

# NCTU\_TaNoShiI

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6.2 Fraction Limit	17	1 const int MEM = 16000004;	
<b>7 Other</b>	<b>17</b>	2 struct Treap {	
7.1 Annealing	17	3 static Treap nil, mem[MEM], *pmem;	
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7.3 MoOnTree	18	5 char val;	
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7.5 Det	19	7 Treap () : l(&nil), r(&nil), size(0) {}	
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8.1 AC	19	9 l(&nil), r(&nil), val(_val), size(1) {}	
8.2 SuffixAutomata	20	10 } Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap	
8.3 Palindromic Tree	21	11 ::	
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8.5 ZvaluePalindromes	21	13 int size(const Treap *t) { return t->size; }	
		14 void pull(Treap *t) {	
		15 if (!size(t)) return;	
		16 t->size = size(t->l) + size(t->r) + 1;	
		17 }	
		18 Treap* merge(Treap *a, Treap *b) {	
		19 if (!size(a)) return b;	
		20 if (!size(b)) return a;	
		21 Treap *t;	

```

21     if (rand() % (size(a) + size(b)) < size(a)) {
22         t = new (Treap::pmem++) Treap(*a);
23         t->r = merge(a->r, b);
24     } else {
25         t = new (Treap::pmem++) Treap(*b);
26         t->l = merge(a, b->l);
27     }
28     pull(t);
29     return t;
30 }
31 void split(Treap *t, int k, Treap *&a, Treap *&b) {
32     if (!size(t)) a = b = &Treap::nil;
33     else if (size(t->l) + 1 <= k) {
34         a = new (Treap::pmem++) Treap(*t);
35         split(t->r, k - size(t->l) - 1, a->r, b);
36         pull(a);
37     } else {
38         b = new (Treap::pmem++) Treap(*t);
39         split(t->l, k, a, b->l);
40         pull(b);
41     }
42 }
43 int nv;
44 Treap *rt[50005];
45 void print(const Treap *t) {
46     if (!size(t)) return;
47     print(t->l);
48     cout << t->val;
49     print(t->r);
50 }
51 int main(int argc, char** argv) {
52     IOS;
53     rt[nv=0] = &Treap::nil;
54     Treap::pmem = Treap::mem;
55     int Q, cmd, p, c, v;
56     string s;
57     cin >> Q;
58     while (Q--) {
59         cin >> cmd;
60         if (cmd == 1) {
61             // insert string s after position p
62             cin >> p >> s;
63             Treap *tl, *tr;
64             split(rt[nv], p, tl, tr);
65             for (int i=0; i<s.size(); i++)
66                 tl = merge(tl, new (Treap::pmem++) Treap
67                     (s[i]));
68             rt[++nv] = merge(tl, tr);
69         } else if (cmd == 2) {
70             // remove c characters starting at
71             position
72             Treap *tl, *tm, *tr;
73             cin >> p >> c;
74             split(rt[nv], p-1, tl, tm);
75             split(tm, c, tm, tr);
76             rt[++nv] = merge(tl, tr);
77         } else if (cmd == 3) {
78             // print c characters starting at
79             position p, in version v
80             Treap *tl, *tm, *tr;
81             cin >> v >> p >> c;
82             split(rt[v], p-1, tl, tm);
83             split(tm, c, tm, tr);
84             print(tm);
85             cout << "n";
86         }
87     }
88     return 0;

```

## 2.2 Pbds Kth

```

1 #include <bits/extc++.h>
2 using namespace __gnu_pbds;
3 typedef tree<int, null_type, less<int>, rb_tree_tag,
4     tree_order_statistics_node_update> set_t;
5 int main()
6 {
7     // Insert some entries into s.
8     set_t s;
9     s.insert(12); s.insert(505);
10    // The order of the keys should be: 12, 505.

```

```

11    assert(*s.find_by_order(0) == 12);
12    assert(*s.find_by_order(3) == 505);
13    // The order of the keys should be: 12, 505.
14    assert(s.order_of_key(12) == 0);
15    assert(s.order_of_key(505) == 1);
16    // Erase an entry.
17    s.erase(12);
18    // The order of the keys should be: 505.
19    assert(*s.find_by_order(0) == 505);
20    // The order of the keys should be: 505.
21    assert(s.order_of_key(505) == 0);
22 }

```

## 2.3 PbdsHeap

```

1 #include <bits/extc++.h>
2 typedef __gnu_pbds::priority_queue<int> heap_t;
3 heap_t a, b;
4 int main() {
5     a.clear(); b.clear();
6     a.push(1); a.push(3);
7     b.push(2); b.push(4);
8     assert(a.top() == 3);
9     assert(b.top() == 4);
10    // merge two heap
11    a.join(b);
12    assert(a.top() == 4);
13    assert(b.empty());
14    return 0;
15 }

