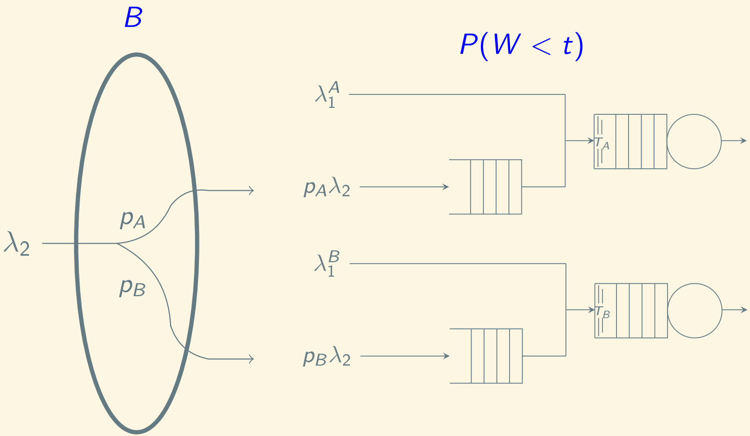
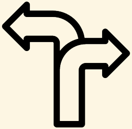
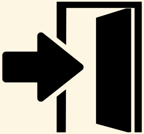
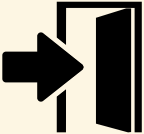
Game - Players and objectives



Game - Strategies



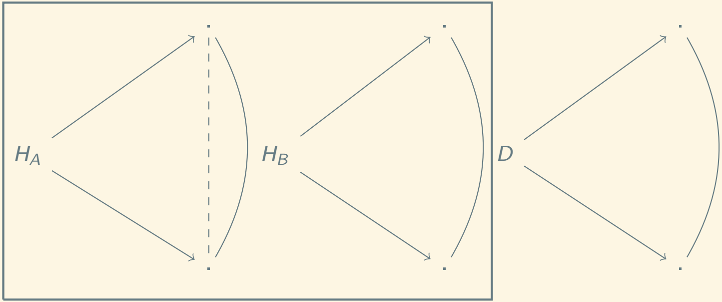
$p_A, p_B \in [0, 1]$

$p_A + p_B = 1$

$T_A \in [1, N_A]$

$T_B \in [1, N_B]$

Game - Formulation



Game - Payoff matrices

$$A = \begin{pmatrix} U^A_{1,1} & U^A_{1,2} & \cdots & U^A_{1,N_B} \\ U^A_{2,1} & U^A_{2,2} & \cdots & U^A_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^A_{N_A,1} & U^A_{N_A,2} & \cdots & U^A_{N_A,N_B} \end{pmatrix}, \quad B = \begin{pmatrix} U^B_{1,1} & U^B_{1,2} & \cdots & U^B_{1,N_B} \\ U^B_{2,1} & U^B_{2,2} & \cdots & U^B_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^B_{N_A,1} & U^B_{N_A,2} & \cdots & U^B_{N_A,N_B} \end{pmatrix}$$

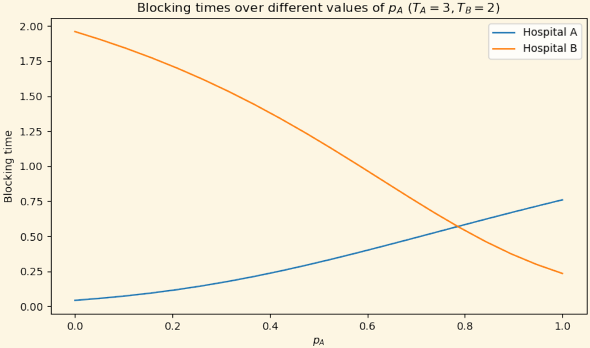
$$R = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N_B} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N_A,1} & p_{N_A,2} & \cdots & p_{N_A,N_B} \end{pmatrix}$$

Ambulance's Decision

$$R = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N_B} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N_A,1} & p_{N_A,2} & \cdots & p_{N_A,N_B} \end{pmatrix}$$

Ambulance's Decision

$$\begin{matrix} T_A = 3 \\ T_B = 2 \end{matrix} \rightarrow \begin{pmatrix} - & - & - & - \\ - & - & - & - \\ - & \mathrm{x} & - & - \\ - & - & - & - \end{pmatrix} \rightarrow B_A(p_A) = B_B(1 - p_A)$$



