Faculty of Computer Science and Information Technology

Discrete Mathematics

Lecture 3

Sets

Sets

- A set is an unordered collection of objects.

The objects in a set are called the elements, or members, of the set. A set is said to contain its elements.

- $\boldsymbol{S = \{a, b, c, d\}}$

- We write $a \in S$ to denote that a is an element of the set S. The notation $e \notin S$ denotes that e is not an element of the set S.

Describing a Set: Roster Method

- The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.
- The set of positive integers less than 100 can be denoted by $\{1, 2, 3, \ldots, 99\}$. ellipses (…)
- Set of all integers less than 0:

S = {…., -3,-2,-1}

- Set of all vowels in the English alphabet:

V = {a,e,i,o,u}

Describing a Set: Set-Builder

- Another way to describe a set is to use set builder notation.

• The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.

$O = \{x \mid x \text{ is an odd positive integer less than } 10\}$

$O = \{x \in \mathbf{Z}^+ \mid x \text{ is odd and } x < 10\}$.

Describing a Set: Set-Builder

- Use set builder notation to give a description of each of these sets:

a) $\{-3, -2, -1, 0, 1, 2, 3\}$

Solution:

A = {x|x is an integer and $-3 \leq x \leq 3$}

b) {m, n, o, p}

Solution:

letters = {x|x is a small English letter and from the letter m to p}

Some Important Sets

- **N** = {0,1,2,3….}, the set of all natural numbers
- **Z** = {…,-3,-2,-1,0,1,2,3,…} the set of all integers
- $\mathbf{Z}^{+}$ = {1,2,3,…..} the set of allpositive integers
- **R** = the set of all real numbers
- $\mathbf{R}^{+}$= the set of all positive real numbers

Example :List the members of these sets (in Roster from)

{x | x is the square of an integer and x < 100}

Solution : {1,4,9,16,25,36,49,64,81}

Interval Notation

- Closed interval $[a, b]$
- Open interval (a, b)
- $[a, b] = \{x \mid a \leq x \leq b\}$
- $[a, b) = \{x \mid a \leq x < b\}$
- $(a, b] = \{x \mid a < x \leq b\}$
- $(a, b) = \{x \mid a < x < b\}$
- **Example :** Which of the intervals (0, 5), (0, 5], [0, 5), [0, 5], (1, 4], [2, 3], (2, 3) contains

a) 0? b) 5?

Solution : a) [0,5), [0, 5] b) (0,5], [0, 5]

Set Equality

- **Definition:** Two sets are equal if and only if they have the same elements.

- Therefore if A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$.
- We write A = B if A and B are equal sets.
- The sets **{1, 3 , 5} and {3, 5 , 1}** are equal, because they have the same elements.
- {1 , 3 , 3 , 5 , 5 , 5} is the same as the set {1, 3 , 5} because they have the same elements.

Empty Set

➢ **Empty Set**

❖ There is a special set that has no elements. This set is called the empty set, or null set, and is denoted by ∅.

❖ The empty set can also be denoted by { }

$\emptyset = \{\ \} \neq \{\ \emptyset\ \}$

Set Cardinality

- **Definition:**

The cardinality is the number of distinct elements in S. The cardinality of S is denoted by |S|.

Examples:

- $S = \{a, b, c, d\}$ $\quad |S| = 4$
- $A = \{1, 2, 3, 7, 9\}$ $\quad |A| = 5$
- $\emptyset = \{\ \}$ $\quad |\emptyset| = 0 \quad |\{\ \}| = 0$

Set Cardinality

Examples:

- $S = \{a, b, c, d, \{2\}\}$ $|S| = 5$
- $A = \{1, 2, 3, \{2,3\}, 9\}$ $|A| = 5$
- $|\{\emptyset\}| = 1$
- Let S be the letters of the English alphabet.