```

## 2.4 KDtree

```

1 struct KDTree {
2     struct Node {
3         int x, y, x1, y1, x2, y2;
4         int id, f;
5         Node *L, *R;
6     } tree[MXN];
7     int n;
8     Node *root;
9     long long dis2(int x1, int y1, int x2, int y2) {
10         long long dx = x1-x2;
11         long long dy = y1-y2;
12         return dx*dx+dy*dy;
13     }
14     static bool cmpx(Node& a, Node& b) { return a.x < b.x; }
15     static bool cmpy(Node& a, Node& b) { return a.y < b.y; }
16     void init(vector<pair<int, int>> ip) {
17         n = ip.size();
18         for (int i=0; i<n; i++) {
19             tree[i].id = i;
20             tree[i].x = ip[i].first;
21             tree[i].y = ip[i].second;
22         }
23         root = build_tree(0, n-1, 0);
24     }
25     Node* build_tree(int L, int R, int dep) {
26         if (L>R) return nullptr;
27         int M = (L+R)/2;
28         tree[M].f = dep%2;
29         nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
30             cmpy : cmpx);
31         tree[M].x1 = tree[M].x2 = tree[M].x;
32         tree[M].y1 = tree[M].y2 = tree[M].y;
33         tree[M].L = build_tree(L, M-1, dep+1);
34         if (tree[M].L) {
35             tree[M].x1 = min(tree[M].x1, tree[M].L->
36                 x1);
37             tree[M].x2 = max(tree[M].x2, tree[M].L->
38                 x2);
39             tree[M].y1 = min(tree[M].y1, tree[M].L->
40                 y1);
41             tree[M].y2 = max(tree[M].y2, tree[M].L->
42                 y2);
43         }
44         tree[M].R = build_tree(M+1, R, dep+1);
45         if (tree[M].R) {
46             tree[M].x1 = min(tree[M].x1, tree[M].R->
47                 x1);

```

```

43     tree[M].x2 = max(tree[M].x2, tree[M].R->
44     x2);
45     tree[M].y1 = min(tree[M].y1, tree[M].R->
46     y1);
47     tree[M].y2 = max(tree[M].y2, tree[M].R->
48     y2);
49     }
50     return tree+M;
51 }
52 int touch(Node* r, int x, int y, long long d2){
53     long long dis = sqrt(d2)+1;
54     if (x<r->x1-dis || x>r->x2+dis || y<r->y1-
55     dis || y>
56     r->y2+dis)
57     return 0;
58     return 1;
59 }
60 void nearest(Node* r, int x, int y, int &mID,
61 long
62 long &md2) {
63     if (!r || !touch(r, x, y, md2)) return;
64     long long d2 = dis2(r->x, r->y, x, y);
65     if (d2 < md2 || (d2 == md2 && mID < r->id))
66     {
67         mID = r->id;
68         md2 = d2;
69     }
70     // search order depends on split dim
71     if ((r->f == 0 && x < r->x) ||
72     (r->f == 1 && y < r->y)) {
73         nearest(r->L, x, y, mID, md2);
74         nearest(r->R, x, y, mID, md2);
75     } else {
76         nearest(r->R, x, y, mID, md2);
77         nearest(r->L, x, y, mID, md2);
78     }
79 }
80 int query(int x, int y) {
81     int id = 1029384756;
82     long long d2 = 102938475612345678LL;
83     nearest(root, x, y, id, d2);
84     return id;
85 }
86 }tree;

```

## 2.5 LCT

```

1  const int MXN = 100005;
2  const int MEM = 100005;
3
4  struct Splay {
5      static Splay nil, mem[MEM], *pmem;
6      Splay *ch[2], *f;
7      int val, rev, size;
8      Splay () : val(-1), rev(0), size(0) {
9          f = ch[0] = ch[1] = &nil;
10     }
11     Splay (int _val) : val(_val), rev(0), size(1) {
12         f = ch[0] = ch[1] = &nil;
13     }
14     bool isr() {
15         return f->ch[0] != this && f->ch[1] != this;
16     }
17     int dir() {
18         return f->ch[0] == this ? 0 : 1;
19     }
20     void setCh(Splay *c, int d) {
21         ch[d] = c;
22         if (c != &nil) c->f = this;
23         pull();
24     }
25     void push() {
26         if (rev) {
27             swap(ch[0], ch[1]);
28             if (ch[0] != &nil) ch[0]->rev ^= 1;
29             if (ch[1] != &nil) ch[1]->rev ^= 1;
30             rev=0;
31         }
32     }
33     void pull() {
34         size = ch[0]->size + ch[1]->size + 1;
35         if (ch[0] != &nil) ch[0]->f = this;
36         if (ch[1] != &nil) ch[1]->f = this;
37     }
38 } Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay
39 :::mem;
40 Splay *nil = &Splay::nil;
41 void rotate(Splay *x) {
42     Splay *p = x->f;
43     int d = x->dir();
44     if (!p->isr()) p->f->setCh(x, p->dir());
45     else x->f = p->f;
46     p->setCh(x->ch[!d], d);
47     x->setCh(p, !d);
48     p->pull(); x->pull();
49 }
50
51 vector<Splay*> splayVec;
52 void splay(Splay *x) {
53     splayVec.clear();
54     for (Splay *q=x;; q=q->f) {
55         splayVec.push_back(q);
56         if (q->isr()) break;
57     }
58     reverse(begin(splayVec), end(splayVec));
59     for (auto it : splayVec) it->push();
60     while (!x->isr()) {
61         if (x->f->isr()) rotate(x);
62         else if (x->dir()==x->f->dir()) rotate(x->f),
63         rotate(x);
64         else rotate(x), rotate(x);
65     }
66 }
67 Splay* access(Splay *x) {
68     Splay *q = nil;
69     for (;x!=nil;x=x->f) {
70         splay(x);
71         x->setCh(q, 1);
72         q = x;
73     }
74     return q;
75 }
76 void evert(Splay *x) {
77     access(x);
78     splay(x);
79     x->rev ^= 1;
80     x->push(); x->pull();
81 }
82 void link(Splay *x, Splay *y) {
83     // evert(x);
84     access(x);
85     splay(x);
86     evert(y);
87     x->setCh(y, 1);
88 }
89 void cut(Splay *x, Splay *y) {
90     // evert(x);
91     access(y);
92     splay(y);
93     y->push();
94     y->ch[0] = y->ch[0]->f = nil;
95 }
96
97 int N, Q;
98 Splay *vt[MXN];
99
100 int ask(Splay *x, Splay *y) {
101     access(x);
102     access(y);
103     splay(x);
104     int res = x->f->val;
105     if (res == -1) res=x->val;
106     return res;
107 }
108 int main(int argc, char** argv) {
109     scanf("%d%d", &N, &Q);
110     for (int i=1; i<=N; i++)
111         vt[i] = new (Splay::pmem++) Splay(i);
112     while (Q--) {
113         char cmd[105];
114         int u, v;
115         scanf("%s", cmd);
116         if (cmd[1] == 'i') {

