## Operational Amplifiers and Applications



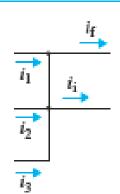


#### Fundamentals of basic circuits and what we learned in Chapter 1

The current i through resistance R is related to its voltage v by Ohm's law

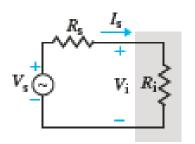
v = iR

According to Kirchoff's current law, the sum of all currents at a node must be zero, or the currents coming in to the node must equal to the current going out



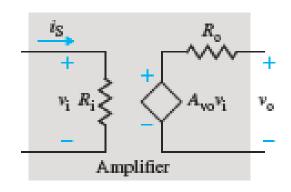
 $i_1 + i_2 + i_3 = i_f + i_i$ 

According to the voltage divider rule between two resistances  $R_s$ and  $R_i$ , the output voltage  $V_i$  across resistance  $R_i$  is related to the input voltage  $V_s$  by



 $V_{\rm i} = \frac{R_{\rm i}}{R_{\rm i} + R_{\rm s}} V_{\rm s}$ 

The model and parameters of a voltage amplifier are



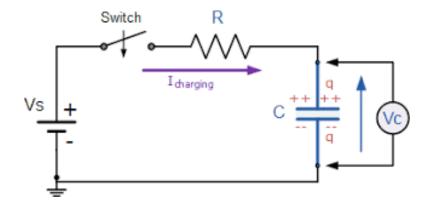
Voltage gain  $A_{vo}$ ,

Input resistance  $R_i$ ,

output resistance  $R_o$ 

If voltage amplifiers with gains of  $A_1, A_2, \ldots A_n$  are cascaded, the overall gain is the product of the gain of each stage.

 $A_{vo} = A_1 A_2 \dots A_n$ 



The instantaneous current i(t) through an RC circuit for a step voltage  $V_{\rm s}$ 

$$i = \frac{V_{\rm S}}{R}(1 - e^{-t/RC})$$

The voltage  $v_c$  of a capacitor C is related to its current i by

$$v_c = \frac{1}{C} \int i \, dt$$

The current i of a capacitor C is related to its voltage  $v_c$  by

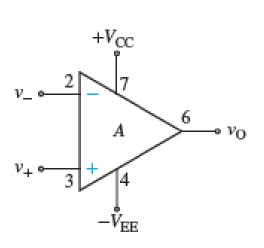
$$i = C \frac{dv_0}{dt}$$



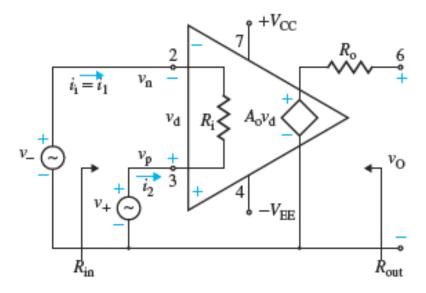
The impedance of a capacitor at a frequency $\omega$ or $f$ , $X_{\rm C}$	$X_{\rm c} = \frac{1}{j\omega C} or \frac{1}{j2\pi fC}$
The cutoff frequency of an RC circuit	$f_{\rm L} = \frac{1}{2\pi RC}$
The cutoff frequency is defined as the frequency at which the gain rises or falls to 70.7% $(1/\sqrt{2})$ of the band-pass gain, $A_{\rm BP}$ .	70.7% (1/√2)
The vector representation of a sinusoidal voltage with a peak magnitude $V_{\rm m}, V_{\rm m} \sin(2\pi f t + \varphi)$	$V_{ m m} ot \varphi$
The derivative of $V_{\mathrm{m}}\sin(\omega t)$ , $\frac{d}{dt}(V_{\mathrm{m}}\sin\omega t)$	$\omega V_{\rm m}(\cos \omega t)$ , or $\omega V_{\rm m} \angle 90^{\circ}$

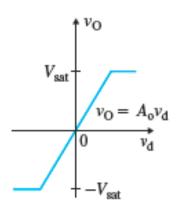
## Characteristics of Ideal Op-Amps - Op-Amp Circuit Model

 The output voltage of an op-amp is directly proportional to the small-signal differential input voltage.