Hospitals' Decision

$$A = \begin{pmatrix} U^A_{1,1} & U^A_{1,2} & \cdots & U^A_{1,N_B} \\ U^A_{2,1} & U^A_{2,2} & \cdots & U^A_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^A_{N_A,1} & U^A_{N_A,2} & \cdots & U^A_{N_A,N_B} \end{pmatrix}, \quad B = \begin{pmatrix} U^B_{1,1} & U^B_{1,2} & \cdots & U^B_{1,N_B} \\ U^B_{2,1} & U^B_{2,2} & \cdots & U^B_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^B_{N_A,1} & U^B_{N_A,2} & \cdots & U^B_{N_A,N_B} \end{pmatrix}$$

$$U^{(i)}_{T_A,T_B} = 1 - \left[(P(X^{(i)} < t) - 0.95)^2 \right]$$

Nash Equilibrium

$$R = \begin{pmatrix} 0.5 & 0.1 & 0 & 0 \\ 0.9 & 0.5 & 0.2 & 0 \\ 1 & 0.8 & 0.5 & 0.3 \\ 1 & 1 & 0.7 & 0.5 \end{pmatrix}$$

$$A = \begin{pmatrix} 8.39 & 8.39 & 8.39 & 8.39 \\ 8.96 & 8.85 & 8.65 & 8.45 \\ 9.95 & 9.87 & 9.6 & 9.2 \\ 4.37 & 5.11 & 8.6 & 9.91 \end{pmatrix}$$

$$B = \begin{pmatrix} 8.39 & 8.96 & 9.95 & 4.37 \\ 8.39 & 8.85 & 9.87 & 5.11 \\ 8.39 & 8.65 & 9.6 & 8.6 \\ 8.39 & 8.45 & 9.2 & 9.91 \end{pmatrix}$$

Nash Equilibria:	A	B
	$(0, 0, 0.4, 0.6)$	$(0, 0, 0.4, 0.6)$

Asymmetric Replicator Dynamics

$$\frac{dx}{dt}_i = x_i((f_x)_i - \phi_x), \quad \text{for all } i$$

$$\frac{dy}{dt}_i = y_i((f_y)_i - \phi_y), \quad \text{for all } i$$

Inefficiency measure

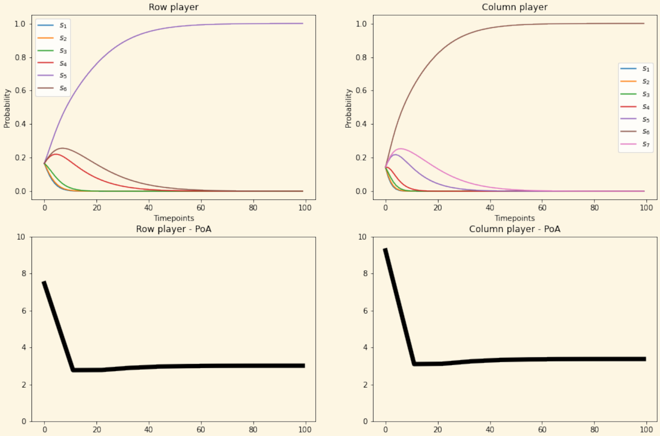
$$PoA = \frac{\max_{s \in E} Cost(s)}{\min_{s \in S} Cost(S)}$$

Inefficiency measure

$$PoA = \frac{\max_{s \in E} Cost(s)}{\min_{s \in S} Cost(S)}$$

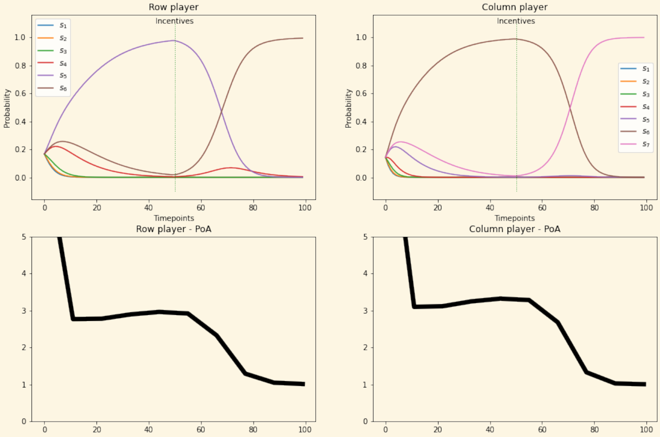
$$PoA_A(s_r) = \frac{Cost(s_r)}{\min_{s \in S} Cost(S)}, \qquad PoA_B(s_c) = \frac{Cost(s_c)}{\min_{s \in S} Cost(S)}$$

Learning algorithms - Asymmetric replicator dynamics



Inefficiencies can be learned and emerge naturally

Learning algorithms - Asymmetric replicator dynamics



Targeted incentivisation of behaviours can help escape learned inefficiencies