3. $\Gamma(n) = (n-1)! = \int_0^\infty t^{n-1} e^{-t}\,dt$ dla $n \in N, n > 0$

Indukcja:

P: $n = 1$

$$\Gamma(1) = \int_0^\infty t^0 e^{-t}\,dt = \int_0^\infty e^{-t}\,dt = \Big|_0^\infty -e^{-t} = \underbrace{\lim_{t\to\infty} -e^{-t}}_{0} + \underbrace{e^0}_{1} = 1 = 0!$$

K: $n \Rightarrow n+1$

$$\Gamma(n+1) = \int_0^\infty t^n e^{-t}\,dt = \Big|_0^\infty -e^{-t}t^n + \int_0^\infty n t^{n-1} e^{-t}\,dt = \underbrace{\lim_{t\to\infty} \frac{-t^n}{e^t}}_{\substack{\text{n-krotna} \\ \text{d'l HpH} \\ \frac{-1}{e^t} \to 0}} + n\underbrace{\int_0^\infty t^{n-1} e^{-t}\,dt}_{(n-1)! \text{ z zał. I.}}$$

D	I
$+t^n$	e^{-t}
$-nt^{n-1}$	$-e^{-t}$

$$= 0 + n \cdot (n-1)! = n! = (n+1-1)!$$