Faculty of Computers and Artificial intelligence

Discrete Mathematics

Lecture 9

Relations

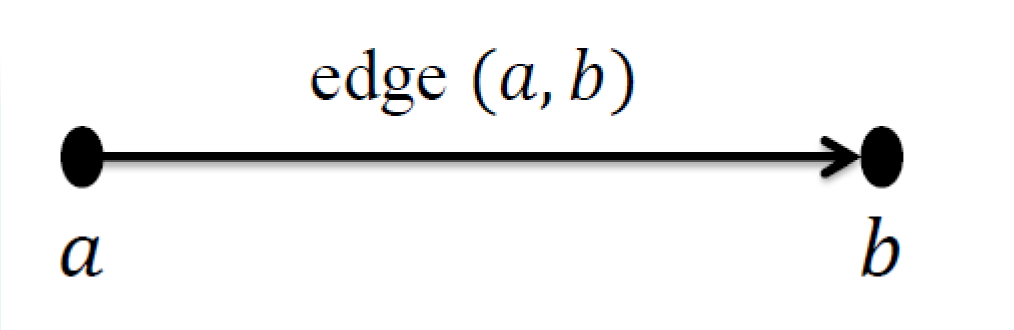
Part 3

Representing Relations

Representing Relations Using Digraphs

A **directed graph**, or **digraph**, consists of a set V of ***vertices*** (or ***nodes***) together with a set E of ordered pairs of elements of V called **edges**. The vertex a is called the ***initial vertex*** of the edge (a, b), and the vertex b is called the ***terminal vertex*** of this edge.