$$v_o = A_o v_d = A_o (v_p - v_n)$$



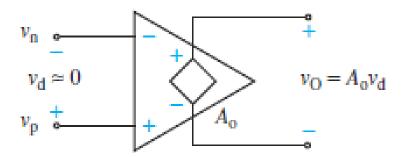




(b) Transfer characteristic

### **Ideal Op-Amp Model**

- The open-loop voltage gain is infinite:  $A_o = \infty$ .
- The input resistance is infinite:  $R_i = \infty$  .
- The amplifier draws no current:  $i_i = 0$ .
- The output resistance is negligible:  $R_o = 0$ .
- The gain A<sub>o</sub> remains constant and is not a function of frequency.
- The output voltage does not change with changes in power supplies. This condition is generally specified in terms of the power supply sensitivity (PSS): PSS = 0.

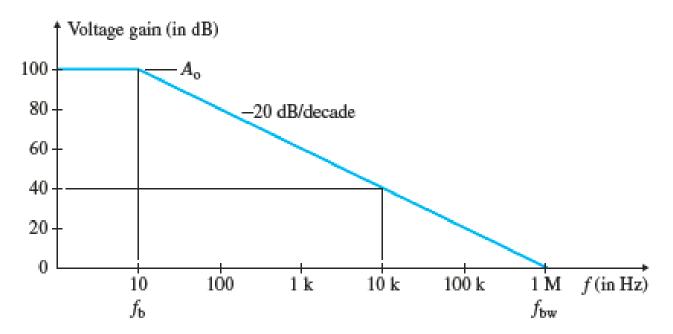




### **Op-Amp Frequency Response**

The differential voltage gain of an op-amp has the highest value at DC or low frequencies. The gain decreases with frequency.

 $A_o(j\omega) = \frac{A_o}{1 + j\omega/\omega_b} = \frac{A_o}{1 + jf/f_b}$ 



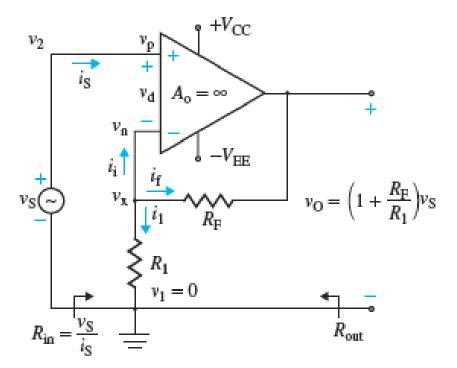


### **Non-inverting Amplifiers**

The input voltage is connected directly to the noninverting

terminal.

Closed-loop voltage gain A<sub>f</sub>:



$$v_x = \frac{R_1}{R_1 + R_f} \ v_o = v_s$$

$$A_{o} = \frac{v_{O}}{v_{d}}$$

$$A_{f} = \frac{v_{O}}{v_{S}}$$

$$A_{f} = \frac{v_{O}}{v_{S}}$$

$$A_{f} = \frac{v_{O}}{v_{S}}$$



(a) Noninverting configuration

(b) Closed-loop feedback

#### **Input and Output Resistances**

 Since the current drawn by the amplifier is zero, the effective input resistance of the amplifier is very high, tending to infinity.

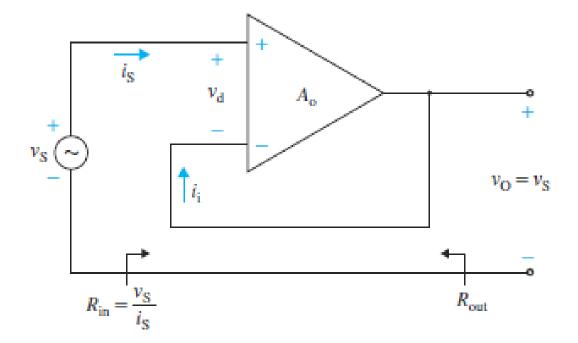
$$R_{in} = \frac{v_S}{i_S} = \infty$$

• The effective output resistance:

$$R_{out} = R_o \approx 0$$

### **Voltage Follower**

- If  $R_F = 0$  or  $R_1 = \infty$  then  $A_f = 1$
- Used as the *buffer stage* between a low impedance load and a source requiring a high impedance load.



