Workforce Behaviours in Healthcare Systems

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Thesis structure

1. Introduction
2. Literature review
3. Queueing theoretic model
4. Game theoretic model
5. Numerical results
6. Agent-based model
7. Conclusions

1. Introduction - Congestion in Healthcare

Patients forced to wait for 24 hours in ambulances, data shows

Ambulance crews forced to wait outside A&Es for 24 hours, according to chiefs

Rebecca Thomas Health Correspondent • Tuesday 17 May 2022 08:26 • Comments

(AFP/Getty)

'Appalling' waits for ambulances in England leaving lives at risk

Exclusive: Royal College of Emergency Medicine president says NHS is breaking its agreement to treat sickest in a timely way

- 'For staff, this is heartbreaking': senior doctor's view on crisis
- 'I feel so let down': long waits for ambulances in south-west

Ambulance handover delays highest since start of winter

A paramedic walks past a line of ambulances

NHS 'on its knees' as ambulance response times for life-threatening calls rise to record high

Average response time to deal with Category 1 cases – such as cardiac arrests – is now nine minutes and 20 seconds with rises across all categories.

1. Introduction - Research contributions

- Queueing model with 2 consecutive waiting spaces.
- Performance measure formulas for queueing model.
- Game theoretic model between the EMS and two EDs.
- Numerical experiments showing emergent behaviour of gaming between EDs and the EMS.
- Reinforcement learning algorithm on ED staff to optimise their behaviour.

2. Literature Review

OR in Healthcare

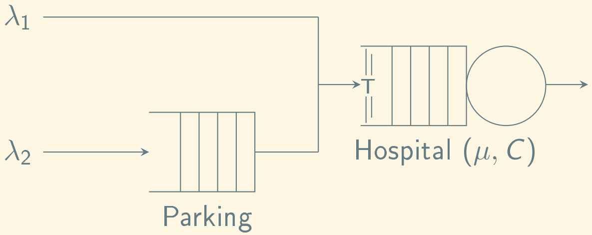
Game Theory and Queueing Theory

Game Theory in Healthcare

Behavioural OR

3. Queueing theoretic model - Motivation

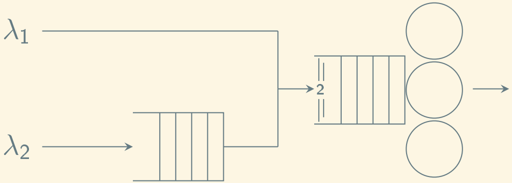
3. Queueing theoretic model - Diagrammatic representation

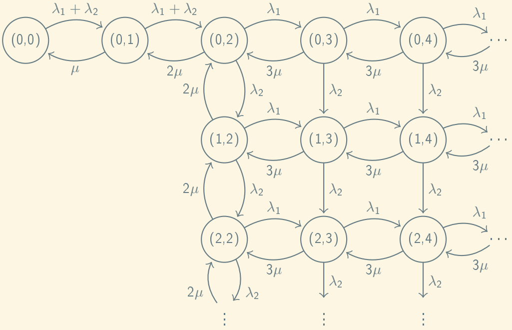


Parameters:

- λ_1: Arrival rate of type 1 individuals
- λ_2: Arrival rate of type 2 individuals
- μ: Service rate
- C: Number of servers
- T: Threshold

3. Queueing theoretic model - Markov Chain model





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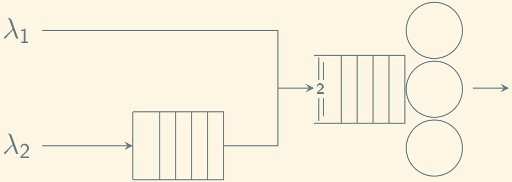

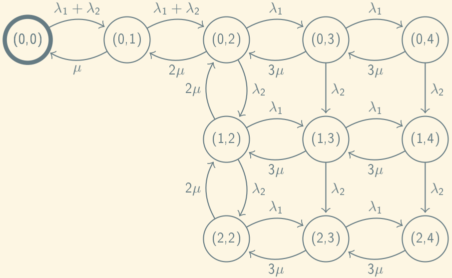

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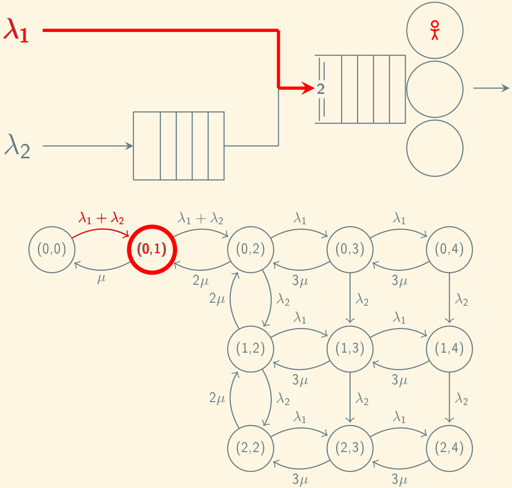

