A game theoretic model between two hospitals

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Ambulance blockage problem in UK

Patients forced to wait for 24 hours in ambulances, data shows

Ambulance crews forced to wait outside A&Es for 24 hours, according to chiefs

Rebecca Thomas Health Correspondent • Tuesday 17 May 2022 08:26 • Comments

(AFP/Getty)

'Appalling' waits for ambulances in England leaving lives at risk

Exclusive: Royal College of Emergency Medicine president says NHS is breaking its agreement to treat sickest in a timely way

- 'For staff, this is heartbreaking': senior doctor's view on crisis
- 'I feel so let down': long waits for ambulances in south-west

Ambulance handover delays highest since start of winter

NHS 'on its knees' as ambulance response times for life-threatening calls rise to record high

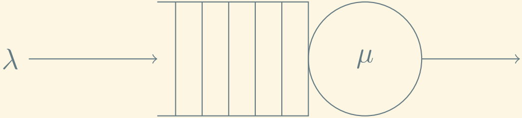
Average response time to deal with Category 1 cases – such as cardiac arrests – is now nine minutes and 20 seconds with rises across all categories.

Queueing theory

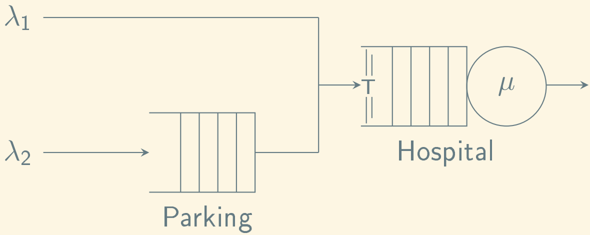
$\times$

Game theory

Queues



Queueing representation of hospital



- λ_1: Arrival rate of non-ambulance patients
- λ_2: Arrival rate of ambulance patients
- μ: Service rate
- T: Threshold

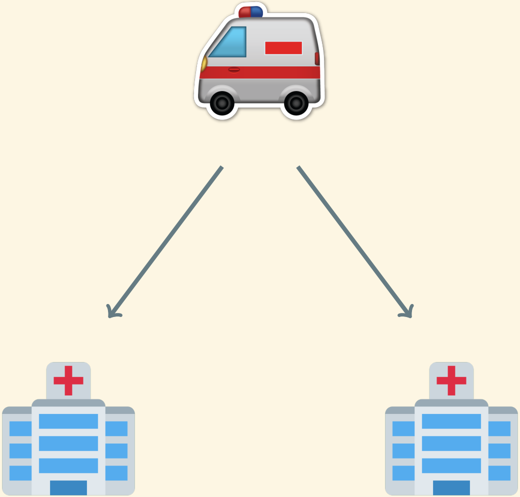
Performance Measures

$$\bar{B} = \frac{\sum_{(u,v)\in S_A^{(2)}} \pi_{(u,v)}\ b(u,v)}{\sum_{(u,v)\in S_A^{(2)}} \pi_{(u,v)}}$$

$$\bar{W} = \frac{\sum_{(u,v)\in S_A^{(2)}} \pi_{(u,v)}\ w(u,v)}{\sum_{(u,v)\in S_A^{(2)}} \pi_{(u,v)}}$$

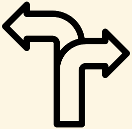
$$P(W < t) = \frac{\lambda_1 P_{L_1'}}{\lambda_2 P_{L_2'} + \lambda_1 P_{L_1'}} P(W^{(1)} < t) + \frac{\lambda_2 P_{L_2'}}{\lambda_2 P_{L_2'} + \lambda_1 P_{L_1'}} P(W^{(2)} < t)$$

