Simulation of the Ising model

These assignments should be carried out by means of a program, written in a language of your choice, preferably C or FORTRAN.

I) Write a program that simulates the Ising model on a 40×40 lattice, with the Metropolis algorithm.

II) *Thermalisation.* Plot the magnetisation and energy as a function of time (measured in Monte Carlo steps per site), for various coupling constants $\beta J = 0.3 \ldots 0.5$. Also, measure in equilibrium the autocorrelation functions of the energy $c_e(\Delta t) = \langle (E(t+\Delta t) - \bar{E}) \cdot (E(t) - \bar{E}) \rangle_t$ as well as the autocorrelation function of the absolute value of the magnetization. Report the required thermalization time, as a function of coupling constant.

III) Plot the average of the absolute value of the magnetisation $|M|$, in equilibrium (i.e. after thermalization!), as a function of inverse temperature βJ; do you see signs of a phase transition?

IV) Plot $\chi = \frac{\beta}{N} \left(\langle M^2 \rangle - \langle |M| \rangle^2 \right)$ as a function of inverse temperature; do you see signs of a phase transition?

V) Implement the Wolff algorithm, and repeat the measurements III and IV. (This allows you to obtain much more accurate results.)

VI) Repeat III and IV also for lattice sizes 20×20, 30×30, 50×50, 60×60 and 100×100. Which changes do you observe?

VII) Try to obtain a data collapse of the curves of χ very close to the critical point; what values for the critical temperature and the critical exponents γ and ν do you obtain?

VIII) Determine the exponents χ, γ and ν in the three-dimensional Ising model (analogous to VII).