

Discrete Math

Ch1: Logic and Proofs

Lab 1

1) Which of these sentences are propositions?
What are the truth values of those that are propositions?

- | | |
|-----------------------------------|-------------------|
| a) Cairo is the capital of Egypt. | “True” |
| b) $2 + 3 = 5$. | “True” |
| c) $5 + 7 = 10$. | “False” |
| d) $x + 2 = 11$. | “Not proposition” |
| e) Answer this question. | “Not proposition” |

Compound Propositions, constructed from logical connectives and other propositions

Conjunction (AND) (\wedge)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (Inclusive or) (\vee)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive or (XOR) (\oplus)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication (if, then) (\rightarrow)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional (if and only if) (\leftrightarrow)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Different Ways of Expressing $p \rightarrow q$

- if p , then q
- if p , q
- q unless $\neg p$
- q if p
- q whenever p
- q follows from p
- p implies q
- p only if q
- q when p
- p is sufficient for q
- q is necessary for p
- a necessary condition for p is q
- a sufficient condition for q is p

Truth table for
1 proposition
($2^1=2$ rows)

P
T
F

Truth table for
2 propositions
($2^2=4$ rows)

P	Q
T	T
T	F
F	T
F	F

Truth table for
3 propositions
($2^3=8$ rows)

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Truth table for
4 propositions
($2^4=16$ rows)

P	Q	R	S
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

5) What is the **negation(\neg)** of each of these propositions?

P	$\neg P$
T	F
F	T

a) Mai has an MP3 player.

“Mai does not have an MP3 player”

b) There is no pollution in New Jersey.

“There is pollution in New Jersey”

c) $2 + 1 = 3$. “ $2 + 1$ not equal 3”

d) The summer in Italy is hot and sunny.

“The summer in Italy is not hot and sunny”

8) Suppose that

Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP;

Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP;

Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP.

Determine the truth value(T/F) of each of these propositions.

a) Smartphone B has the most RAM of these three smartphones.

b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

c) Smartphone B has more RAM, more ROM, **and** a higher resolution camera than Smartphone A.

d) **If** Smartphone B has more RAM and more ROM than Smartphone C, **then** it also has a higher resolution camera.

e) Smartphone A has more RAM than Smartphone B **if and only if** Smartphone B has more RAM than Smartphone A.

13) Let p and q be the propositions

p : It is below freezing.

q : It is snowing.

Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing *and* snowing.

$"p \wedge q"$

b) It is below freezing *but not* snowing.

$"p \wedge \neg q"$

c) It is *not* below freezing *and* it is *not* snowing.

$"\neg p \wedge \neg q"$

d) It is either snowing *or* below freezing.

$"p \vee q"$

e) *If* it is below freezing, it is also snowing.

$"p \rightarrow q"$

f) *Either* it is below freezing *or* it is snowing, *but it is not snowing if* it is below freezing.

$"(p \vee q) \wedge (p \rightarrow \neg q)"$

g) That it is below freezing *is necessary and sufficient* for it to be snowing.

$"p \leftrightarrow q"$

10) Let p and q be the propositions
 p : I bought a lottery ticket this week.
 q : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

a) $\neg p$

“I **didn't** buy a lottery ticket this week.”

b) $p \vee q$

“I bought a lottery ticket this week **OR** I won the million dollar jackpot.”

c) $p \rightarrow q$ “**if** I bought a lottery ticket this week, **then** I won the million dollar jackpot.”

d) $p \wedge q$

e) $p \leftrightarrow q$ “I bought a lottery ticket this week **if and only if** I won the million dollar jackpot”

f) $\neg p \rightarrow \neg q$ “**if** I **didn't** buy a lottery ticket this week, **then** I **didn't** win the million dollar jackpot”

g) $\neg p \wedge \neg q$

h) $\neg p \vee (p \wedge q)$

“I **didn't** buy a lottery ticket this week **OR** I bought a lottery ticket this week **AND** I won the million dollar jackpot “

18) Determine whether these bi-conditionals are true or false.

a) $2 + 2 = 4$ if and only if $1 + 1 = 2$. " $T \leftrightarrow T = T$ "

b) $1 + 1 = 2$ if and only if $2 + 3 = 4$. " $T \leftrightarrow F = F$ "

c) $1 + 1 = 3$ if and only if monkeys can fly. " $F \leftrightarrow F = T$ "

d) $0 > 1$ if and only if $2 > 1$. " $F \leftrightarrow T = F$ "

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

21) For each of these sentences, determine whether an inclusive or, or an exclusive.

- a) **Coffee or tea comes with dinner. “Exclusive or”**
- b) **A password must have at least three digits or be at least eight characters long. “Inclusive or”**
- c) **The prerequisite for the course is a course in number theory or a course in cryptography.**
- d) **You can pay using U.S. dollars or euros.**

Inclusive OR

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

c and d, Assignment

29) State the converse, contrapositive, and inverse of each of these conditional statements.

$q \rightarrow p$,is the **converse** of $p \rightarrow q$

$\neg q \rightarrow \neg p$,is the **contrapositive** of $p \rightarrow q$

$\neg p \rightarrow \neg q$,is the **inverse** of $p \rightarrow q$

a) If it snows today, I will ski tomorrow.

“converse: if I will ski tomorrow, it snows today “

“contra-positive: if I won't ski tomorrow, then it doesn't snow today “

“inverse: If it doesn't snow today, then I will not ski tomorrow ”

b) I come to class whenever there is going to be a quiz.

Precedence of Logical Operators

Operator	Precedence
\neg	1
\wedge	2
\vee	3
\rightarrow	4
\leftrightarrow	5

33) Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$

b) $p \vee \neg p$

c) $(p \vee \neg q) \rightarrow q$

d) $(p \vee q) \rightarrow (p \wedge q)$

e) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

f) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

E and F
Assignment

a) $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

b) $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

c) $(p \vee \neg q) \rightarrow q$

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d) $(p \vee q) \rightarrow (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T