

1. One sentence for each of the four major theories of computation:
 - a. **Language Theory** describes, classifies, and manipulates sets of strings (languages), syntax, and semantics using grammars, automata, and algebraic structures.
 - b. **Automata Theory** looks at what problems can be solved by machines like finite automata or Turing machines.
 - c. **Computability Theory** tries to determine which problems can be solved algorithmically, regardless of efficiency.
 - d. **Complexity Theory** classifies what makes certain problems computationally hard or easy in terms of time, space, or other resources.
2. Given $L_1 = \{0, 011, 10\}$ and $L_2 = \{10, 1\}$. What are:
 - a. $L_1 \cup L_2 = \{0, 011, 10, 1\}$
 - b. $L_1 \cap L_2 = \{10\}$
 - c. $L_1 L_2 = \{010, 01, 01110, 0111, 1010, 101\}$
 - d. $L_2^* = \{\epsilon, 10, 1, 1010, 101, 110, 11, \dots\}$
3. Generative grammars for the following languages:
 - a. Empty Language
 - $S \rightarrow \epsilon$
 - b. $\{0^i 1^j 2^k \mid i=j \vee j=k\}$
 - $S \rightarrow A 2^* \mid 0^* B \mid \epsilon$
 - $A \rightarrow 0 A 1 \mid \epsilon$
 - $B \rightarrow 1 B 2 \mid \epsilon$
 - c. $\{w \in \{0,1\}^* \mid w \text{ does not contain the substring } 000\}$
 - $S \rightarrow A \mid \epsilon$
 - $A \rightarrow 1 A \mid 0 B \mid \epsilon$
 - $B \rightarrow 1 A \mid 0 C \mid \epsilon$

- $C \rightarrow 1 A \mid \epsilon$

d. $\{w \in \{a,b\}^* \mid w \text{ has twice as many } a\text{'s as } b\text{'s}\}$

- $S \rightarrow X \mid \epsilon$

- $X \rightarrow X a X a X b X \mid X a X b X a X \mid X b X a X a X \mid \epsilon$

e. $\{a^n b^n a^n b^n \mid n \geq 0\}$

- $S \rightarrow a X b Y a Z b \mid \epsilon$

- $X \rightarrow X A \mid A \mid \epsilon$

- $A a \rightarrow a A$

- $A b \rightarrow a b B$

- $A Z \rightarrow a Z b$

- $B b \rightarrow b B$

- $B Y \rightarrow b Y A$

- $Y \rightarrow \epsilon$

- $Z \rightarrow \epsilon$

4. The grammar for floating-point numerals:

$$n \rightarrow d+f?e?$$

$$f \rightarrow "."d+$$

$$e \rightarrow ("E"|"e")("+|-")?d+$$

$$d \rightarrow "0".. "9"$$

(V, Σ, R, S) -Definition of this grammar:

- $V \rightarrow \{n, f, e, d\}$

- $\Sigma \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ., +, -, E, e\}$

- $R \rightarrow \{ \text{I spent too much time trying to figure this out, I give up} \}$

- $S \rightarrow n$

5. See other pdf

6. See other pdf

7. For the JavaScript/Python expression $5 * 3 - 1 ** 3$,
- Show a 3AC machine program to evaluate this expression, leaving the result in r_0
 - MUL 5, 3, r_1
 - POW 1, 3, r_2
 - SUB r_1 , r_2 , r_0
 - Show a Stack machine program to evaluate this expression, leaving the result on the top of the stack.
 - Push 5
 - Push 3
 - MUL
 - Push 1
 - Push 3
 - Pow
 - Sub
8. Characterize each of the following languages as either (a) regular, (b) context-free but not regular, (c) recursive but not context-free, (d) recursively enumerable but not recursive, or (e) not even recursively enumerable.
- $\{a^i b^j c^k \mid i > j > k\}$
 - (c) recursive but not context-free
 - $\{a^i b^j c^k \mid i > j \wedge k \leq i - j\}$
 - (b) context-free but not regular
 - $\{\langle M \rangle \cdot w \mid M \text{ accepts } w\}$
 - (d) recursively enumerable but not recursive
 - $\{G \mid G \text{ is context-free} \wedge L(G) = \emptyset\}$
 - (d) recursively enumerable but not recursive
 - $\{a, b\}^* \{b\}^+$

- (a) regular

f. $\{\langle M \rangle \mid M \text{ does not halt}\}$

- not even recursively enumerable

g. $\{w \mid w \text{ is a decimal numeral divisible by 7}\}$

- (a) regular

h. $\{www \mid w \text{ is a string over the Unicode alphabet}\}$

- (c) recursive but not context-free