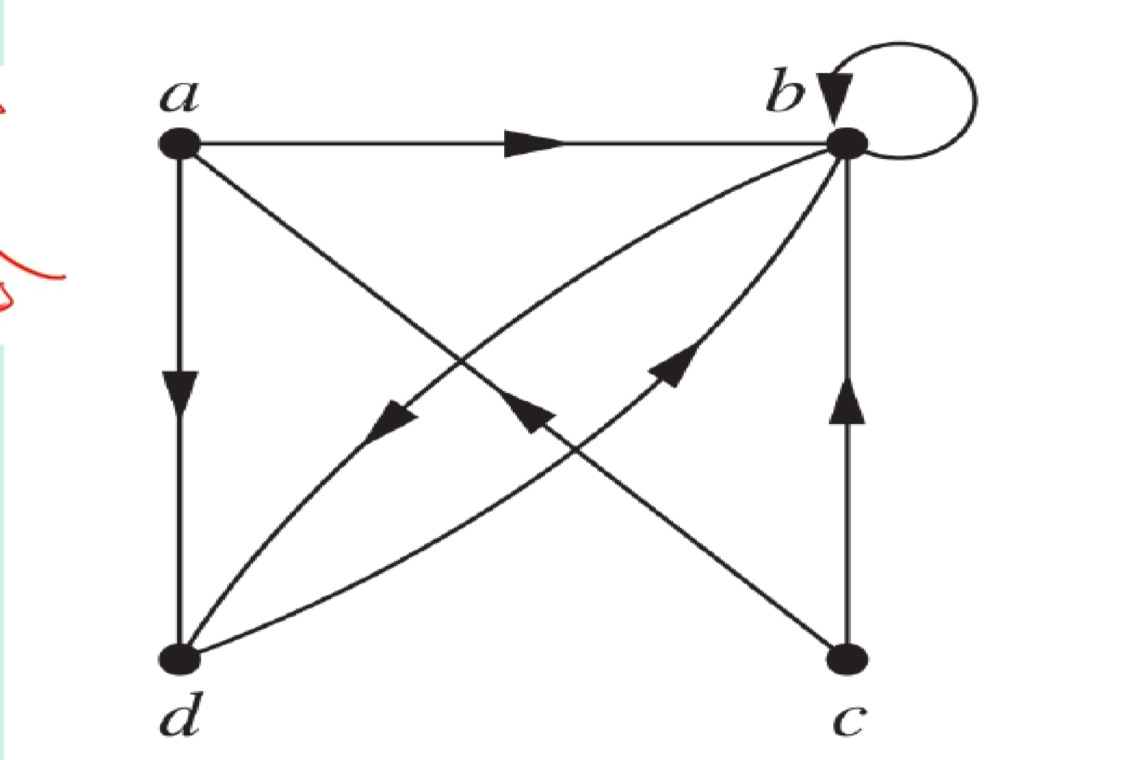
$(a, b) \in R$



Representing Relations

- **Example 1:**

$$R = \{(a,b),(a,d),(b,b),(b,d),(c,a),(c,b),(d,b)\}$$



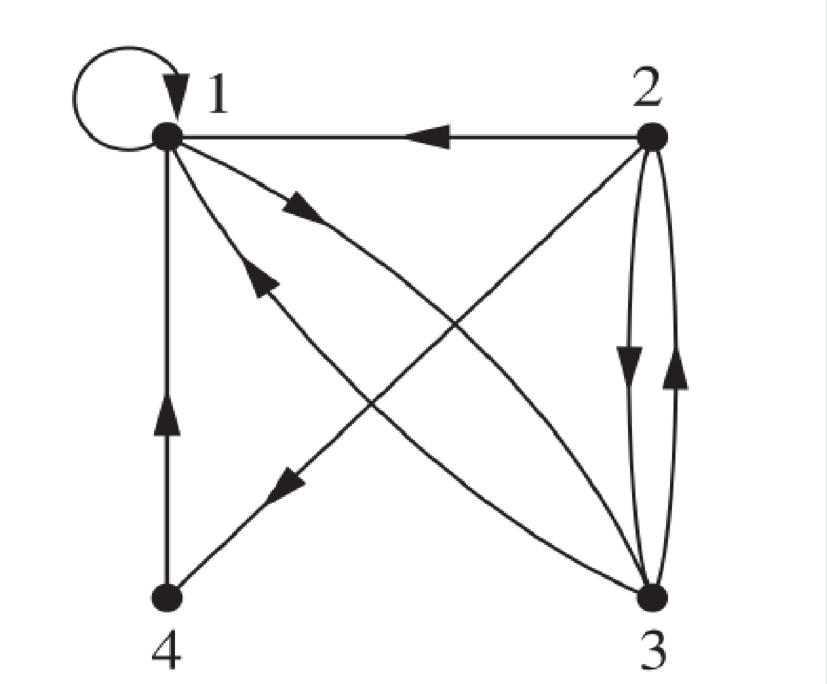
Representing Relations

- **Example 2:**

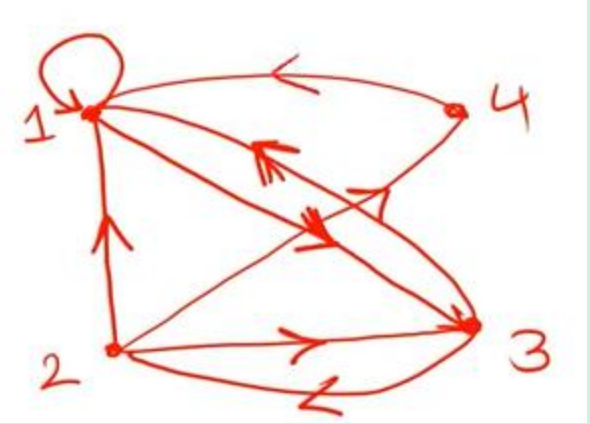
The directed graph of the relation

$$R = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)\}$$

on the set $\{1, 2, 3, 4\}$ is



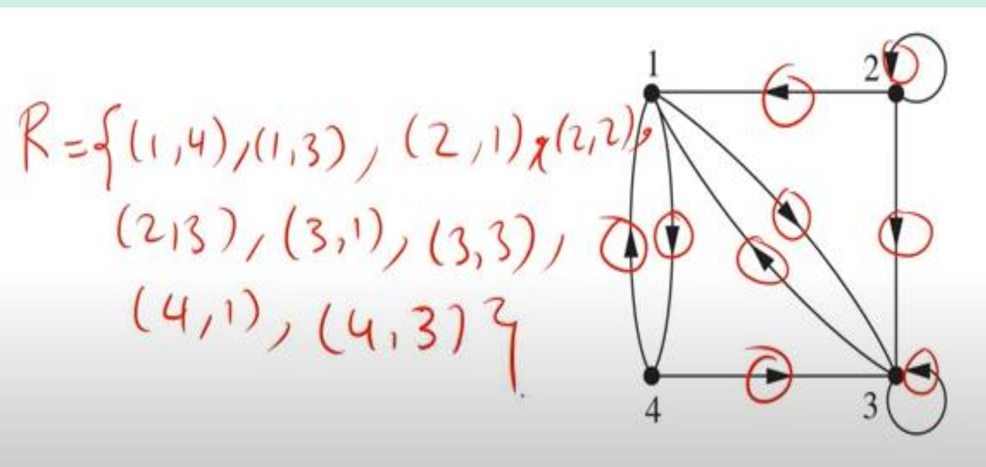
Solution:

