9.

$$B(p,q) = \int_0^1 t^{p-1}(1-t)^{q-1}\,dt$$

$$\Gamma(p) = \int_0^\infty t^{p-1} e^{-t}\,dt$$

$$\Gamma(p)\Gamma(q) = \int_0^\infty t^{p-1} e^{-t}\,dt \int_0^\infty u^{q-1} e^{-u}\,du =$$

$$= \int_0^\infty \int_0^\infty t^{p-1} e^{-t} u^{q-1} e^{-u}\,du\,dt = \int_0^\infty \int_0^\infty t^{p-1} u^{q-1} e^{-(t+u)}\,du\,dt$$

Stosujemy podstawienie $u = xy$, $t = x(1-y)$, bo chcemy $t+u = x$

$t \in [0,\infty)$, $u \in [0,\infty)$, więc $x \in [0,\infty)$

$xy \in [0,\infty)$, $x(1-y) \in [0,\infty)$, więc $y \in [0,1)$

Liczymy Jacobian

$$|J| = \begin{vmatrix} \frac{\partial t}{\partial x} & \frac{\partial t}{\partial y} \\ \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \end{vmatrix} = \begin{vmatrix} 1-y & -x \\ y & x \end{vmatrix} = (1-y)x + xy = x$$

$$\int_0^\infty \int_0^1 x^{p-1}(1-y)^{p-1} x^{q-1} y^{q-1} e^{-x} x\,dy\,dx = \int_0^\infty \int_0^1 x^{p+q-1} e^{-x} y^{q-1}(1-y)^{p-1}\,dy\,dx =$$

$$= \int_0^\infty x^{p+q-1} e^{-x}\,dx \int_0^1 y^{q-1}(1-y)^{p-1}\,dy = \Gamma(p+q)\,B(q,p) = \Gamma(p+q)\,B(p,q)$$