Magnetic Field Structure

The magnetic field structure proposed is a model containing only azimuthal (ϕ) component, setting every other one equal to zero. This is done, in the first step, for purposes of simplicity and easiness on the essential calculations taking place to set up the model. In other words, the magnetic field structure is:

$$B(\varpi) = \begin{cases} B_\phi(\varpi) = -\dfrac{\gamma B_0\,(\varpi/\varpi_0)}{1+(\varpi/\varpi_0)^2} \\ B_\varpi(\varpi) = B_z(\varpi) = 0 \end{cases}$$

Whereas, the γ is Lorentz factor, B_0 is the magnitude of the field at distance ϖ_0 measured from jet's axis. The parameter ϖ_0 plays the role of a characteristic distance for the internal structure of the jet. This particular radius is the limit of the internal part of the outflow, also referred as "spine" in the bibliography, enveloped by the outer region, best known in the models as "sheathe".

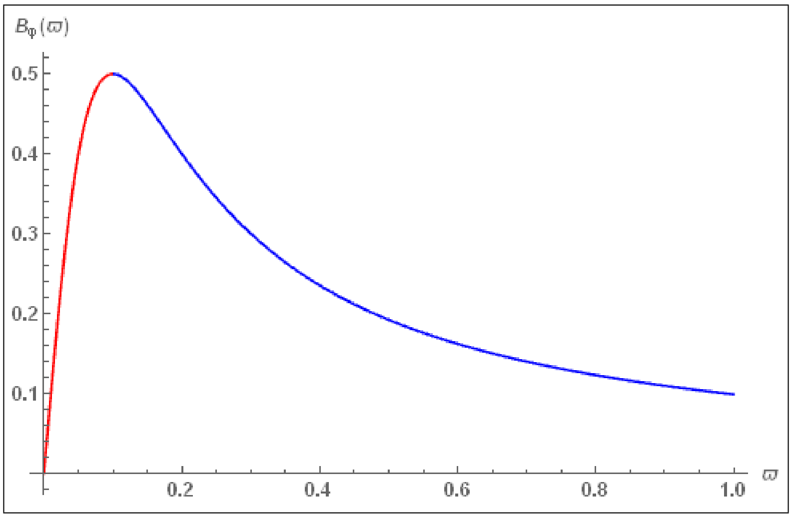

Figure 1: The plot of the magnetic field's absolute value as discussed above. The "spine" region is the red line and the "sheathe" region the blue. Jet's radius has been given the value 1 ($\varpi_j = 1$) for this example.

The ϖ_0 parameter equals 0.1 (or 10% of the jet's radius) and provides the distance from the axis that the magnetic field achieves it's maximum value. In the above case the magnitude is random.

The correct way to define the magnitude of the magnetic field (B_0) is through the equilibrium of pressure at the boundary of the jet with the ISM, in order for the system to be dynamically stable. Mathematically, we can write:

$$\Pi\,|_{\varpi_j} = P_{\text{gas}}$$

Where, $\Pi = P_j + \underbrace{\dfrac{B^2 - E^2}{2}}_{\text{magnetic pressure}}$ and P is thermal pressure. The subscript gas (from now on

g) refers to the ISM gas. The electric field is provided by:

$$\mathbf{E} = -\left(\mathbf{u} \times \mathbf{B}\right)$$

For the implied jet model we also have:

- $\mathbf{u} = (0, 0, u_z)$
- $\gamma = 4\ ,\ \gamma = \dfrac{1}{\sqrt{1-\mathbf{u}^2}}$
- $P_g = 10^{-8}$
- $\rho_{\text{ISM}} = 1,\ \rho_j = 10^{-3}$

Thus, the magnetic pressure takes the form:

$$\frac{B^2 - E^2}{2} = \frac{B^2}{2\gamma^2}$$

Finally,

$$\boxed{\Pi = P_j + \frac{B^2}{2\gamma^2}}$$

The last step is to define plasma beta, which is the ratio of the thermal pressure over the magnetic pressure. We want the beta parameter to range between $10^{-2} - 10^{-1}$, resulting to a magnetically defined jet. Also, we will set beta (at $\varpi = \varpi_\star$, whereas $\varpi_\star$ some particular distance from the axis of the jet):

$$\beta|_{\varpi=\varpi_\star} = \frac{P_j}{B(\varpi_0)^2} = \frac{4P_j}{B_\star^2}$$

The above relation gives us B_0:

$$\boxed{B_0 = \frac{\sqrt{2P_j}}{\sqrt{\beta|_{\varpi=\varpi_\star}}} \cdot \frac{1 + \left(\varpi_\star/\varpi_0\right)^2}{\left(\varpi_\star/\ \varpi_0\right)}}$$

The final step is to find the value of the jet's pressure. The relation of boundary equilibrium gives:

$$\begin{aligned}
&\frac{B(\varpi_j)^2}{2\gamma^2} + P_j = P_g \Rightarrow \\
&\frac{\gamma^2 B_0^2 \left(\varpi_j/\varpi_0\right)^2}{2\gamma^2 \left[1 + \left(\varpi_j/\varpi_0\right)^2\right]^2} + P_j = P_g \Rightarrow \\
&\Rightarrow \ldots \Rightarrow \\
&P_j = \frac{P_g}{1 + \left[\dfrac{1 + \left(\varpi_\star/\varpi_0\right)^2}{1 + \left(\varpi_j/\varpi_0\right)^2}\right]^2 \left[\dfrac{\left(\varpi_j/\ \varpi_0\right)}{\left(\varpi_\star/\ \varpi_0\right)}\right]^2} \Rightarrow \\
&\boxed{P_j = \frac{P_g}{1 + \left[\dfrac{1 + \left(\varpi_\star/\varpi_0\right)^2}{1 + \left(\varpi_j/\varpi_0\right)^2}\right]^2 \left(\varpi_j/\varpi_\star\right)^2}}
\end{aligned}$$

Finally, we have established the analytical expressions for P_j, B_0 depending on P_g, ϖ_j, ϖ_0, $\varpi_\star$ and $\beta|_{\varpi=\varpi_\star}$ which are parameters given by the user.

