Abstract

To be written.

This MMSC thesis will further explore general kernel spectral methods for finding equilibrium measures where initial progress made in Gutleb, Carrillo and Olver 2020 and Gutleb, Carrillo and Olver 2021.

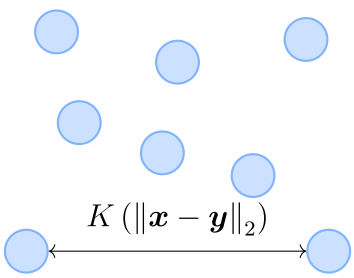
Keywords: Equilibrium Measures

Languages: C++, Julia, Python

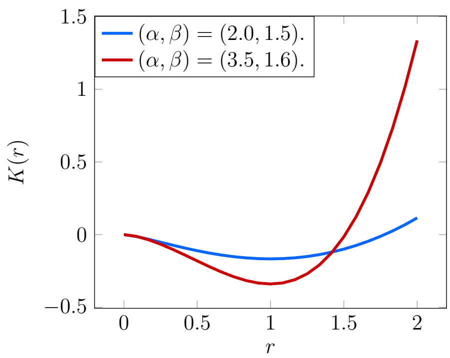
Contents

Chapter 1

Introduction



(a) $N = 8$ particles interacting with one another through the potential $K(r)$.



(b) Plot of attractive-repulsive potential functions $K(r) = \frac{r^\alpha}{\alpha} - \frac{r^\beta}{\beta}$ for different α, β.

Cf. Figure 1.1a and Figure 1.1b.

Just Notes

This chapter's purpose is for the collection of notes, and it will not be included in the final dissertation.

Special Functions we like

Pochhammer's falling symbol $(x)_n := \prod_{k=0}^{n-1}(x-k)\,.$

Pochhammer's rising symbol $(x)^n := \prod_{k=0}^{n-1}(x+k)\,.$

Hypergeometric series

$$ {}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z) := \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n}{(b_1)_n \cdots (b_q)_n} \frac{z^n}{n!}.. $$

Hypergeometric function

$$ {}_2F_1(a,-n;c;z) = \sum_{j=0}^{n} (-1)^j \binom{n}{j} \frac{(a)_j}{(c)_j} z^j\,. $$

(A special case of the hypergeometric series with $p=2$, $q=1$).

Jacobi (=hypergeometric) polynomials

$$ P_n^{(\alpha,\beta)}(z) := \frac{(\alpha+1)_n}{n!}\,{}_2F_1\left(-n, 1+\alpha+\beta+n; \alpha+1; \tfrac{1}{2}(1-z)\right)\,. $$

Gegenbauer (=ultraspherical) polynomials

$$ C_n^{(\lambda)}(z) := \frac{(2\lambda)_n}{n!}\,{}_2F_1\left(-n, 2\lambda+n; \lambda+\frac{1}{2}; \frac{1-z}{2}\right) = \frac{(2\lambda)_n}{(\lambda+\frac{1}{2})_n} P_n^{(\lambda-1/2,\lambda-1/2)}(x)\,. $$

They satisfy a three-term recurrence relation (as all orthogonal polynomials do!)

$$\begin{aligned} C_0^{(\lambda)}(x) &= 1 \\ C_1^{(\lambda)}(x) &= 2\lambda x \\ (n+1)C_{n+1}^{(\lambda)}(x) &= 2(n+\lambda)xC_n^{(\lambda)}(x) - (n+2\lambda-1)C_{n-1}^{(\lambda)}(x). \end{aligned} \quad .$$

Chapter 2

Conclusion

In the present thesis, we explored the interesting realm of particle-particle interactions. Next to the written part, the reader will find an implementation of the particle simulator, including a Graphical User Interface (GUI), as well as the numerical solver.

Acronyms, Definitions and Theorems

GUI	Graphical User Interface	6

Definitions

Theorems

Lemmata

Remarks

Bibliography

Gutleb, Timon S., José A. Carrillo and Sheehan Olver (Oct. 2020). 'Computing Equilibrium Measures with Power Law Kernels'. In: *arXiv*. DOI: 10.1090/mcom/3740. eprint: 2011.00045.

– (Sept. 2021). 'Computation of Power Law Equilibrium Measures on Balls of Arbitrary Dimension'. In: *arXiv*. DOI: 10.1007/s00365-022-09606-0. eprint: 2109.00843.

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Appendix A – Supplemental Proofs