  Then $|S| = 26$

Infinite

Infinite

A set is said to be infinite if it is not finite. The set of positive integers is infinite.

$$Z^{+} = \{1,2,3, \ldots\}$$

Subset

- **Definition:**

- The set A is said to be a subset of B if and only if every element of A is also an element of B .
- We use the notation $A \subseteq B$ to indicate that A is a subset of the set B .

$$(A \subseteq B) \equiv (B \supseteq A)$$

$$A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$$

Proper Subset

- The set A is a subset of the set B but that $A \neq B$, we write $A \subset B$

and say that A is a **proper subset** of B.

$$A \subset B \leftrightarrow (\forall x\ (x \in A \rightarrow x \in B \wedge \exists x\ (x \in B \wedge x \notin A))$$

Example

For each of the following sets, determine whether 3 is an element of that set.

$\{1,2,3,4\}$ $\{\{1\},\{2\},\{3\},\{4\}\}$

$\{1, 2, \{1,3\}\}$

Proper Subset

Example

Solution:

{1,2,**3**,4} **√**

{{1} ,{2}, {3}, {4}} **X**

{1} **{2}** **{3} ≠3** **{4}**

{1 , 2, **{1,3}**} **X**

1

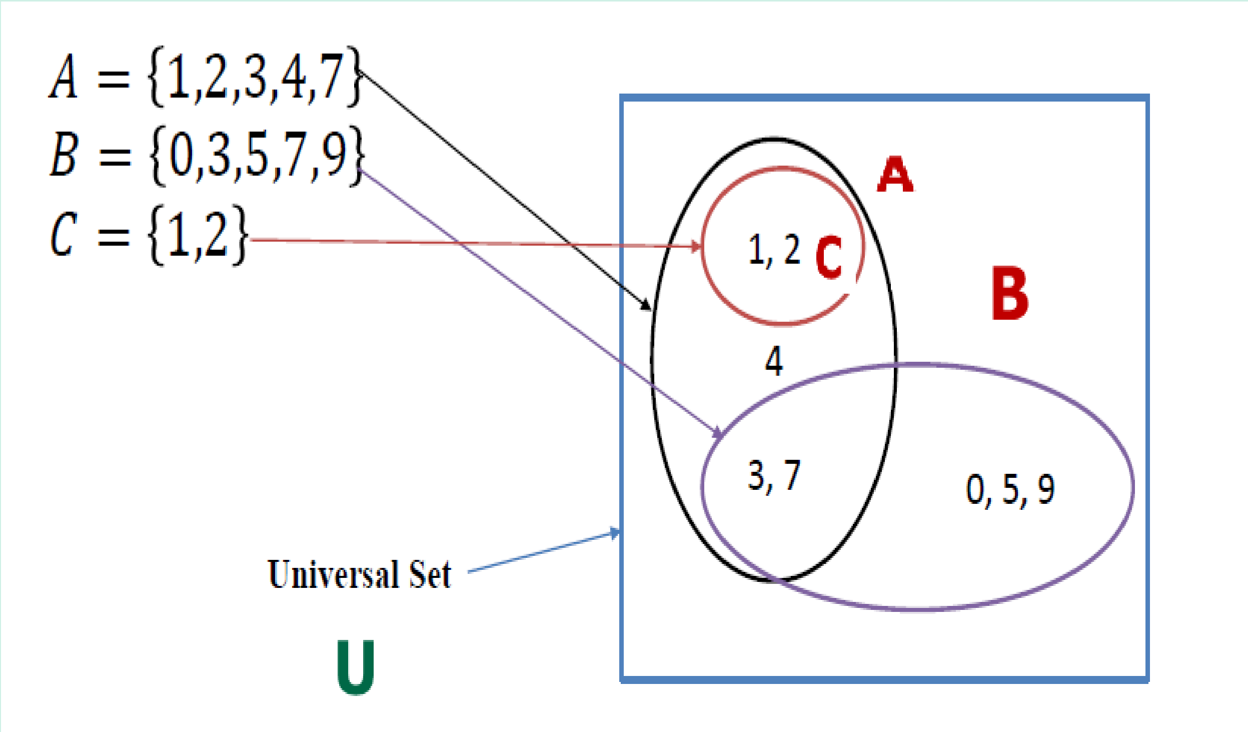
2

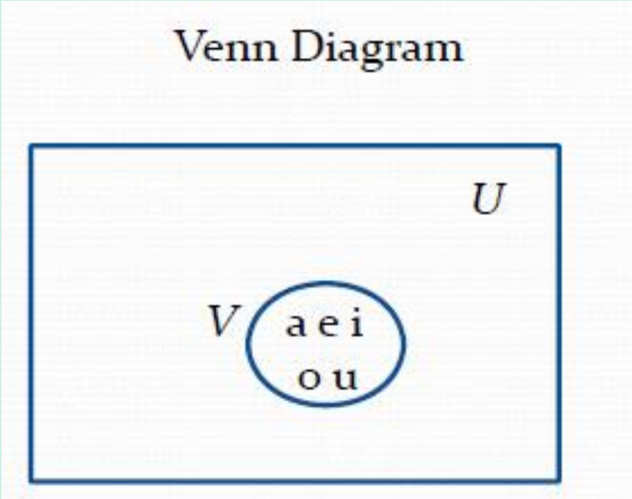
{1,3} ≠3

Universal Set

- The **universal set U** is the set containing everything currently under consideration.

- **Venn Diagram**





Power Set

- **The set of all subsets.** If the set is S. The power set of S is denoted by $P(S)$. The number of elements in the power set is $2^{|S|}$
- $\boldsymbol{P(S) \equiv 2^{S}}$

S = {1,2,3,4}

A= {1 , 2}   B= {3}   ∅ = { }

C = {1 , 4}   Z = {1 , 2,3 ,4}

{ , , , } $2^{|S|}$

Power Set

- **Examples:**

1. S = {1,2,3} |S| = 3

1. $P(S) = 2^{S}$

2. $|P(S)| = 2^{|S|} = 2^{3} = 8$

$$|P(S)| = 2^3 = 8 \text{ elements}$$

$= \{\emptyset, \{1\} \ , \{2\}, \{3\}, \{1,2\} \ , \{1,3\} \ , \{2,3\} \ \{1 \ ,2,3\}$

