جامعة الأهرام الكندية
AHRAM CANADIAN UNIVERSITY

Faculty of Computer Science and Information Technology

Discrete Mathematics

Lecture 1

Logic and Proofs

Lectures Reference

- **Kenneth H. Rosen. Discrete Mathematics and Its Applications, Eighth Edition:**

https://imanulhuq.yolasite.com/resources/Discrete%20Mathematics%20and%20Its%20Applications%20-%208e%20%28Kenneth%20Rosen%29%20%5B9781259676512%5D_compressed-compressed.pdf

Course Objectives

- Learn how to think mathematically.
- • Grasp the basic logical and reasoning mechanisms of mathematical thought.
- • Improve problem-solving skills.
- • Grasp the basic elements of induction, recursion, combination and discrete structures.

DM is a Gateway Course

- Topics in discrete mathematics will be important in many courses that you will learn in the future:

• Computer Science: Computer Architecture, Data Structures, Algorithms, Programming Languages, Compilers, Computer Security, Databases, Artificial Intelligence, Networking, Graphics, Game Design, Theory of Computation, ……

• Mathematics: Logic, Set Theory, Probability, Number Theory, Abstract Algebra, Graph Theory, Game Theory, Network Optimization, …

Introduction to Propositional Logic

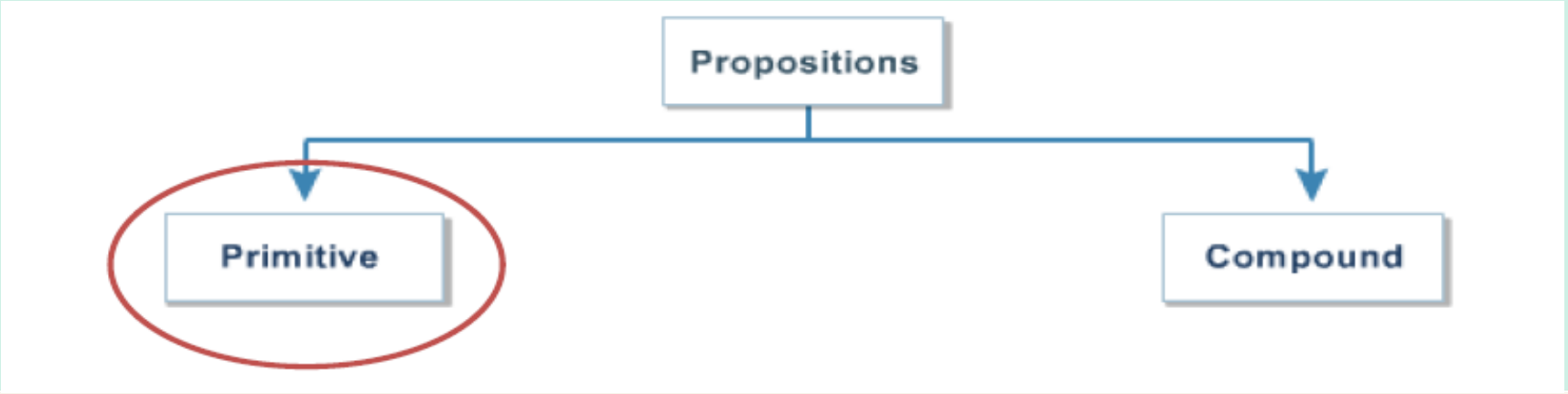
- What is Logic?

- Logic is the science that deals with the methods of reasoning.

- Logical reasoning is used in mathematics to prove theorems.

Introduction to Propositional Logic

- The basic building blocks of logic is Proposition (الاقتراح)
- A proposition (or statement (إفادة)) is a declarative (تصريحي) sentence that is either true or false, but not both.
- The area of logic that deals with propositions is called propositional logics.



Introduction to Propositional Logic

- **Examples of propositions:**

$2 + 3 = 5$ **True**

$5 - 2 = 1$ **False**

Today is Friday **False**

$x + 3 = 7$, for $x = 4$ **True**

Cairo is the capital of Egypt **True**

Examples that are not propositions.