```

```

117     scanf("%d%d", &u, &v);
118     link(vt[v], vt[u]);
119 } else if (cmd[0] == 'c') {
120     scanf("%d", &v);
121     cut(vt[1], vt[v]);
122 } else {
123     scanf("%d%d", &u, &v);
124     int res=ask(vt[u], vt[v]);
125     printf("%d\n", res);
126 }
127 }
128
129 return 0;
130 }

```

### 3 Flow

#### 3.1 Minimum Weight Matching

```

1 struct Graph {
2     // Minimum General Weighted Matching (Perfect
3     Match) clique
4     static const int MXN = 105;
5     int n, edge[MXN][MXN];
6     int match[MXN], dis[MXN], onstk[MXN];
7     vector<int> stk;
8     void init(int _n) {
9         n = _n;
10        MEM(edge);
11    }
12    void add_edge(int u, int v, int w) {
13        edge[u][v] = edge[v][u] = w;
14    }
15    bool SPFA(int u) {
16        if (onstk[u]) return true;
17        stk.pb(u);
18        onstk[u] = 1;
19        for (int v=0; v<n; v++){
20            if (u != v && match[u] != v && !onstk[v])
21                int m = match[v];
22                if (dis[m] > dis[u] - edge[v][m] +
23                    edge[u][v]){
24                    dis[m] = dis[u] - edge[v][m] +
25                    edge[u][v];
26                    onstk[v] = 1;
27                    stk.pb(v);
28                    if (SPFA(m)) return true;
29                    stk.pop_back();
30                    onstk[v] = 0;
31                }
32                onstk[u] = 0;
33                stk.pop_back();
34                return false;
35    }
36    int solve() {
37        // find a match
38        for (int i=0; i<n; i+=2){
39            match[i] = i+1;
40            match[i+1] = i;
41        }
42        while (true){
43            int found = 0;
44            MEM(dis); MEM(onstk);
45            for (int i=0; i<n; i++){
46                stk.clear();
47                if (!onstk[i] && SPFA(i)){
48                    found = 1;
49                    while (stk.size()>=2){
50                        int u = stk.back(); stk.
51                        pop_back();
52                        int v = stk.back(); stk.
53                        pop_back();
54                        match[u] = v;
55                        match[v] = u;
56                    }
57                    if (!found) break;
58                }
59            }
60            int ret = 0;

```

```

59     for (int i=0; i<n; i++)
60         ret += edge[i][match[i]];
61     ret /= 2;
62     return ret;
63 }
64 }graph;

```

#### 3.2 CostFlow

```

1 struct CostFlow {
2     static const int MXN = 205;
3     static const long long INF = 102938475610293847
4     LL;
5     struct Edge {
6         int v, r;
7         long long f, c;
8         Edge(int a, int b, int _c, int d):v(a),r(b),f(
9         _c),c(d){
10    }
11    };
12    int n, s, t, prv[MXN], prvl[MXN], inq[MXN];
13    long long dis[MXN], fl, cost;
14    vector<Edge> E[MXN];
15    void init(int _n, int _s, int _t) {
16        n = _n; s = _s; t = _t;
17        for (int i=0; i<n; i++) E[i].clear();
18        fl = cost = 0;
19    }
20    void add_edge(int u, int v, long long f, long
21    long c)
22    {
23        E[u].pb(Edge(v, E[v].size(), f, c));
24        E[v].pb(Edge(u, E[u].size()-1, 0, -c));
25    }
26    pll flow() {
27        while (true) {
28            for (int i=0; i<n; i++) {
29                dis[i] = INF;
30                inq[i] = 0;
31            }
32            dis[s] = 0;
33            queue<int> que;
34            que.push(s);
35            while (!que.empty()) {
36                int u = que.front(); que.pop();
37                inq[u] = 0;
38                for (int i=0; i<E[u].size(); i++) {
39                    int v = E[u][i].v;
40                    long long w = E[u][i].c;
41                    if (E[u][i].f > 0 && dis[v] >
42                        dis[u] + w) {
43                        prv[v] = u; prvl[v] = i;
44                        dis[v] = dis[u] + w;
45                        if (!inq[v]) {
46                            inq[v] = 1;
47                            que.push(v);
48                        }
49                    }
50                }
51            }
52            if (dis[t] == INF) break;
53            long long tf = INF;
54            for (int v=t, u, l; v!=s; v=u) {
55                u=prv[v]; l=prvl[v];
56                tf = min(tf, E[u][l].f);
57            }
58            for (int v=t, u, l; v!=s; v=u) {
59                u=prv[v]; l=prvl[v];
60                E[u][l].f -= tf;
61                E[v][E[u][l].r].f += tf;
62            }
63            cost += tf * dis[t];
64            fl += tf;
65        }
66        return {fl, cost};
67    }
68 }flow;

