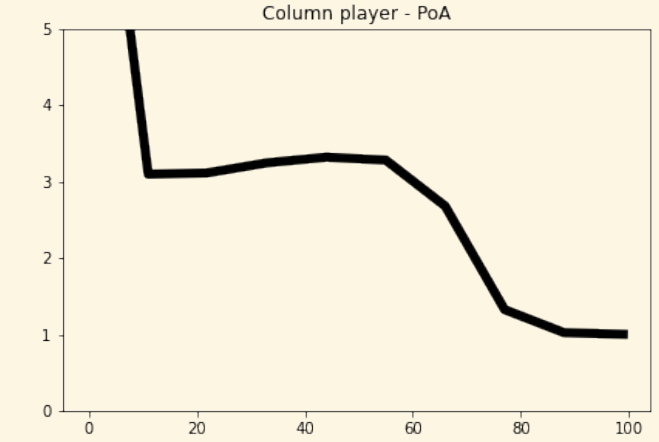
Inefficiencies can be learned and emerge naturally

Learning algorithms - Asymmetric replicator dynamics









Targeted incentivisation of behaviours can help escape learned inefficiencies

Thank you!

$ pip install ambulance_game

https://github.com/11michalis11/AmbulanceDecisionGame

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