2. If S = {a,b} then

P(S) = {ø, {a},{b},{a,b}}

Power Set

- **Examples:**

3. What is the power set of the empty set ?

$\emptyset = \{\ \}$ $\quad 2^{|\emptyset|} = 2^{\mathbf{0}} = 1$

$P(\emptyset) = 2^{\emptyset} = \{\emptyset\}$

4. What is the power set of the set {ø}?

A = {ø} $\quad |A| = 1 \quad 2^{\mathbf{1}} = 2$

P(A) = {ø, {ø} }

$\boldsymbol{P(\{ø\}) = \{ø, \{ø\}\ \}}$

The ordered n-tuple

The ordered n-tuple $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, … , and a_n as its nth element.

In particular, ordered 2-tuples are called ordered pairs (e.g., the ordered pairs (a, b))

tuple $(1,2,2,3) \neq (1,2,3)$; but set $\{1,2,2,3\} = \{1,2,3\}$.

tuple $(1,2,3) \neq (3,2,1)$, but set $\{1,2,3\} = \{3,2,1\}$

Cartesian Products

- **Let A and B be sets.**

The **Cartesian product** of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence, $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$.

Example:

Let A = {1,2} , and B = {a,b,c}

A × B = {(1,a),(1,b),(1,c), (2,a),(2,b),(2,c)}

$|A \times B| = |A| * |B| = 2 * 3 = 6$

The Cartesian product of more than two sets

The Cartesian product of the sets $A_1, A_2, \ldots, A_n$, denoted by $A_1 \times A_2 \times \cdots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots, a_n)$, where a_i belongs to A_i for $i = 1, 2, \ldots, n$. In other words,

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}.$$

The Cartesian product of more than two sets

- **Example:**

What is $A \times B \times C$, where $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$

Solution:

$A \times B \times C = \{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$

$$|A \times B \times C| = |A| * |B| * |C| = 2 * 2 * 3 = 12$$

Questions?