```

#### 3.3 Mincut Tree

```

1 set<int> temp;
2 int Vis[3005];
3 int cvis[3005];

```

```

4 void dfs(int n){
5     Vis[n]=1;
6     for(auto it=v[n].begin();it!=v[n].end();it++){
7         if(val[n][*it]>flow[n][*it]&&!Vis[*it]){
8             dfs(*it);
9             if(cvis[*it])
10                temp.insert(*it);
11        }
12    }
13 }
14 int n;
15 int dc(set<int> s,int flag){
16     if(s.size()==1)
17         return *s.begin();
18     for(int i=0;i<n;i++){
19         for(auto it=v[i].begin();it!=v[i].end();it++){
20             flow[i][*it]=0;
21         }
22         for(auto it=s.begin();it!=s.end();it++){
23             cvis[*it]=1;
24         }
25         int res=Flow(*s.begin(),*s.rbegin());
26         MEM(Vis);
27         dfs(*s.begin());
28         temp.insert(*s.begin());
29         for(auto it=s.begin();it!=s.end();it++){
30             cvis[*it]=0;
31         }
32         set<int> s1,s2;
33         swap(s1,temp);
34         temp.clear();
35         for(auto it=s1.begin();it!=s1.end();it++){
36             s.erase(*it);
37             swap(s2,s);
38             int x=dc(s1,0);
39             int y=dc(s2,1);
40             vt[x].pb(mp(y,res));
41             vt[y].pb(mp(x,res));
42             if(flag==0)
43                 return x;
44             else
45                 return y;
46         }
47     }
48 }

```

### 3.4 Dinic

```

1 struct Dinic{
2     static const int MXN = 10000;
3     struct Edge{ int v,f,re; Edge(int a,int b,int c)
4         :v(a),f(b),re(c){}};
5     int n,s,t,level[MXN];
6     vector<Edge> E[MXN];
7     void init(int _n, int _s, int _t){
8         n = _n; s = _s; t = _t;
9         for (int i=0; i<=n; i++) E[i].clear();
10    }
11    void add_edge(int u, int v, int f){
12        E[u].pb(Edge(v,f,E[v].size()));
13        E[v].pb(Edge(u,0,E[u].size()-1)); //direct
14    }
15    bool BFS(){
16        MEM(level);
17        queue<int> que;
18        que.push(s);
19        level[s] = 0;
20        while (!que.empty()){
21            int u = que.front(); que.pop();
22            for (auto it : E[u]){
23                if (it.f > 0 && level[it.v] == -1){
24                    level[it.v] = level[u]+1;
25                    que.push(it.v);
26                }
27            }
28        }
29        return level[t] != -1;
30    }
31    int DFS(int u, int nf){
32        if (u == t) return nf;
33        int res = 0;
34        for (auto &it : E[u]){
35            if (it.f > 0 && level[it.v] == level[u]
36                ]+1){
37                int tf = DFS(it.v, min(nf,it.f));
38                res += tf; nf -= tf; it.f -= tf;
39            }
40        }
41    }
42 }

```

```

37         E[it.v][it.re].f += tf;
38         if (nf == 0) return res;
39     }
40 }
41 if (!res) level[u] = -1;
42 return res;
43 }
44 int flow(int res=0){
45     while ( BFS() )
46         res += DFS(s,2147483647);
47     return res;
48 }
49 }flow;

```

### 3.5 GeneralGraphmatch

```

1 struct GenMatch { // 1-base
2     static const int MAXN = 505;
3     int V;
4     bool el[MAXN][MAXN];
5     int pr[MAXN];
6     bool inq[MAXN],inp[MAXN],inb[MAXN];
7     queue<int> qe;
8     int st,ed;
9     int nb;
10    int bk[MAXN],djs[MAXN];
11    int ans;
12    void init(int _V) {
13        V = _V;
14        MEM(el); MEM(pr);
15        MEM(inq); MEM(inp); MEM(inb);
16        MEM(bk); MEM(djs);
17        ans = 0;
18    }
19    void add_edge(int u, int v) {
20        el[u][v] = el[v][u] = 1;
21    }
22    int lca(int u,int v) {
23        memset(inp,0,sizeof(inp));
24        while(1) {
25            u = djs[u];
26            inp[u] = true;
27            if(u == st) break;
28            u = bk[pr[u]];
29        }
30        while(1) {
31            v = djs[v];
32            if(inp[v]) return v;
33            v = bk[pr[v]];
34        }
35        return v;
36    }
37    void upd(int u) {
38        int v;
39        while(djs[u] != nb) {
40            v = pr[u];
41            inb[djs[u]] = inb[djs[v]] = true;
42            u = bk[v];
43            if(djs[u] != nb) bk[u] = v;
44        }
45    }
46    void blo(int u,int v) {
47        nb = lca(u,v);
48        memset(inb,0,sizeof(inb));
49        upd(u); upd(v);
50        if(djs[u] != nb) bk[u] = v;
51        if(djs[v] != nb) bk[v] = u;
52        for(int tu = 1; tu <= V; tu++){
53            if(inb[djs[tu]]) {
54                djs[tu] = nb;
55                if(!inq[tu]){
56                    qe.push(tu);
57                    inq[tu] = 1;
58                }
59            }
60        }
61    }
62    void flow() {
63        memset(inq,false,sizeof(inq));
64        memset(bk,0,sizeof(bk));
65        for(int i = 1; i <= V;i++){
66            djs[i] = i;
67            while(qe.size()) qe.pop();
68            qe.push(st);
69        }
70    }
71 }