What time is it?

Read this carefully.

$x + 3 = 7$ find x?

Introduction to Propositional Logic

- We use letters to denote propositional variables

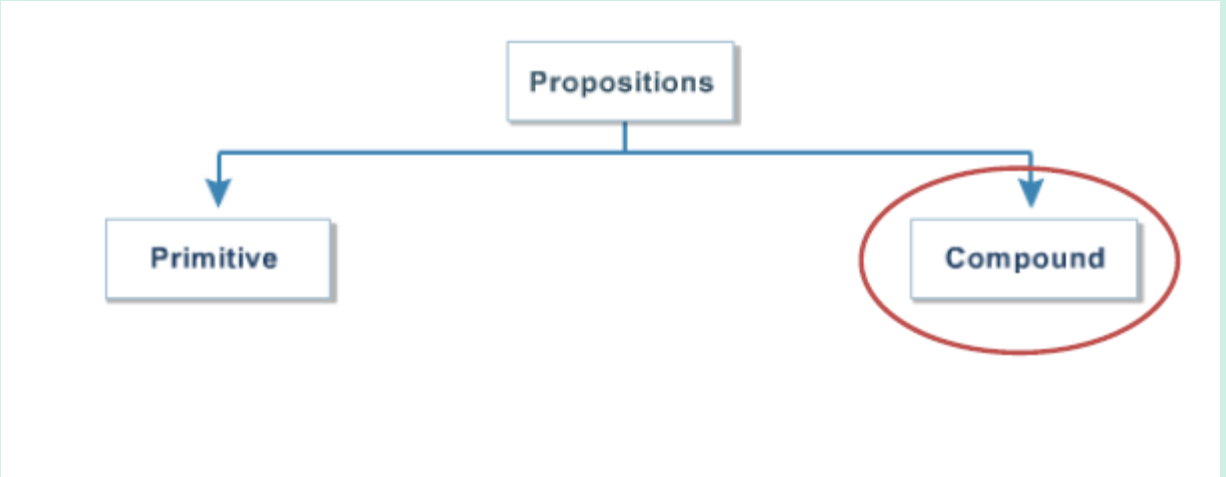
$p, q, r, s, \ldots$

- The truth value of a proposition is true, denoted by T, if it is a true proposition and false, denoted by F, if it is a false proposition.

Compound Propositions

➢ Compound Proposition

- Compound Propositions are formed from existing propositions using **logical operators**.
- Negation ¬
- Conjunction ∧
- Disjunction ∨
- Implication →
- Biconditional ↔



Compound Propositions

➢ **Negation** (not)

- The negation of a proposition p is denoted by ¬p and has this truth table:

p	$\neg p$
T	F
F	T

- Example: If p denotes "Cairo is the capital of Egypt.", then ¬p denotes "It is not the case that Cairo is the capital of Egypt," or more simply "Cairo is not the capital of Egypt." The proposition ¬p is read " not p "

Compound Propositions

➢ Conjunction (and)

• The conjunction of propositions p and q is denoted by p ∧ q and has this truth table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

•The conjunction p ∧ q is true when both p and q are true and is false otherwise .

• Example: p : Today is Friday.

q : It is raining today.

p ∧q : Today is Friday and it is raining today.

Compound Propositions

- **Disjunction(or)**
- • The disjunction of propositions p and q is denoted by p v q and has this truth table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

•The disjunction p v q is false when both p and q are false and is true otherwise .

• Example: p : Today is Friday.

q : It is raining today.

p vq : Today is Friday or it is raining today.

Compound Propositions

➢ Exclusive Or (Xor)

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$ (or p XOR q), is the proposition that is true when exactly one of p and q is true and is false otherwise.

The truth table for $\oplus$ is:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

- Example

p: They are parents.

q : They are children.

p⊕q: They are parents or children but not both.

Questions?