

A game theoretic model between two Emergency Departments and the Emergency Medical Services

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THIS.

Supervisors:

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Prof. Paul Harper

Ambulance blockage problem in UK

Patients forced to wait for 24 hours in ambulances, data shows

Ambulance crews forced to wait outside A&Es for 24 hours, according to chiefs

Rebecca Thomas Health Correspondent • Tuesday 17 May 2022 08:26 • Comments



(AFP/Getty)

'Appalling' waits for ambulances in England leaving lives at risk

Exclusive: Royal College of Emergency Medicine president says NHS is breaking its agreement to treat sickest in a timely way
The staff, this is heart-breaking: senior doctor's view on crisis
I feel so let down: long waits for ambulances on the south-west



Ambulance handover delays highest since start of winter
© iStockphoto.com



NHS 'on its knees' as ambulance response times for life-threatening calls rise to record high

Average response time to deal with Category 1 cases – such as cardiac arrest – is now nine minutes and 20 seconds, with rises across all categories.

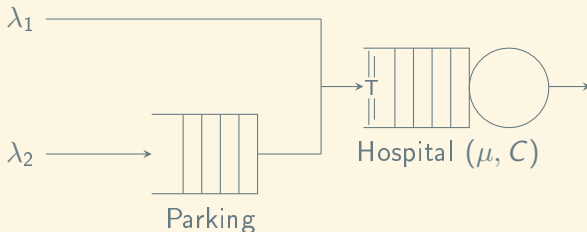


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Queues - Custom network of queues



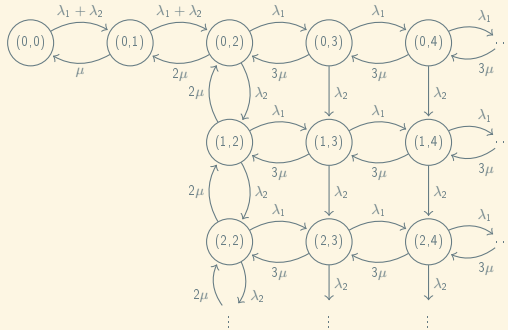
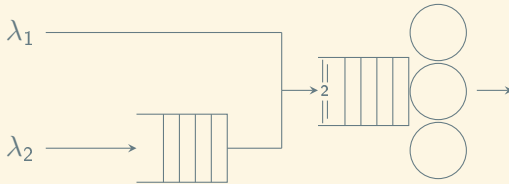
Queues - Custom network of queues



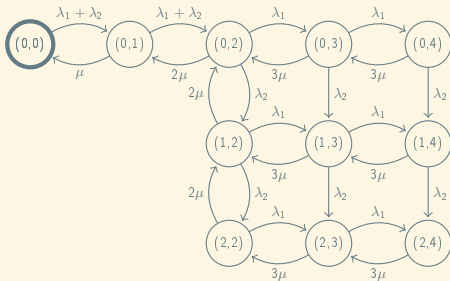
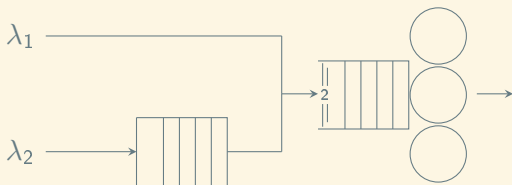
Parameters:

- ▶ λ_1 : Arrival rate of type 1 individuals
- ▶ λ_2 : Arrival rate of type 2 individuals
- ▶ μ : Service rate
- ▶ C : Number of servers
- ▶ T : Threshold

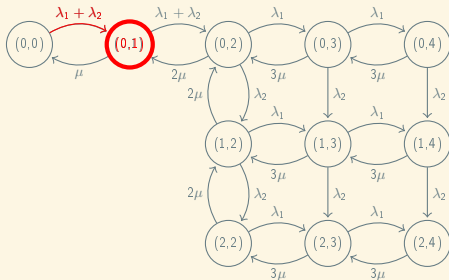
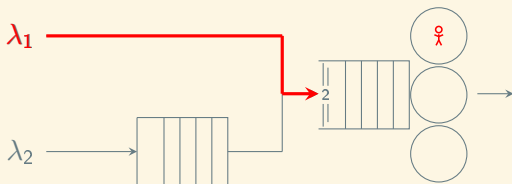
Markov Chain - Custom network