```



```

68     inq[st] = 1;
69     ed = 0;
70     while(qe.size()) {
71         int u = qe.front(); qe.pop();
72         for(int v = 1; v <= V; v++)
73             if(el[u][v] && (djs[u] != djs[v]) && (pr
74 [u] !=
75 v)) {
76             if((v == st) || ((pr[v] > 0) && bk[
77 pr[v]] >
78 0))
79                 blo(u,v);
80             else if(bk[v] == 0) {
81                 bk[v] = u;
82                 if(pr[v] > 0) {
83                     if(!inq[pr[v]]) qe.push(pr[v
84 ]));
85                 } else {
86                     ed = v;
87                     return;
88                 }
89             }
90         }
91     }
92 void aug() {
93     int u,v,w;
94     u = ed;
95     while(u > 0) {
96         v = bk[u];
97         w = pr[v];
98         pr[v] = u;
99         pr[u] = v;
100         u = w;
101     }
102 int solve() {
103     memset(pr,0,sizeof(pr));
104     for(int u = 1; u <= V; u++)
105         if(pr[u] == 0) {
106             st = u;
107             flow();
108             if(ed > 0) {
109                 aug();
110                 ans ++;
111             }
112         }
113     return ans;
114 }
115 }gp;

```

### 3.6 KM

```

1 struct KM{
2     // Maximum Bipartite Weighted Matching (Perfect
3     Match)
4     static const int MXN = 650;
5     const int INF = 2147483647; //LL
6     int px[MXN],py[MXN],match[MXN],par[MXN],n;
7     int g[MXN][MXN],lx[MXN],ly[MXN],slack_y[MXN]
8 ];
9     // ^^^^ long long
10 void init(int _n){
11     n = _n;
12     for (int i=0; i<n; i++)
13         for (int j=0; j<n; j++)
14             g[i][j] = 0;
15 }
16 void add_edge(int x, int y, int w){ // LL
17     g[x][y] = w;
18 }
19 void adjust(int y){
20     match[y]=py[y];
21     if(px[match[y]]!=-2)
22         adjust(px[match[y]]);
23 }
24 bool dfs(int x){
25     for(int y=0;y<n;++y){
26         if(py[y]!=-1)continue;
27         int t=lx[x]+ly[y]-g[x][y]; //LL
28         if(t==0){
29             py[y]=x;
30             if(match[y]==-1){

```

```

29         adjust(y);
30         return 1;
31     }
32     if(px[match[y]]!=-1)continue;
33     px[match[y]]=y;
34     if(dfs(match[y]))return 1;
35 }else if(slack_y[y]>t){
36     slack_y[y]=t;
37     par[y]=x;
38 }
39 }
40 return 0;
41 }
42 int solve(){//LL
43     fill(match,match+n,-1);
44     fill(ly,ly+n,0);
45     for(int i=0;i<n;++i){
46         lx[i]=-INF;
47         for(int y=0;y<n;++y){
48             lx[i]=max(lx[i],g[i][y]);
49         }
50     }
51     for(int i=0;i<n;++i){
52         for(int j=0;j<n;++j)slack_y[j]=INF;
53         fill(px,px+n,-1);
54         fill(py,py+n,-1);
55         px[i]=-2;
56         if(dfs(i))continue;
57         bool flag=1;
58         while(flag){
59             int cut=INF; //LL
60             for(int j=0;j<n;++j)
61                 if(py[j]==-1)cut=min(cut,slack_y
62 [j]);
63             for(int j=0;j<n;++j){
64                 if(px[j]!=-1)lx[j]-=cut;
65                 if(py[j]!=-1)ly[j]+=cut;
66                 else slack_y[j]-=cut;
67             }
68             for(int y=0;y<n;++y){
69                 if(py[y]==-1&&slack_y[y]==0){
70                     py[y]=par[y];
71                     if(match[y]==-1){
72                         adjust(y);
73                         flag=0;
74                         break;
75                     }
76                     px[match[y]]=y;
77                     if(dfs(match[y])){
78                         flag=0;
79                         break;
80                     }
81                 }
82             }
83         }
84     }
85     int res=0;//LL
86     for(int i=0;i<n;++i)
87         res+=g[match[i]][i];
88     return res;
89 }graph;

```

### 3.7 SWmincut

```

1 struct SW{ // 0(V^3)
2     static const int MXN = 514;
3     int n,vst[MXN],del[MXN];
4     int edge[MXN][MXN],wei[MXN];
5     void init(int _n){
6         n = _n;
7         MEM(edge);
8         MEM(del);
9     }
10 void add_edge(int u, int v, int w){
11     edge[u][v] += w;
12     edge[v][u] += w;
13 }
14 void search(int &s, int &t){
15     MEM(vst); MEM(wei);
16     s = t = -1;
17     while (true){
18         int mx=-1, cur=0;