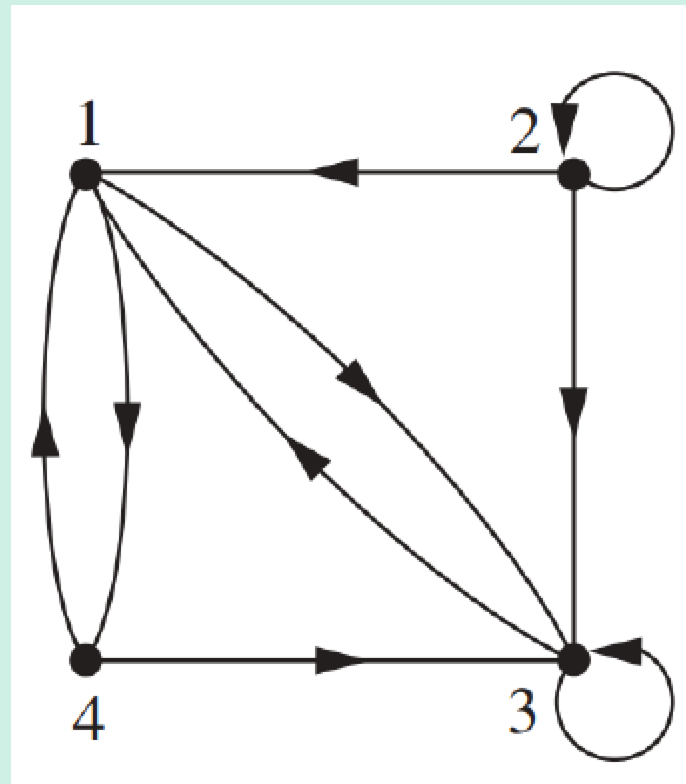


Representing Relations

- **Example 3:**

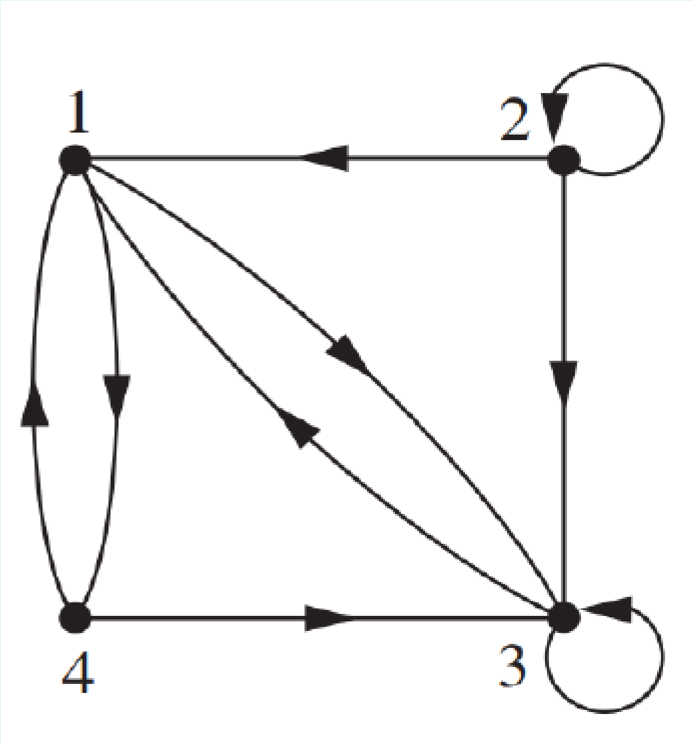
What are the ordered pairs in the relation R represented by the directed graph shown in





Representing Relations

- **Example 3:**



Solution:

$$R = \{(1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,3), (4,1), (4,3)\}$$

Equivalence Relations

❑ Definition

A relation on a set A is called an **equivalence relation** if it is reflexive, symmetric, and transitive.

Example 1:

Which of these relations on {0, 1, 2, 3} are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) {(0, 0), (1, 1), (2, 2), (3, 3)}

b) {(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}

c) {(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)}

d) {(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

e) {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)}

Equivalence Relations

Example 1:

a) $\boxed{\{(0, 0), (1, 1), (2, 2), (3, 3)\}}$ Equivalence

b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$

c) $\boxed{\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}}$ Equivalence

d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

Equivalence Relations

Example 2:

$\mathbb{R}$

Show that a relation on the set of real numbers

$R = \{(a, b) \mid (a - b) \text{ is an integer}\}$ is an equivalence relation.

Solution:

$a - a = 0$ is an integer, then $(a, a) \in R$ for all a.
So, R is **reflexive**.

If $(a, b) \in R$, then $a - b$ is an integer, therefore, $b - a$ is also an integer, i.e., $(b, a) \in R$. So, R is **symmetric**. $7-5=2$ $5-7=-2$

If $(\overset{a-b}{a, b})$ and $(\overset{b-c}{b, c}) \in R$, then $a - b$ and $b - c$ are integers, therefore, $a - b + b - c = a - c$ is also an integer, i.e., $(a, c) \in R$. So, R is **transitive**. $a-b$ ✓ $+$ $b-c$ ✓

Questions?