The game



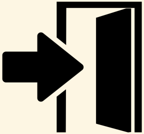
Players - Strategies - Objectives



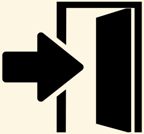


$p_A, p_B \in [0, 1]$

$p_A + p_B = 1$



$T_A \in [1, N_A]$



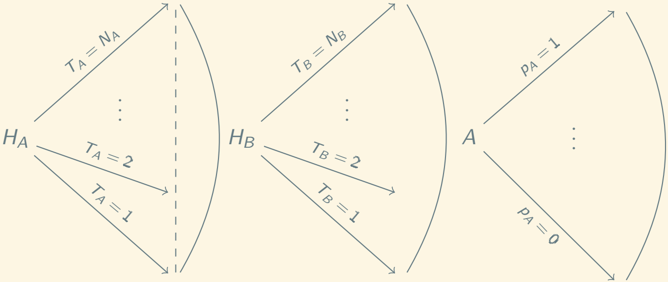
$T_B \in [1, N_B]$

$\min \bar{B}$

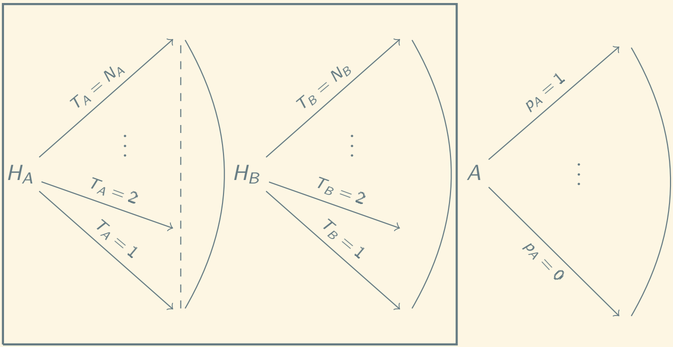
$P(W^{(A)} < t) > 0.95$

$P(W^{(B)} < t) > 0.95$

Imperfect information extensive form game



Imperfect information extensive form game



Routing matrix

$$R = \begin{pmatrix} (p^A_{1,1}, p^B_{1,1}) & (p^A_{1,2}, p^B_{1,2}) & \dots & (p^A_{1,N_B}, p^B_{1,N_B}) \\ (p^A_{2,1}, p^B_{2,1}) & (p^A_{2,2}, p^B_{2,2}) & \dots & (p^A_{2,N_B}, p^B_{2,N_B}) \\ \vdots & \vdots & \ddots & \vdots \\ (p^A_{N_A,1}, p^B_{N_A,1}) & (p^A_{N_A,2}, p^B_{N_A,2}) & \dots & (p^A_{N_A,N_B}, p^B_{N_A,N_B}) \end{pmatrix}$$

Hospital's utility

$$U_{T_A,T_B}^{(i)} = 1 - \left[\left(P(W^{(i)} < t) - 0.95 \right)^2 \right]$$

$$A = \begin{pmatrix} U_{1,1}^{A} & U_{1,2}^{A} & \cdots & U_{1,N_B}^{A} \\ U_{2,1}^{A} & U_{2,2}^{A} & \cdots & U_{2,N_B}^{A} \\ \vdots & \vdots & \ddots & \vdots \\ U_{N_A,1}^{A} & U_{N_A,2}^{A} & \cdots & U_{N_A,N_B}^{A} \end{pmatrix}, \quad B = \begin{pmatrix} U_{1,1}^{B} & U_{1,2}^{B} & \cdots & U_{1,N_B}^{B} \\ U_{2,1}^{B} & U_{2,2}^{B} & \cdots & U_{2,N_B}^{B} \\ \vdots & \vdots & \ddots & \vdots \\ U_{N_A,1}^{B} & U_{N_A,2}^{B} & \cdots & U_{N_A,N_B}^{B} \end{pmatrix}$$

Exploring the game

Nash equilibrium

- Lemke-Howson algorithm
- Support enumeration

Learning algorithms

- Fictitious play
- Replicator dynamics

Nash Equilibrium

$$A = \begin{pmatrix} 8.39 & 8.39 & 8.39 & 8.39 \\ 8.96 & 8.85 & 8.65 & 8.45 \\ 9.95 & 9.87 & 9.6 & 9.2 \\ 4.37 & 5.11 & 8.6 & 9.91 \end{pmatrix} \qquad B = \begin{pmatrix} 8.39 & 8.96 & 9.95 & 4.37 \\ 8.39 & 8.85 & 9.87 & 5.11 \\ 8.39 & 8.65 & 9.6 & 8.6 \\ 8.39 & 8.45 & 9.2 & 9.91 \end{pmatrix}$$

Nash Equilibria:

A	B	*Method*
$(0, 0, 1, 0)$	$(0, 0, 1, 0)$	S.E., L.H.
$(0, 0, 0, 1)$	$(0, 0, 0, 1)$	S.E., L.H.
$(0, 0, 0.4, 0.6)$	$(0, 0, 0.4, 0.6)$	S.E.