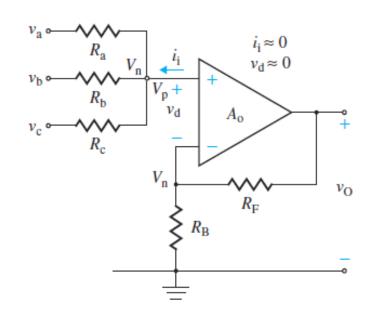


#### **Non-inverting Summing Amplifiers**

$$v_{\rm p} = \frac{R_{\rm b} \| R_{\rm c}}{R_{\rm a} + R_{\rm b} \| R_{\rm c}} v_{\rm a} + \frac{R_{\rm a} \| R_{\rm c}}{R_{\rm b} + R_{\rm a} \| R_{\rm c}} v_{\rm b} + \frac{R_{\rm a} \| R_{\rm b}}{R_{\rm c} + R_{\rm a} \| R_{\rm b}} v_{\rm c}$$

$$v_o = (1 + \frac{R_F}{R_B})(\frac{R_A}{R_a}v_a + \frac{R_A}{R_b}v_b + \frac{R_A}{R_c}v_c)$$

where 
$$R_A = (R_a \parallel R_b \parallel R_c)$$

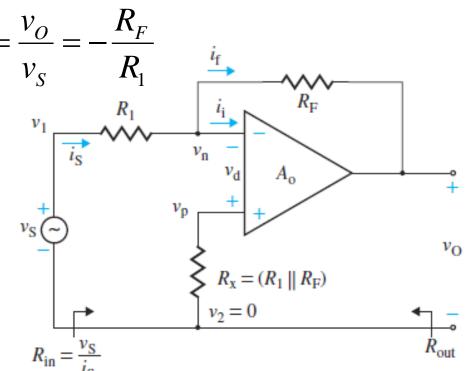




### **Inverting Amplifiers**

- The input voltage is connected to the inverting terminal and the non-inverting terminal is connected to the ground directly or through a resistance.
- Closed-Loop Gain:

$$\frac{v_o}{R_F} = -\frac{v_S}{R_1}$$

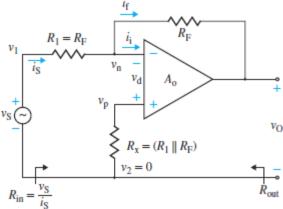


#### **Input and Output Resistances**

• The effective input resistance R<sub>in</sub> of the amplifier:

$$R_{in} = \frac{v_S}{i_S} = \frac{v_S}{(v_S + v_d)/R_1} \approx R_1$$

• **Op-Amp Inverter**: An inverter is often used to invert the polarity of a signal.





# Effect of Finite Op-Amp Gain on Closed-Loop Gain

Closed-loop voltage gain:

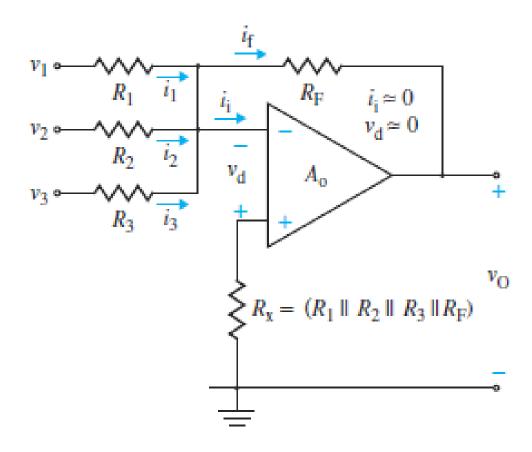
$$A_f = \frac{v_O}{v_S} = -\frac{R_F / R_1}{1 + (1 + R_F / R_1) / A_o} = -\frac{R_F}{R_1 (1 + x)}$$



#### **Inverting Summing Amplifiers**

Output voltage:

$$v_o = -(\frac{R_F}{R_1}v_1 + \frac{R_F}{R_2}v_2 + \frac{R_F}{R_3}v_3)$$

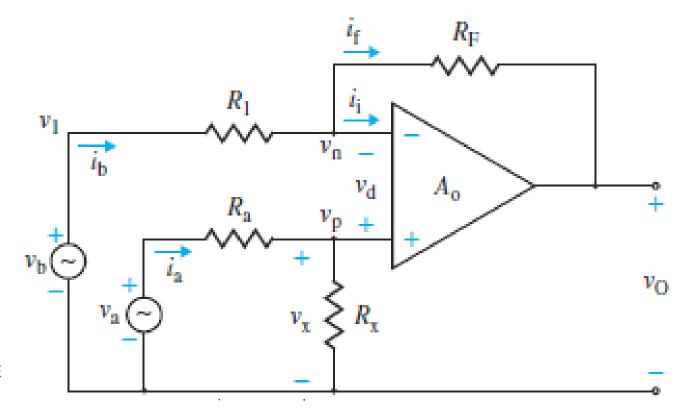




### **Difference Amplifiers**

- Two input voltages (v<sub>a</sub> and v<sub>b</sub>) are applied—one to the non-inverting terminal and another to the inverting terminal.
- Closed-Loop Gain of Difference Amplifiers:

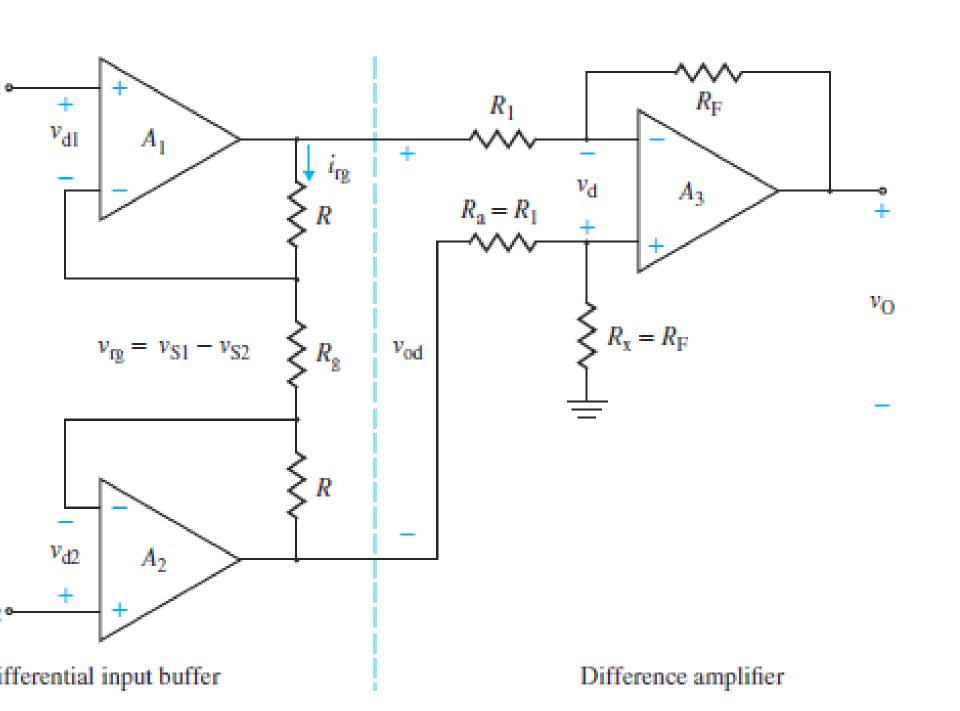
$$v_o = -\frac{R_F}{R_1} v_b + (1 + \frac{R_F}{R_1})(\frac{R_x}{R_x + R_a})v_a$$



#### **Instrumentation Amplifiers**

- Instrumentation amplifier: dedicated difference amplifier with an extremely high input impedance.
- Its gain can be precisely set by a single resistance.
- Closed-Loop Gain of Instrumentation Amplifiers:





#### **Instrumentation Amplifiers**

$$v_{\text{od}} = i_{\text{rg}}(R_{\text{g}} + 2R) = \frac{v_{\text{S1}} - v_{\text{S2}}}{R_{\text{g}}}(R_{\text{g}} + 2R) = (v_{\text{S1}} - v_{\text{S2}}) \left(1 + \frac{2R}{R_{\text{g}}}\right)$$

Using Eq. (3.82), we can calculate the output voltage  $v_0$  as

$$v_{\rm O} = -v_{\rm od} \frac{R_{\rm F}}{R_1} = -(v_{\rm S1} - v_{\rm S2}) \left(1 + \frac{2R}{R_{\rm g}}\right) \left(\frac{R_{\rm F}}{R_1}\right)$$