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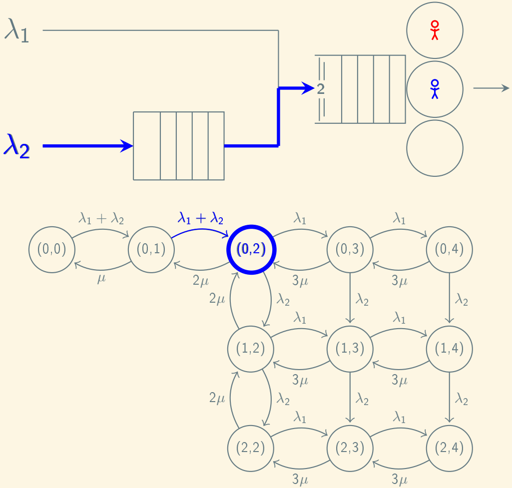

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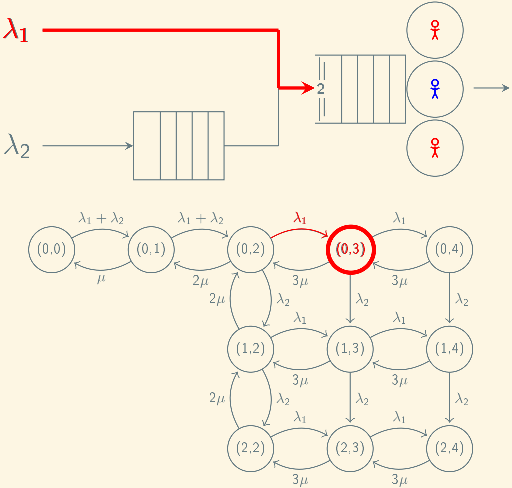

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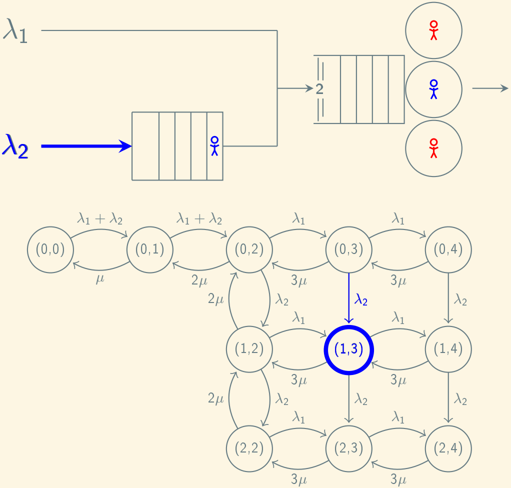

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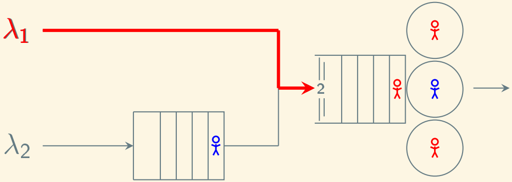

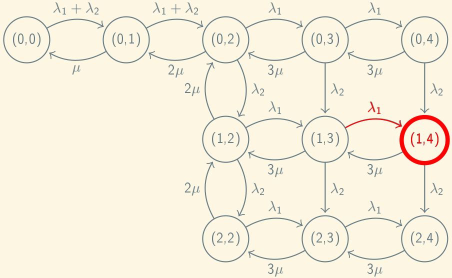

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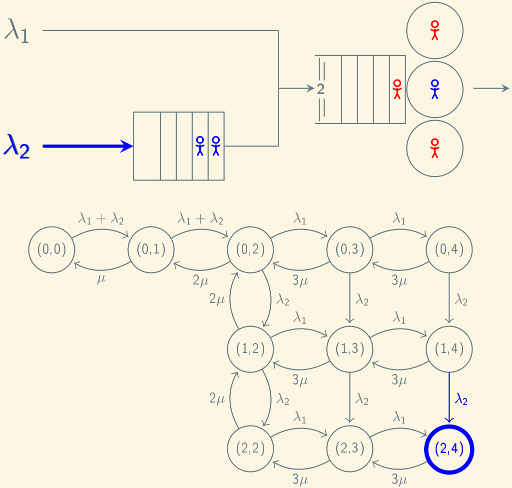