```

```

19     for (int i=0; i<n; i++)
20     if (!del[i] && !vst[i] && mx<wei[i])
21     cur = i, mx = wei[i];
22     if (mx == -1) break;
23     vst[cur] = 1;
24     s = t;
25     t = cur;
26     for (int i=0; i<n; i++)
27     if (!vst[i] && !del[i]) wei[i] += edge[
cur][i];
28 }
29 }
30 int solve(){
31     int res = 2147483647;
32     for (int i=0,x,y; i<n-1; i++){
33         search(x,y);
34         res = min(res,wei[y]);
35         del[y] = 1;
36         for (int j=0; j<n; j++)
37             edge[x][j] = (edge[j][x] += edge[y][j]);
38     }
39     return res;
40 }
41 }graph;

```

## 4 Geometry

### 4.1 Circleintersection

```

1 using ld = double;
2 vector<pdd> interCircle(pdd o1, double r1, pdd o2,
3 double r2) {
4     ld d2 = (o1 - o2) * (o1 - o2);
5     ld d = sqrt(d2);
6     if (d > r1+r2) return {};
7     pdd u = 0.5*(o1+o2) + ((r2*r2-r1*r1)/(2*d2))*(o1
-o2);
8     double A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d)
*(-r1+r2+d));
9     pdd v = A / (2*d2) * pdd(o1.S-o2.S, -o1.F+o2.F);
10    return {u+v, u-v};
11 }
12 }

```

### 4.2 Fermat's Point

```

1 If a angle greater or equal than degree 120
2 return this point
3 else
4 make regular triangle ABC' BCA' CAB'
5 interaction AA' BB' CC'

```

### 4.3 Pointoperators

```

1 #define x first
2 #define y second
3 #define cpdd const pdd
4 struct pdd : pair<double, double> {
5     using pair<double, double>::pair;
6     pdd operator + (cpdd &p) const {
7         return {x+p.x, y+p.y};
8     }
9     pdd operator - ( ) const {
10        return {-x, -y};
11    }
12    pdd operator - (cpdd &p) const {
13        return (*this) + (-p);
14    }
15    pdd operator * (double f) const {
16        return {f*x, f*y};
17    }
18    double operator * (cpdd &p) const {
19        return x*p.x + y*p.y;
20    }
21 };
22 double abs(cpdd &p) { return hypot(p.x, p.y); }
23 double arg(cpdd &p) { return atan2(p.y, p.x); }
24 double cross(cpdd &p, cpdd &q) { return p.x*q.y - p.
y*q
.x; }
25 .x; }
26 double cross(cpdd &p, cpdd &q, cpdd &o) { return
cross(
cross(
p-o, q-o); }
27 p-o, q-o); }
28 pdd operator * (double f, cpdd &p) { return p*f; }
29 //!! Not f*p !!

```

### 4.4 3DConvexHull

```

1 int flag[MXN][MXN];
2 struct Point{
3     ld x,y,z;
4     Point operator - (const Point &b) const {
5         return (Point){x-b.x,y-b.y,z-b.z};
6     }
7     Point operator * (const ld &b) const {
8         return (Point){x*b,y*b,z*b};
9     }
10    ld len() const { return sqrtl(x*x+y*y+z*z); }
11    ld dot(const Point &a) const {
12        return x*a.x+y*a.y+z*a.z;
13    }
14    Point operator * (const Point &b) const {
15        return (Point){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y
-b.x*y
};
16    }
17 }
18 };
19 Point ver(Point a, Point b, Point c) {
20     return (b - a) * (c - a);
21 }
22 vector<Face> convex_hull_3D(const vector<Point> pt)
{
23     int n = SZ(pt);
24     REP(i,n) REP(j,n)
25     flag[i][j] = 0;
26     vector<Face> now;
27     now.push_back((Face){0,1,2});
28     now.push_back((Face){2,1,0});
29     int ftop = 0;
30     for (int i=3; i<n; i++){
31         ftop++;
32         vector<Face> next;
33         REP(j, SZ(now)) {
34             Face& f=now[j];
35             ld d=(pt[i]-pt[f.a]).dot(ver(pt[f.a], pt
[f.b], pt
[f.c]));
36             if (d <= 0) next.push_back(f);
37             int ff = 0;
38             if (d > 0) ff=ftop;
39             else if (d < 0) ff=-ftop;
40             flag[f.a][f.b] = flag[f.b][f.c] = flag[f
.c][f.a]
= ff;
41             }
42             REP(j, SZ(now)) {
43                 Face& f=now[j];
44                 if (flag[f.a][f.b] > 0 and flag[f.a][f.b
] != flag
[f.b][f.a])
45                 next.push_back((Face){f.a,f.b,i});
46                 if (flag[f.b][f.c] > 0 and flag[f.b][f.c
] != flag
[f.c][f.b])
47                 next.push_back((Face){f.b,f.c,i});
48                 if (flag[f.c][f.a] > 0 and flag[f.c][f.a
] != flag
[f.a][f.c])
49                 next.push_back((Face){f.c,f.a,i});
50             }
51             now=next;
52         }
53     }
54     return now;
55 }
56 }
57 }
58 }
59 }

```

### 4.5 Halfplaneintersection

```

1 typedef pdd Point;
2 typedef vector<Point> Polygon;
3 typedef pair<Point,Point> Line;
4 #define N 10
5 #define p1 first
6 #define p2 second
7 pdd operator-(const pdd &a,const pdd &b){
8     return mp(a.x-b.x,a.y-b.y);
9 }
10 pdd operator+(const pdd &a,const pdd &b){
11     return mp(a.x+b.x,a.y+b.y);
12 }
13 pdd operator*(const pdd &a,const double &b){