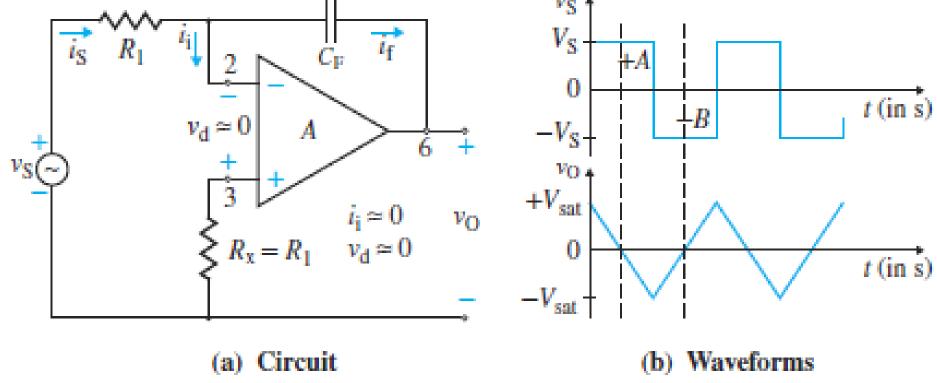
$$A_f = (1 + \frac{2R}{R_g})(\frac{R_F}{R_1})$$



#### **Integrators**

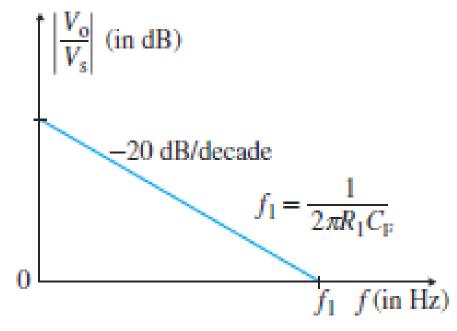
• If the resistance  $R_F$  in the inverting amplifier is replaced by a capacitance  $C_F$ , the circuit will operate as an integrator.

$$v_O(t) = -\frac{1}{R_1 C_F} \int_0^{\tau} v_S dt - v_C(t=0)$$



#### Integrators

$$v_{O}(t) = -\frac{1}{R_{1}C_{F}} \int_{0}^{\tau} v_{S} dt - v_{C}(t=0)$$



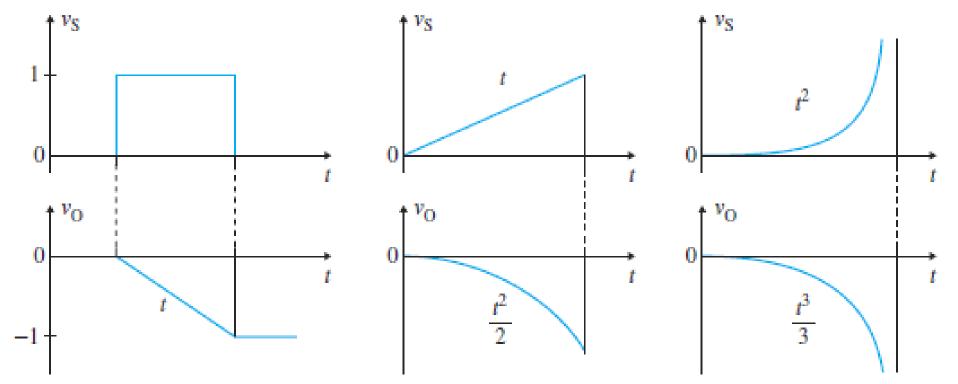


(c) Magnitude plot

# Frequency Response of an Ideal Integrator

• The voltage gain:

$$A_f(j\omega) = \frac{V_o(j\omega)}{V_s(j\omega)} = -\frac{1}{j\omega C_F R_1}$$



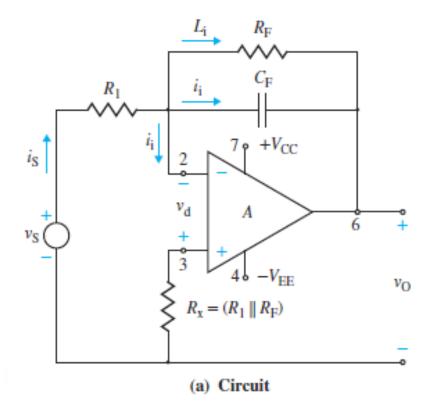
# Closed-Loop Gain of a Practical Integrator