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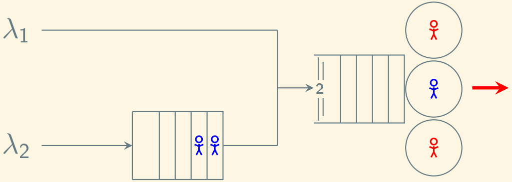

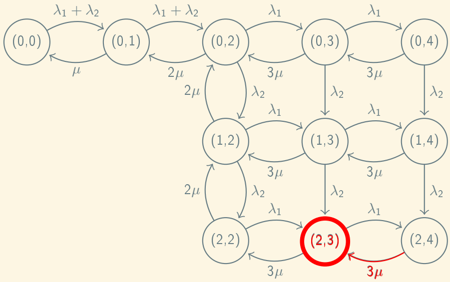

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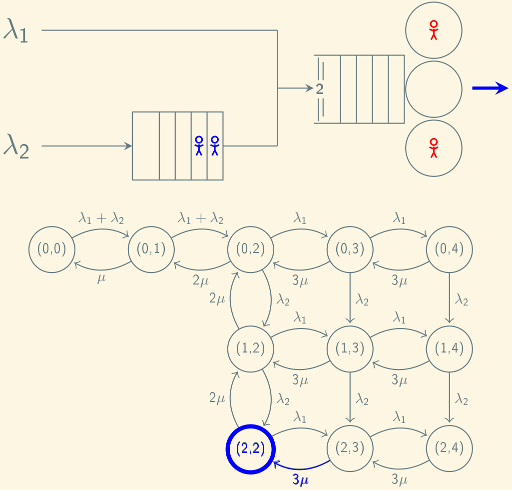

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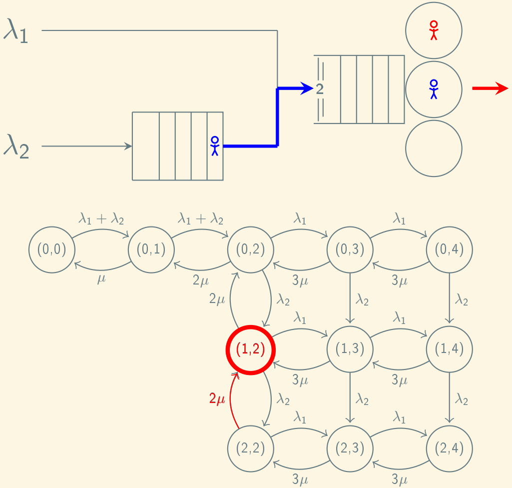

3. Queueing theoretic model - Steady state probabilities

From \To	(0,0)	(0,1)	(0,2)		(2,3)	(2,4)
(0,0)	$-\lambda_1 - \lambda_2$	$\lambda_1 + \lambda_2$	0	$\dots$	0	0
(0,1)	μ	$-\mu - \lambda_1 - \lambda_2$	$\lambda_1 + \lambda_2$	$\dots$	0	0
(0,2)	0	2μ	$-2\mu - \lambda_1 - \lambda_2$	$\dots$	0	0
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
(2,3)	0	0	0	$\dots$	$-\lambda_1 - 3\mu$	λ_1
(2,4)	0	0	0	$\dots$	3μ	-3μ

$$\frac{d\pi}{dt} = \pi Q = 0, \quad \sum \pi_{(u,v)} = 1$$

- Numerical integration
- Linear algebraic method
- Least squares method
- Closed-form approach

3. Queueing theoretic model - Performance measures

- **Waiting time**
  - Recursive formula
  - Direct formula
  - Closed-form formula
- **Blocking time**
  - Direct formula
  - Closed-form formula
- **Proportion of individuals within target**
  - Generic Ψ function
  - Specific Ψ function