```

```

14 return mp(b*a.x,b*a.y);
15 }
16 double cross(Point a, Point b){
17     return a.x * b.y - a.y * b.x;
18 }
19 double cross(Point o, Point a, Point b){
20     return cross(a-o,b-o);
21 }
22 double cross(Line l, Point p){
23     return cross(l.p1, l.p2, p);
24 }
25 double arg(const pdd &a){
26     return atan2(a.y,a.x);
27 }
28 bool parallel(Line l1, Line l2){
29     return cross(l1.p2 - l1.p1, l2.p2 - l2.p1) < 1e-8 && cross(l1.p2 - l1.p1, l2.p2 - l2.p1) > -1e-8;
30 }
31 Point intersection(Line l1, Line l2){
32     Point& a1 = l1.p1, &a2 = l1.p2;
33     Point& b1 = l2.p1, &b2 = l2.p2;
34     Point a = a2 - a1, b = b2 - b1, s = b1 - a1;
35     return a1 + a * (cross(b, s) / cross(b, a));
36 }
37 bool cmp(Line l1, Line l2){
38     return arg(l1.p2 - l1.p1) < arg(l2.p2 - l2.p1);
39 }
40 Polygon halfplane_intersection(vector<Line> hp){
41     sort(hp.begin(), hp.end(), cmp);
42     int L = 0, R = 0;
43     vector<Line> l(N);
44     vector<Point> p(N);
45     l[R] = hp[0];
46     for (int i=1; i<hp.size(); i++){
47         while (L < R && cross(hp[i], p[R-1]) < 0) R--;
48         while (L < R && cross(hp[i], p[L]) < 0) L++;
49         l[++R] = hp[i];
50         if (parallel(l[R-1], hp[i]) && cross(l[R-1], hp[i].p1) > 0) l[R] = hp[i];
51     ];
52     if (L < R) p[R-1] = intersection(l[R], l[R-1]);
53     while (L < R && cross(l[L], p[R-1]) < 0) R--;
54     if (R-L <= 1) return Polygon();
55     if (L < R) p[R] = intersection(l[L], l[R]);
56     Polygon ch;
57     for (int i=L; i<=R; i++) ch.push_back(p[i]);
58     ch.resize(unique(ch.begin(), ch.end()) - ch.begin());
59     if (ch.size() > 1 && ch.front() == ch.back()) ch.pop_back();
60     return ch;
61 }
62 double cal(Polygon p){
63     if(p.empty()) return 0;
64     p.pb(*p.begin());
65     double ans=0;
66     for(int i=0;i<p.size()-1;i++){
67         ans+=p[i].x*p[i+1].y;
68         ans-=p[i].y*p[i+1].x;
69     }
70     ans/=2;
71     ans=abs(ans);
72     return ans;
73 }

```

#### 4.6 ConvexHull

```

1 sort(p,p+n);
2 pii ans[N];
3 ans[0]=p[0];
4 int k=0,now=0;
5 for(int tt=0;tt<2;tt++){
6     for(int i=1;i<n;i++){
7         while(now!=k&&cross(ans[now],p[i],ans[now-1])<0) now--;
8         ans[++now]=p[i];

```

```

9 }
10 k=now;
11 reverse(p,p+n);
12 }

```

#### 4.7 Triangulation

```

1 bool inCircle(pdd a, pdd b, pdd c, pdd d) {
2     b = b - a;
3     c = c - a;
4     d = d - a;
5     if (cross(b, c) < 0) swap(b, c);
6     double m[3][3] = {
7         {b.x, b.y, b*b},
8         {c.x, c.y, c*c},
9         {d.x, d.y, d*d}
10    };
11    double det = m[0][0] * (m[1][1]*m[2][2] - m[1][2]*m[2][1])
12    + m[0][1] * (m[1][2]*m[2][0] - m[1][0]*m[2][2])
13    + m[0][2] * (m[1][0]*m[2][1] - m[1][1]*m[2][0]);
14    return det < 0;
15 }
16 bool intersect(pdd a, pdd b, pdd c, pdd d) {
17     return cross(b, c, a) * cross(b, d, a) < 0 and cross(d, a, c) * cross(d, b, c) < 0;
18 }
19 const double EPS = 1e-12;
20 struct Triangulation {
21     static const int MXN = 1e5+5;
22     int N;
23     vector<int> ord;
24     vector<pdd> pts;
25     set<int> E[MXN];
26     vector<vector<int>> solve(vector<pdd> p) {
27         N = SZ(p);
28         ord.resize(N);
29         for (int i=0; i<N; i++) {
30             E[i].clear();
31             ord[i] = i;
32         }
33         sort(ALL(ord), [&p](int i, int j) {
34             return p[i] < p[j];
35         });
36         pts.resize(N);
37         for (int i=0; i<N; i++) pts[i] = p[ord[i]];
38         go(0, N);
39         vector<vector<int>> res(N);
40         for (int i=0; i<N; i++) {
41             int o = ord[i];
42             for (auto x: E[i]) {
43                 res[o].PB(ord[x]);
44             }
45         }
46         return res;
47     }
48     void add_edge(int u, int v) {
49         E[u].insert(v);
50         E[v].insert(u);
51     }
52     void remove_edge(int u, int v) {
53         E[u].erase(v);
54         E[v].erase(u);
55     }
56     void go(int l, int r) {
57         int n = r - l;
58         if (n <= 3) {
59             for (int i=l; i<r; i++)
60                 for (int j=i+1; j<r; j++) add_edge(i, j);
61             return;
62         }
63         int md = (l+r)/2;
64         go(l, md);
65         go(md, r);
66         int il = l, ir = r-1;
67         while (1) {
68             int nx = -1;
69             for (auto i: E[il]) {
70                 double cs = cross(pts[il], pts[i],