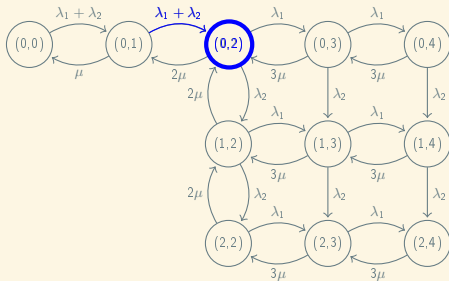
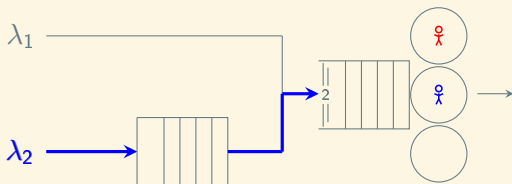
Markov Chain - Custom network - $N = 4, M = 2$



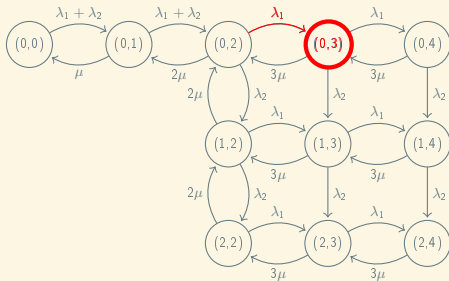
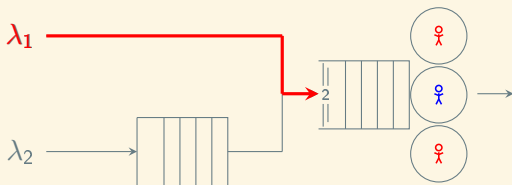
Markov Chain - Custom network - $N = 4, M = 2$



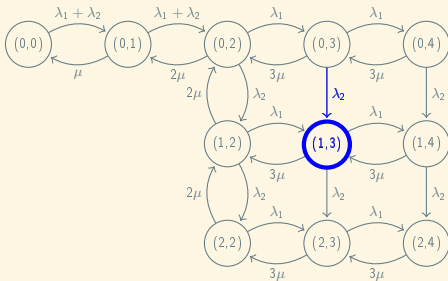
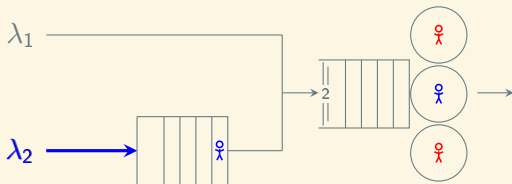
Markov Chain - Custom network - $N = 4, M = 2$



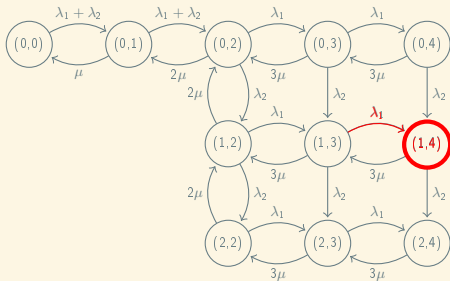
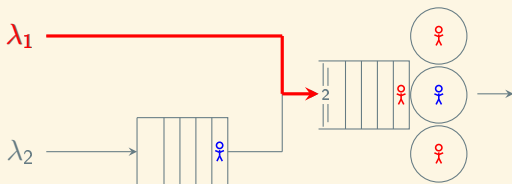
Markov Chain - Custom network - $N = 4, M = 2$