4. Game theoretic model - Outline

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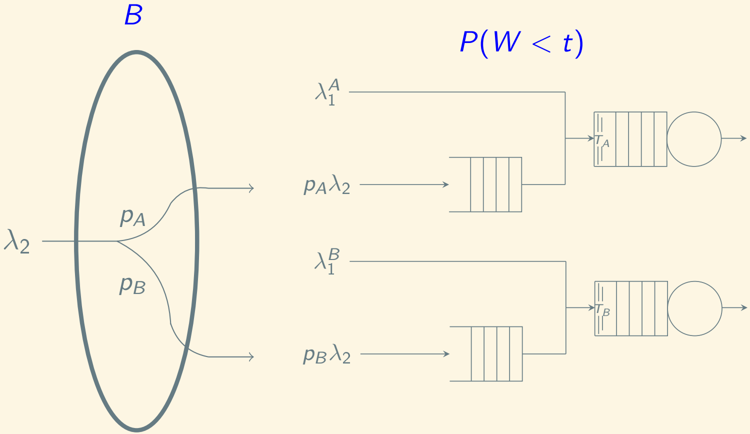
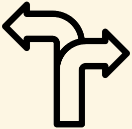
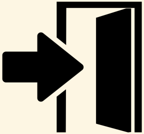
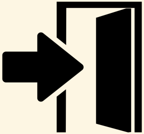
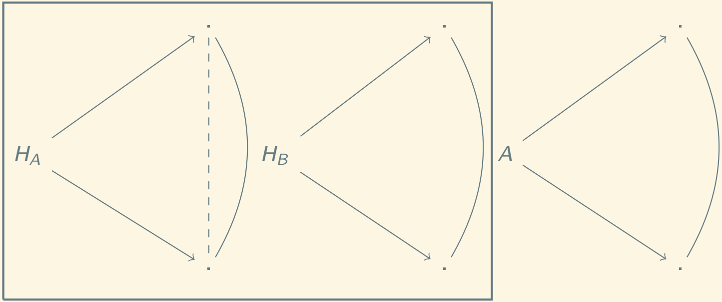


4. Game theoretic model - Players, Strategies, Objectives

		
		
$p_A, p_B \in [0, 1]$ $p_A + p_B = 1$	$T_A \in [1, N_A]$	$T_B \in [1, N_B]$
$\min B$	$P(W^{(A)} < t) > 0.95$	$P(W^{(B)} < t) > 0.95$

4. Game theoretic model - Imperfect information game



4. Game theoretic model - Utilities

$$U^{(i)}_{T_A,T_B} = 1 - \left[\left(P(W^{(i)} < t) - 0.95 \right)^2 \right]$$

$$A = \begin{pmatrix} U^A_{1,1} & U^A_{1,2} & \cdots & U^A_{1,N_B} \\ U^A_{2,1} & U^A_{2,2} & \cdots & U^A_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^A_{N_A,1} & U^A_{N_A,2} & \cdots & U^A_{N_A,N_B} \end{pmatrix}, \quad B = \begin{pmatrix} U^B_{1,1} & U^B_{1,2} & \cdots & U^B_{1,N_B} \\ U^B_{2,1} & U^B_{2,2} & \cdots & U^B_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^B_{N_A,1} & U^B_{N_A,2} & \cdots & U^B_{N_A,N_B} \end{pmatrix}$$

$$R = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N_B} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N_A,1} & p_{N_A,2} & \cdots & p_{N_A,N_B} \end{pmatrix}$$

4. Game theoretic model - Asymmetric replicator Dynamics

$$\frac{dx}{dt}_i = x_i((f_x)_i - \phi_x), \quad \text{for all } i$$

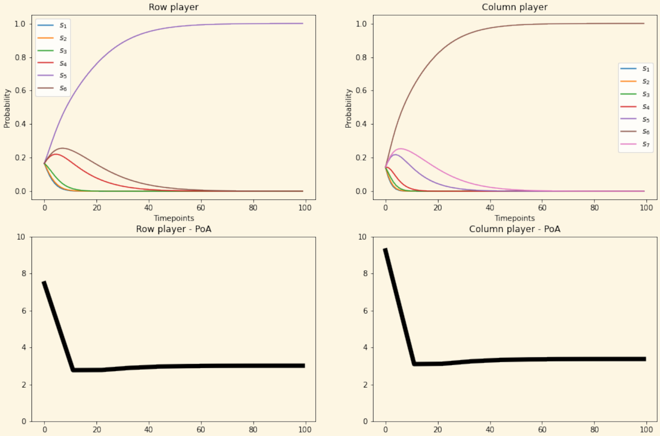
$$\frac{dy}{dt}_i = y_i((f_y)_i - \phi_y), \quad \text{for all } i$$