Now, giving the values $\beta|_{\varpi=\varpi_\star} = 0.05,\ \varpi_0 = 0.2 \cdot \varpi_j,\ \varpi_\star = 0.04 \cdot \varpi_j$ we have:

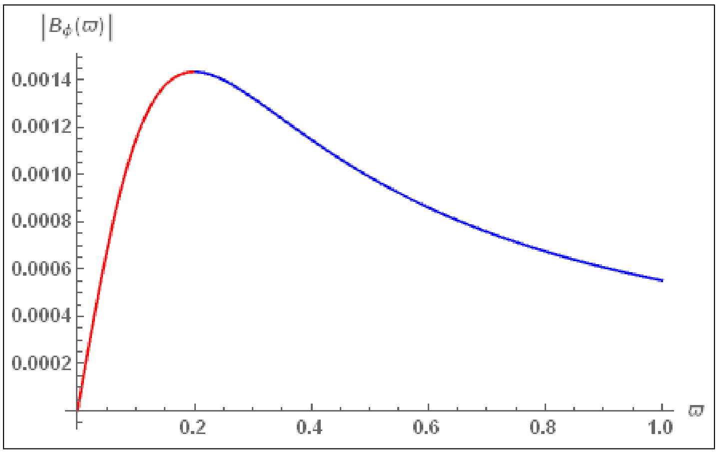

Figure 2: The magnetic field of the jet produced by the equality of the total pressure of the jet and the pressure of the gas and the particular β mentoned above.

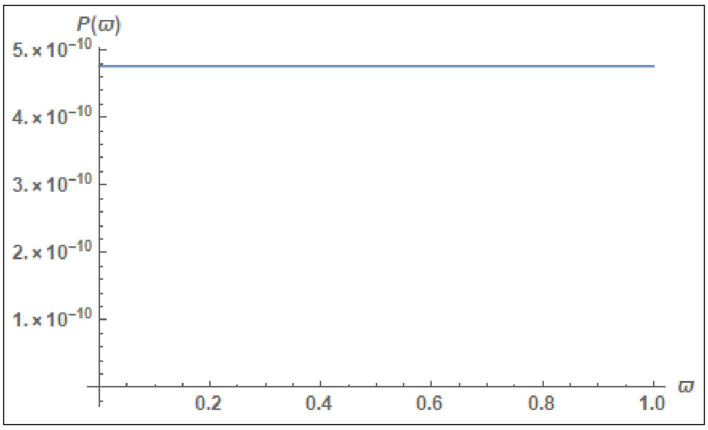

Figure 3: The thermal pressure profile. We provide a constant profile with a numerical value equals to $4.7619 \cdot 10^{-10}$.

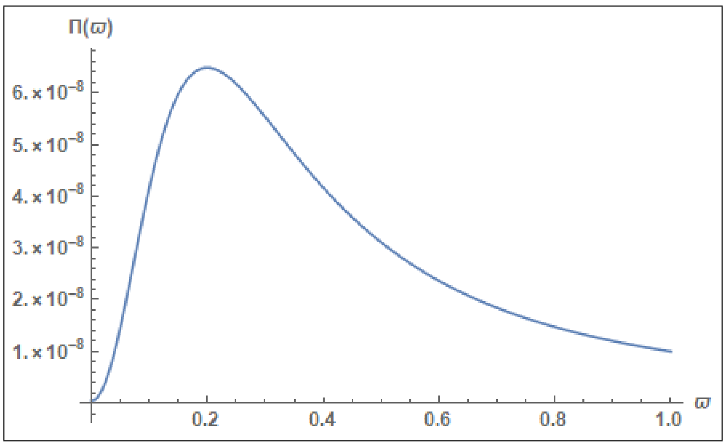

Figure 4: The total pressure profile.

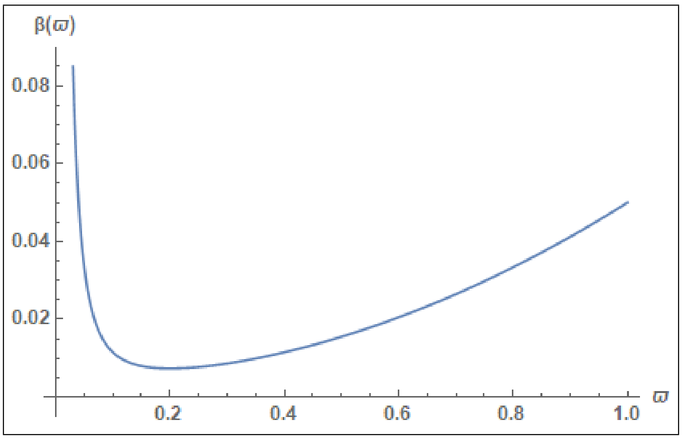

Figure 5: The plasma β parameter. Near axis region is dominated by the thermal pressure, due to the fact that the magnetic field becomes zero for $\varpi \to 0$. Also, The values are below 0.05 for the majority of the axis (except for $\varpi \leq \varpi_\star$).