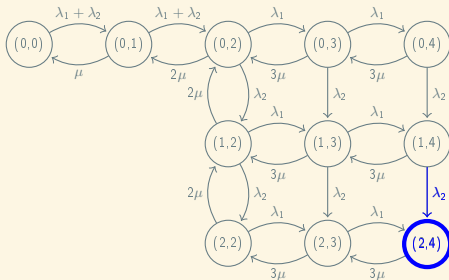
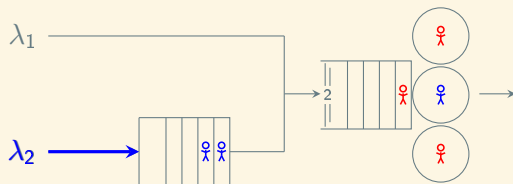
Markov Chain - Custom network - $N = 4, M = 2$



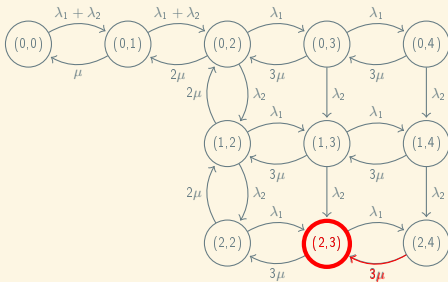
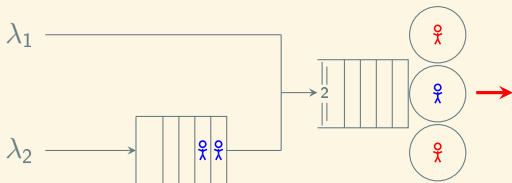
Markov Chain - Custom network - $N = 4, M = 2$



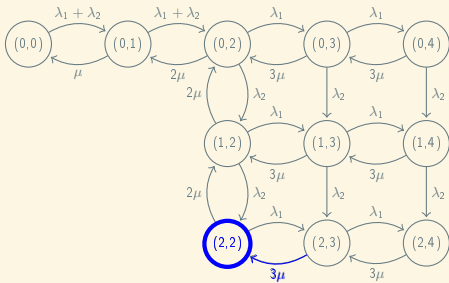
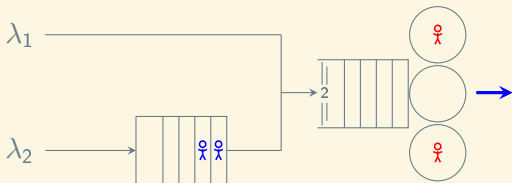
Markov Chain - Custom network - $N = 4, M = 2$



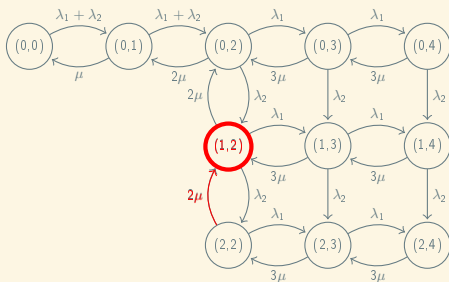
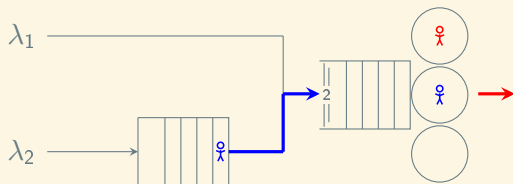
Markov Chain - Custom network - $N = 4, M = 2$



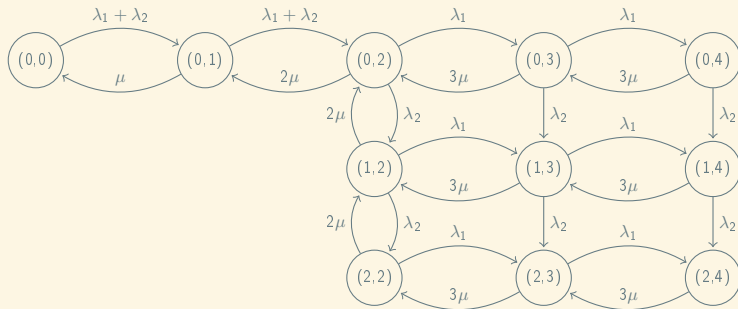
Markov Chain - Custom network - $N = 4, M = 2$