Replicator Dynamics

$$A \in \mathbb{R}^{n \times n}$$

$$x = [x_1, \ldots, x_n], \quad \sum x_i = 1$$

$$f = Ax$$
$$\phi = x^T f$$

$$\frac{dx_i}{dt} = x_i(f_i - \phi)$$

Asymmetric Replicator Dynamics

$$A \in \mathbb{R}^{n \times m} \qquad B \in \mathbb{R}^{n \times m}$$

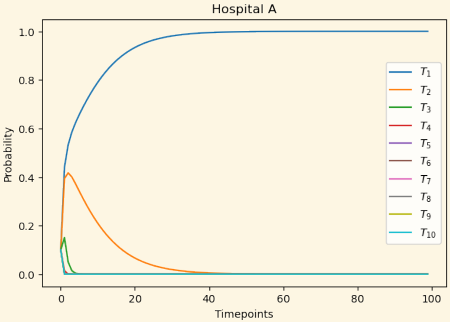
$$x = [x_1, \ldots, x_n] \qquad y = [y_1, \ldots, y_m]$$

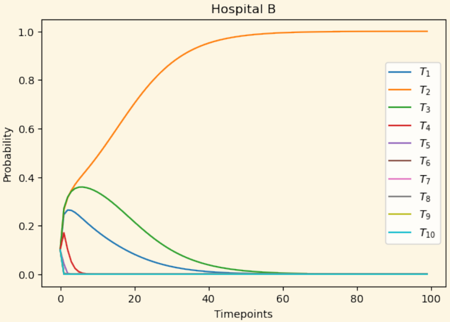
$$f_x = Ay \qquad f_y = x^T B$$
$$\phi_x = f_x x^T \qquad \phi_y = f_y y$$

$$\frac{dx_i}{dt} = x_i((f_x)_i - \phi_x) \qquad \frac{dy_i}{dt} = y_i((f_y)_i - \phi_y)$$

Evolutionary Stable Strategies

Asymmetric replicator dynamics - $t = 1.5$

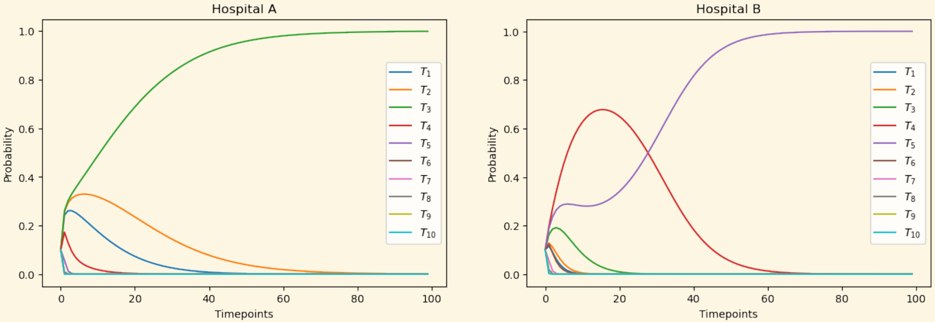




$T_A = 1$

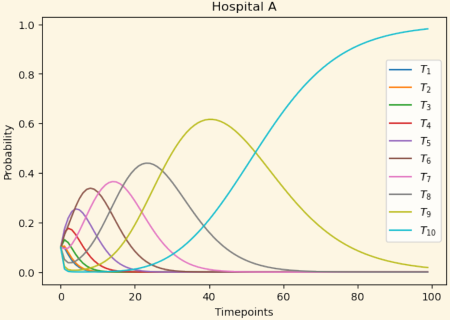
$T_B = 2$

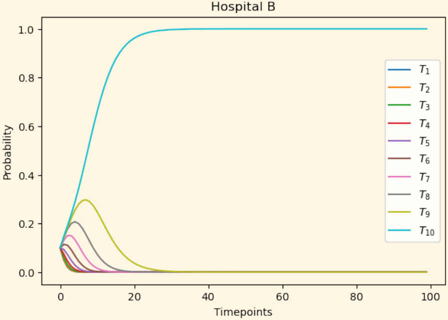
Asymmetric replicator dynamics - $t = 1.7$



$T_A = 3$ $\qquad$ $T_B = 5$

Asymmetric replicator dynamics - $t = 2$





$T_A = 10$

$T_B = 10$

Thank you!

$ pip install ambulance_game

https://github.com/MichalisPanayides/AmbulanceDecisionGame

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