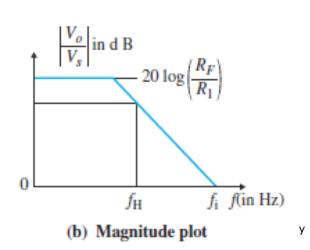
Voltage gain:

$$A_f(j\omega) = \frac{V_o(j\omega)}{V_s(j\omega)} = -\frac{R_F/R_1}{1 + j\omega C_F R_F}$$

• The cutoff frequency:

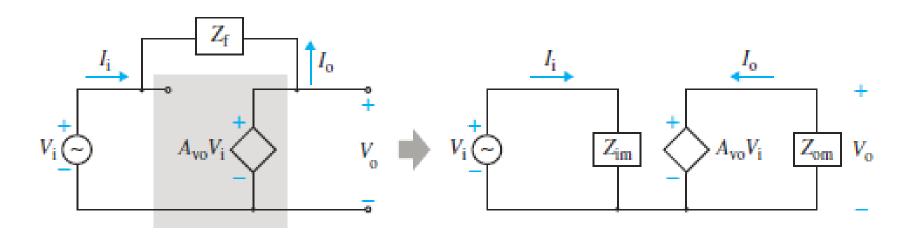
$$f_H = -\frac{1}{2\pi R_E C_E}$$





### Miller's Theorem and Capacitance

- Miller's theorem simplifies the analysis of feedback amplifiers.
- If an impedance is connected between the input side and the output side of a voltage amplifier, this impedance can be replaced by two equivalent impedances—one connected across the input and the other connected across the output terminals.



(a) Feedback amplifier

(b) Miller equivalent



# Miller's Theorem and Capacitance (cont'd)

Output impedance Z<sub>om :</sub>

$$Z_{om} = \frac{V_o}{I_o} = \frac{Z_f}{1 - 1/A_{vo}}$$

Equivalent Miller's input and output capacitances:

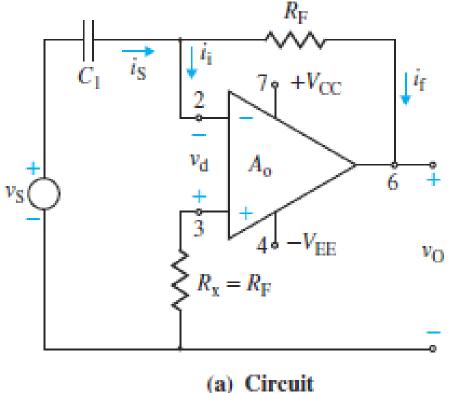
$$C_{im} = (1 - A_{vo})C_F$$
  
 $C_{om} = (1 - 1/A_{vo})C_F$ 

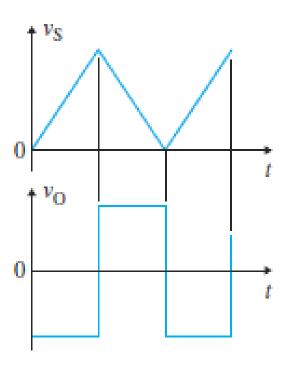
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#### **Differentiators**

- If the resistance  $R_1$  in the inverting amplifier is replaced by a capacitance  $C_1$
- Closed-Loop Gain of an Ideal Differentiator:

$$v_O = -R_F C_1 \frac{dv_S}{dt}$$

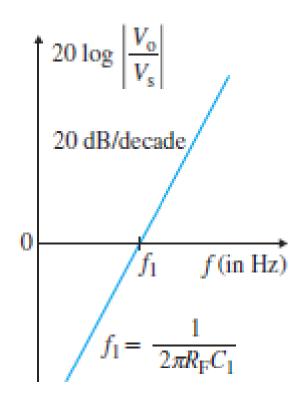




Waveforms

#### **Differentiators**

- If the resistance  $R_1$  in the inverting amplifier is replaced by a capacitance  $C_1$
- Closed-Loop Gain of an Ideal Differentiator:

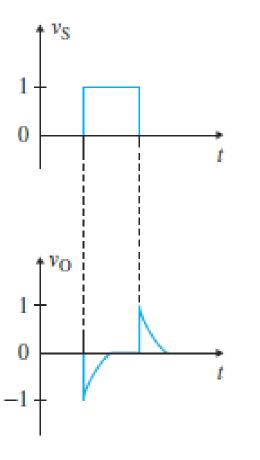


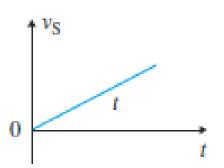
$$v_O = -R_F C_1 \frac{dv_S}{dt}$$

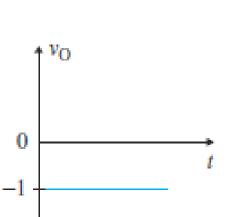


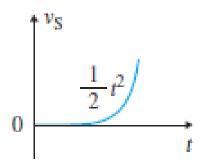
# Frequency Response of an Ideal Differentiator $A_f(j\omega) = -j\omega C_1 R_F$

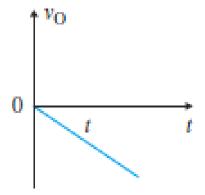
Voltage gain:







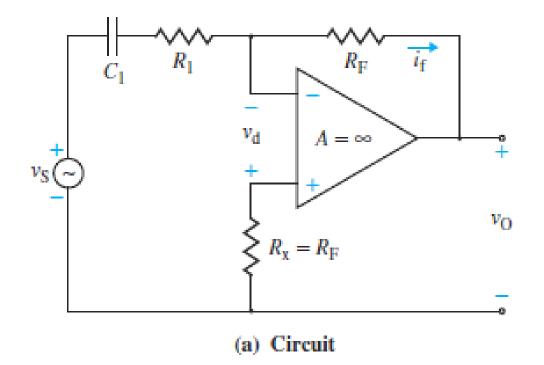


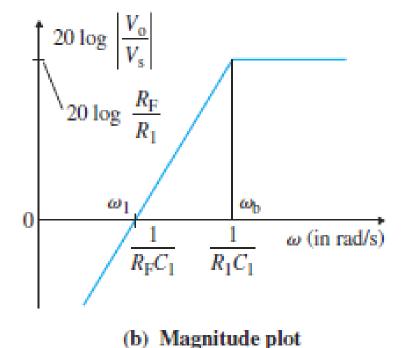




## Closed-Loop Gain of a Practical Differentiator $R_F C_1 j \omega$

$$A_f(j\omega) = -\frac{R_F C_1 j\omega}{1 + j\omega R_1 C_1}$$



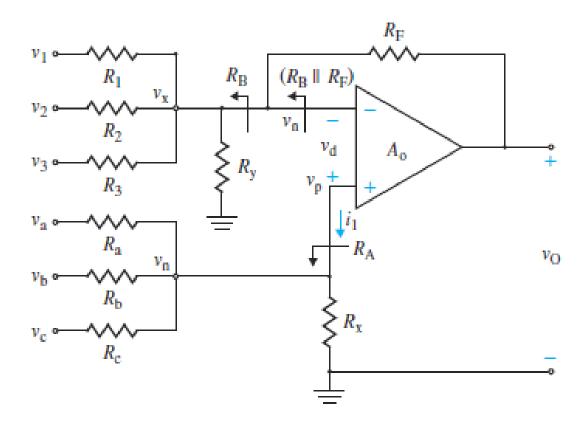


### Addition-Subtraction Amplifiers

$$v_o = (1 + \frac{R_F}{R_B})(\frac{R_A}{R_a}v_a + \frac{R_A}{R_b}v_b + \frac{R_A}{R_c}v_c) - (\frac{R_F}{R_1}v_1 + \frac{R_F}{R_2}v_2 + \frac{R_F}{R_3}v_3)$$