Markov Chain - Custom network - $N = 4, M = 2$



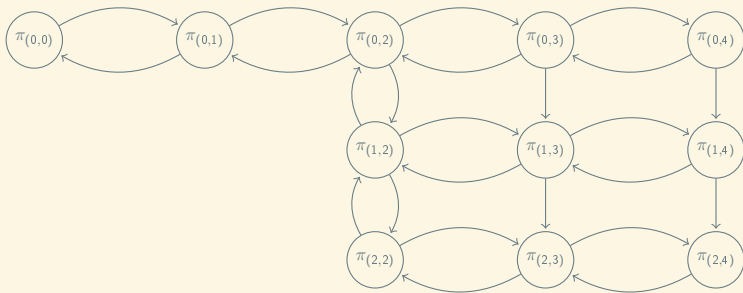
Steady state probabilities - Generator matrix



From \ To	(0,0)	(0,1)	(0,2)	(2,3)	(2,4)
(0,0)	$-\lambda_1 - \lambda_2$	$\lambda_1 + \lambda_2$	0	...	0
(0,1)	μ	$-\mu - \lambda_1 - \lambda_2$	$\lambda_1 + \lambda_2$...	0
(0,2)	0	2μ	$-2\mu - \lambda_1 - \lambda_2$...	0
...
(2,3)	0	0	0	$-\lambda_1 - 3\mu$	λ_1
(2,4)	0	0	0	3μ	-3μ

Steady state probabilities - Generator matrix (Q)

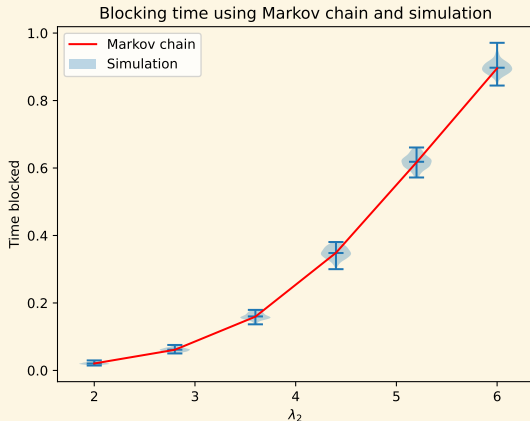
$$\pi = [\pi_{(0,0)} \quad \pi_{(0,1)} \quad \pi_{(0,2)} \quad \dots \quad \pi_{(2,3)} \quad \pi_{(2,4)}], \quad \sum \pi_{(u,v)} = 1$$



$$\frac{d\pi}{dt} = \pi Q = 0$$

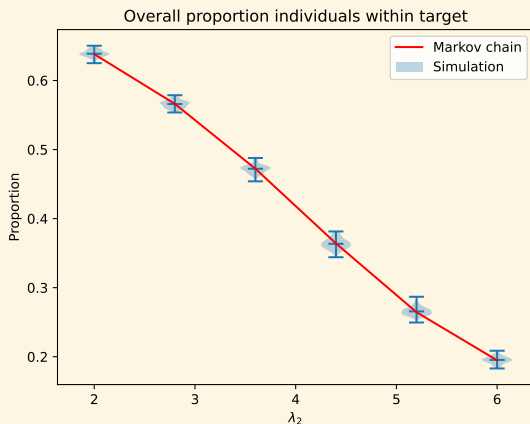
Performance Measures - Blocking time

$$B = \frac{\sum_{(u,v) \in S_A^{(2)}} \pi(u,v) b(u,v)}{\sum_{(u,v) \in S_A^{(2)}} \pi(u,v)}$$