4. Game theoretic model - Compartmentalised price of anarchy

$$PoA = \frac{\max_{s \in E} Cost(s)}{\min_{s \in S} Cost(S)}$$

$$PoA_A(s_r) = \frac{Cost(s_r)}{\min_{s \in S} Cost(S)}, \qquad PoA_B(s_c) = \frac{Cost(s_c)}{\min_{s \in S} Cost(S)}$$

5. Numerical results - Asymmetric replicator dynamics



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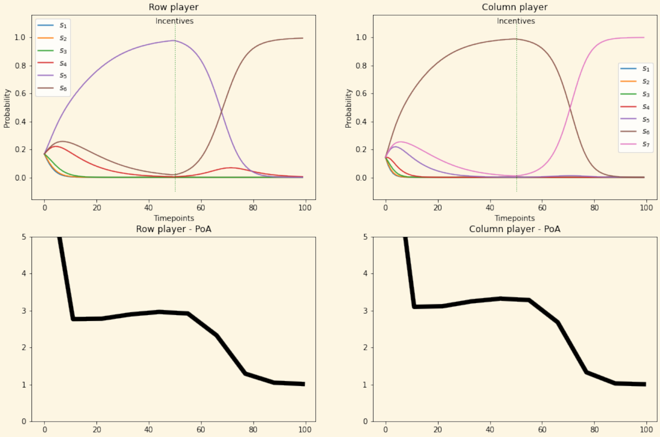
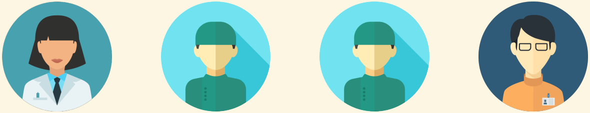


6. Agent-based model - Server's priority

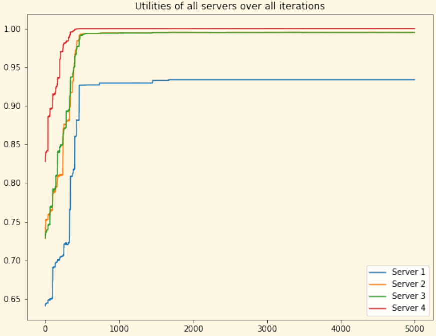


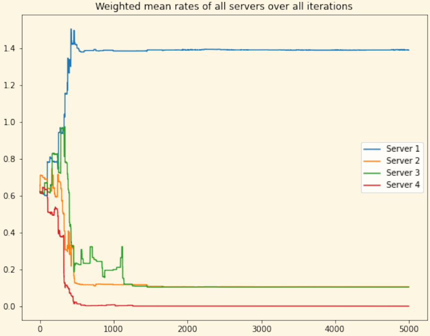
1 2 2 3

Utility

Idle time Served individuals

6. Agent-based model - Reinforcement learning





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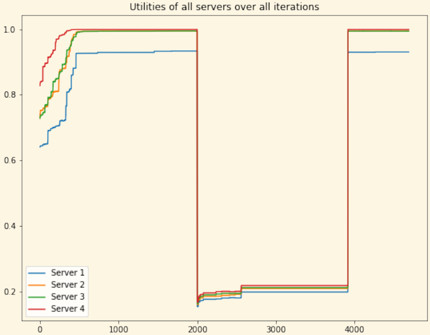

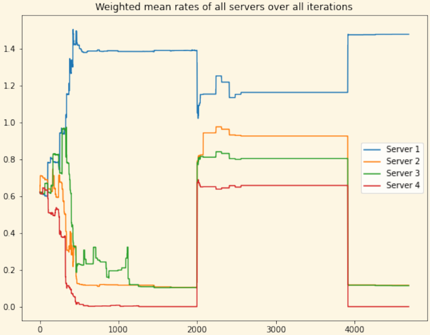

7. Conclusions

Inefficient behaviour can be learned and emerge naturally

Targeted incentivisation of behaviours can help escape learned inefficiencies.

Thank you!

$ pip install ambulance_game

https://github.com/11michalis11/AmbulanceDecisionGame

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