$$R_A = (R_a \parallel R_b \parallel R_c \parallel R_x)$$

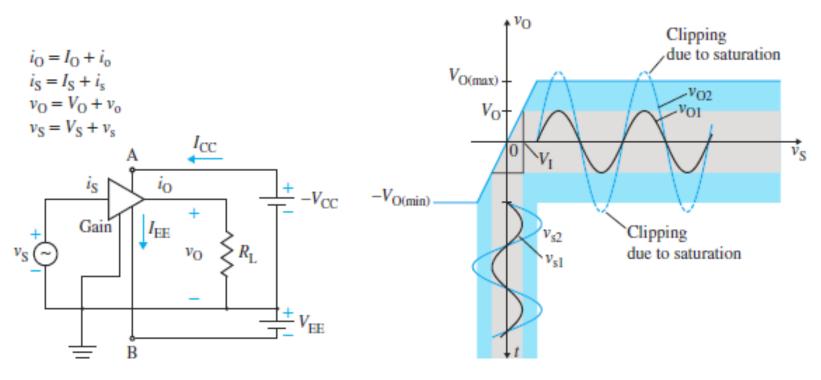
$$R_B = (R_1 || R_2 || R_3 || R_y)$$





#### Large-Signal Operations -Output Voltage Saturation

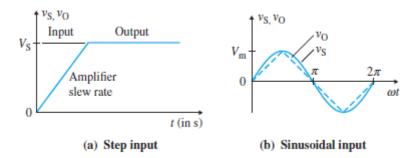
- The output voltage of the amplifier cannot exceed the positive saturation limit  $V_{O(max)}$  and cannot decrease below the negative saturation limit  $-V_{O(min)}$ .
- As long as the amplifier operates with the saturation limits, the voltage gain can normally be assumed to be linear.





#### Some Definitions on Op-Amps

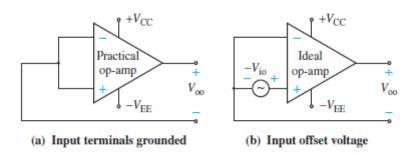
- **Rise Time:** the time required for the output voltage to rise from 10% to 90% of the steady-state value.
- **Slew rate (SR):** the maximum rate of rise of the output voltage per unit time, and it is measured in volts per microsecond.





### **Input Offset Voltage**

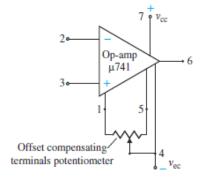
- If the input terminals of an op-amp are tied together and connected to the ground, a certain DC voltage exits at the output. This voltage is called the output offset voltage  $V_{oo}$ .
- Any differential signal between the input terminals is amplified and gives the output voltage  $V_{oo}$ .



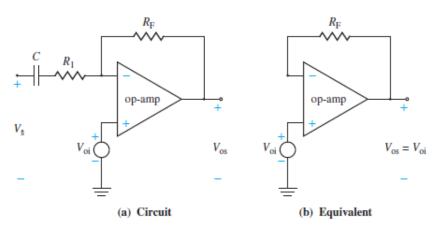


# Minimizing Output Offset Voltage

- Two ways to minimize the output offset voltage:
- (a) Compensating Network:



• (b) Capacitor Coupling:





### **Input Offset Current**

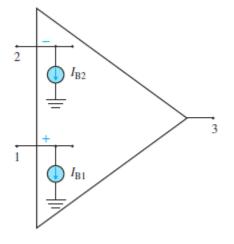
 The DC input base currents will be equal only if the internal transistors have equal current gains (betas).

 The average value I<sub>B</sub> is called the input bias current and is given by:

 $I_{B} = \frac{I_{B1} + I_{B2}}{2}$ 

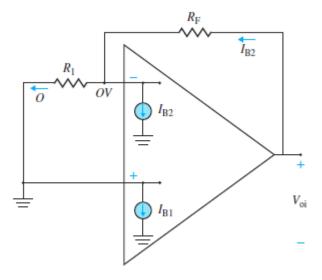
Input offset current

$$I_{i0} = I_{R1} - I_{R2}$$



### **Effect of Input Offset Current**

$$V_{+} = -R_{x}I_{B1}$$





#### **Op-Amp Circuit Design**

#### The design sequence:

- **Step 1.** Study the problem.
- Step 2. Create a block diagram of the solution.
- Step 3. Find a hand-analysis circuit-level solution.
- Step 4. Use PSpice/Multisim/SPICE for verification.
- Step 5. Construct the circuit in the lab and take measurements.



### **Chapter 2 Summary**

- An op-amp is a high-gain differential amplifier that can perform various functions in electronic circuits.
- The analysis of an op-amp circuit can be simplified by assuming ideal characteristics. An ideal op-amp has a very high voltage gain, a very high input resistance, a very low output resistance, and a negligible input current.
- The characteristics of practical op-amps differ from the ideal characteristics, but analyses based on the ideal conditions are valid for many applications and provide the starting point for practical circuit design.
- Although the DC model of op-amps can be used to analyze complex op-amp circuits, it does not take into account the frequency dependence and op-amp nonlinearities.