Performance Measures - Proportion within target

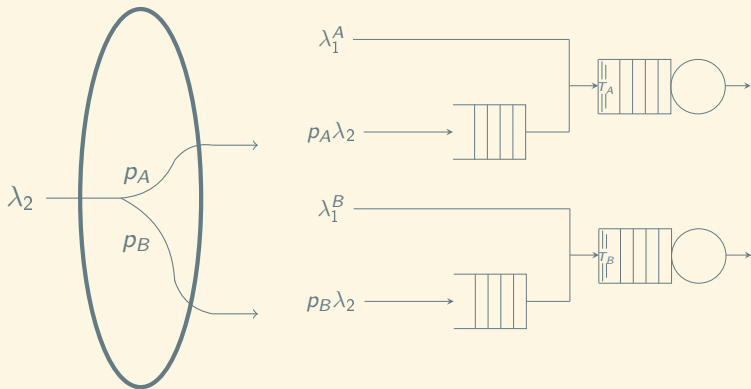
$$P(W < t) = \frac{\lambda_1 P_{L'_1}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(W^{(1)} < t) + \frac{\lambda_2 P_{L'_2}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(W^{(2)} < t)$$



Game - Definition

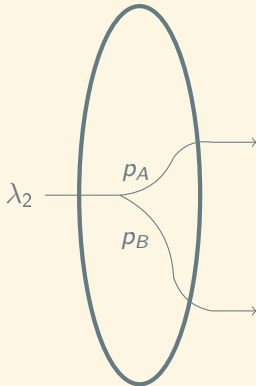


Game - Diagrammatic representation

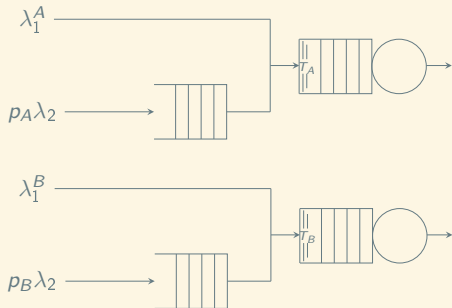


Game - Diagrammatic representation

Blocking time



Proportion of individuals within target

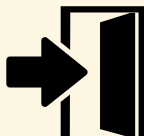


Game - Players, Strategies and Objectives



$$p_A, p_B \in [0, 1]$$
$$p_A + p_B = 1$$

$\min B$



$$T_A \in [1, N_A]$$

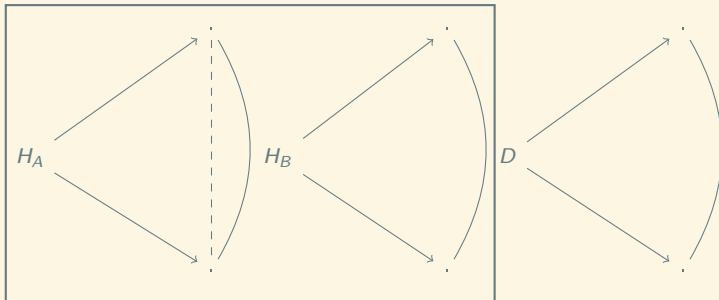
$$P(W^{(A)} < t) > 0.95$$



$$T_B \in [1, N_B]$$

$$P(W^{(B)} < t) > 0.95$$

Game - Formulation



Hospital's utility

$$U_{T_A, T_B}^{(i)} = 1 - \left[(P(W^{(i)} < t) - 0.95)^2 \right]$$

Game - Payoff matrices

$$A = \begin{pmatrix} U_{1,1}^A & U_{1,2}^A & \cdots & U_{1,N_B}^A \\ U_{2,1}^A & U_{2,2}^A & \cdots & U_{2,N_B}^A \\ \vdots & \vdots & \ddots & \vdots \\ U_{N_A,1}^A & U_{N_A,2}^A & \cdots & U_{N_A,N_B}^A \end{pmatrix}, \quad B = \begin{pmatrix} U_{1,1}^B & U_{1,2}^B & \cdots & U_{1,N_B}^B \\ U_{2,1}^B & U_{2,2}^B & \cdots & U_{2,N_B}^B \\ \vdots & \vdots & \ddots & \vdots \\ U_{N_A,1}^B & U_{N_A,2}^B & \cdots & U_{N_A,N_B}^B \end{pmatrix}$$

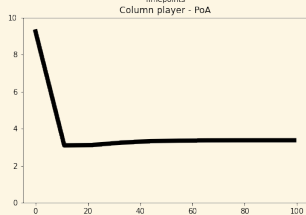
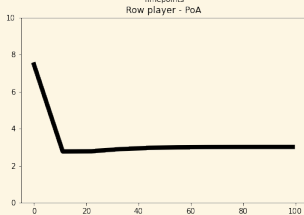
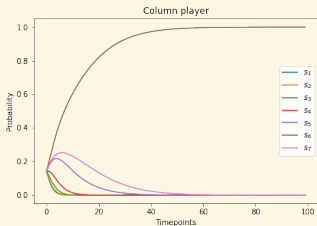
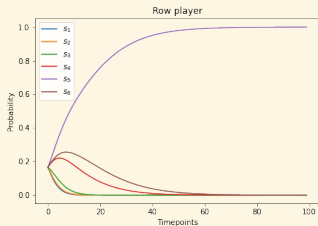
$$R = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N_B} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N_A,1} & p_{N_A,2} & \cdots & p_{N_A,N_B} \end{pmatrix}$$

Asymmetric Replicator Dynamics

$$\frac{dx}{dt}_i = x_i((f_x)_i - \phi_x), \quad \text{for all } i$$

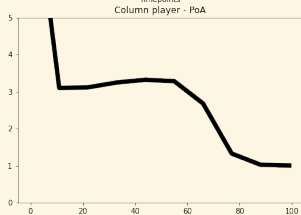
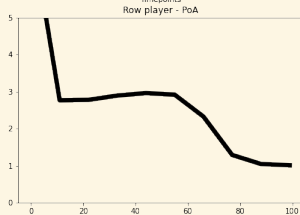
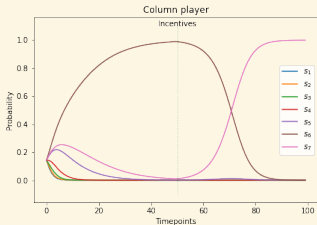
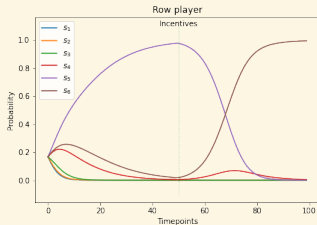
$$\frac{dy}{dt}_i = y_i((f_y)_i - \phi_y), \quad \text{for all } i$$

Learning algorithms - Asymmetric replicator dynamics



Inefficiencies can be learned and
emerge naturally

Learning algorithms - Asymmetric replicator dynamics



Targeted incentivisation of
behaviours can help escape
learned inefficiencies

Reinforcement Learning - Server's behaviour



1



2



2



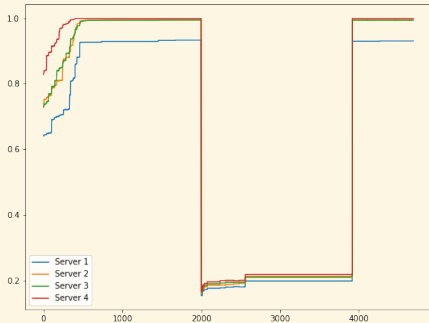
3

Utility

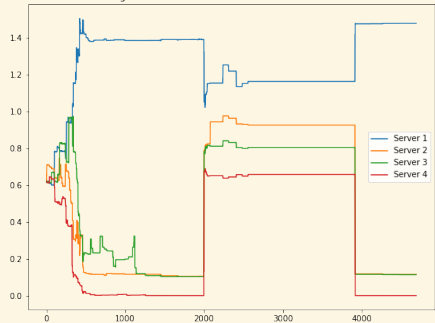
Idle time + Lost inds

Reinforcement Learning - Server's behaviour

Utilities of all servers over all iterations



Weighted mean rates of all servers over all iterations



Thank you!

```
$ pip install ambulance_game  
https://github.com/11michalis11/AmbulanceDecisionGame
```

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