```

pts[



```

76         ir]);
77         if (cs > EPS ||
78             (abs(cs) < EPS and abs(pts[i]-pts[
79             ir]) < abs(pts[il]-pts[ir]))) {
80             nx = i;
81             break;
82         }
83     }
84     if (nx != -1) {
85         il = nx;
86         continue;
87     }
88     for (auto i: E[ir]) {
89         double cs = cross(pts[ir], pts[i],
90             pts[
91             il]);
92         if (cs < -EPS ||
93             (abs(cs) < EPS and abs(pts[i]-pts[
94             il]) < abs(pts[ir]-pts[il]))) {
95             nx = i;
96             break;
97         }
98     }
99     if (nx != -1) {
100         ir = nx;
101     } else break;
102 }
103 add_edge(il, ir);
104 while (1) {
105     int nx = -1;
106     bool is2 = false;
107     National Taiwan University
108     AcThPaUNpPuAmCmBkCfEsFmMdNoLr 19
109     for (int i: E[il]) {
110         if (cross(pts[il], pts[i], pts[ir])
111             < -
112             EPS and
113             (nx == -1 or inCircle(pts[il], pts[
114             ir], pts[nx], pts[i]))) nx = i;
115         }
116         for (int i: E[ir]) {
117             if (cross(pts[ir], pts[i], pts[il])
118                 >
119                 EPS and
120                 (nx == -1 or inCircle(pts[il], pts[
121                 ir], pts[nx], pts[i]))) nx = i,
122                 is2 = 1;
123         }
124         if (nx == -1) break;
125         int a = il, b = ir;
126         if (is2) swap(a, b);
127         for (auto i: E[a]) {
128             if (intersect(pts[a], pts[i], pts[b]
129             ],
130             pts[nx])) {
131                 remove_edge(a, i);
132             }
133         }
134         if (is2) {
135             add_edge(il, nx);
136             ir = nx;
137         } else {
138             add_edge(ir, nx);
139             il = nx;
140         }
141     }
142 }
143 } tri;

```

#### 4.8 Minkowskisum

```

1 vector<Pt> minkowski(vector<Pt> p, vector<Pt> q){
2     int n = p.size(), m = q.size();
3     Pt c = Pt(0, 0);
4     for( int i = 0; i < m; i++) c = c + q[i];
5     c = c / m;
6     for( int i = 0; i < m; i++) q[i] = q[i] - c;
7     int cur = -1;
8     for( int i = 0; i < m; i++)
9         if( (q[i] ^ (p[0] - p[n-1])) > -eps)
10            if( cur == -1 || (q[i] ^ (p[0] - p[n-1])) >
11                (q[cur] ^ (p[0] - p[n-1])) )
12                cur = i;

```

```

13 vector<Pt> h;
14 p.push_back(p[0]);
15 for( int i = 0; i < n; i++)
16     while( true ){
17         h.push_back(p[i] + q[cur]);
18         int nxt = (cur + 1 == m ? 0 : cur + 1);
19         if((q[cur] ^ (p[i+1] - p[i])) < -eps) cur =
20             nxt;
21         else if( (q[nxt] ^ (p[i+1] - p[i])) >
22                 (q[cur] ^ (p[i+1] - p[i])) ) cur =
23             nxt;
24         else break;
25     }
26     for(auto &&i : h) i = i + c;
27     return convex_hull(h);
28 }

```

#### 4.9 K-closet Pair

```

1 #define F(n) Fi(i,n)
2 #define Fi(i,n) Fl(i,0,n)
3 #define Fl(i,l,n) for(int i=(l);i<(int)(n);++i)
4 #include <bits/stdc++.h>
5 // #include <ext/pb_ds/assoc_container.hpp>
6 // #include <ext/pb_ds/priority_queue.hpp>
7 using namespace std;
8 // using namespace __gnu_pbds;
9 typedef long long ll;
10 struct point {
11     point(ll x_ = 0, ll y_ = 0): x(x_), y(y_) {} ll x
12     , y;
13     inline bool operator<(const point &e_) const {
14         return (x != e_.x ? x < e_.x : y < e_.y);
15     }
16     inline friend istream& operator>>(istream &is_,
17         point& e_) {
18         is_ >> e_.x >> e_.y;
19         return is_;
20     }
21 };
22 int k;
23 priority_queue<ll> PQ;
24 inline ll dist2(const point &e1, const point &e2) {
25     ll res = (e1.x-e2.x)*(e1.x-e2.x)+(e1.y-e2.y)*(e1.y-
26     e2.y);
27     PQ.push(res);
28     if (PQ.size() > k) {
29         PQ.pop();
30     }
31     return res;
32 }
33 #define N 500005
34 point p[N];
35 queue<point> Q;
36 ll closet_point(int l, int m, int r, ll delta2) {
37     ll xmid = p[m-1].x;
38     while (!Q.empty()) {
39         Q.pop();
40     }
41     for (int i = l, j = m; i < m; ++i) {
42         if ((p[i].x-xmid)*(p[i].x-xmid) >= delta2) {
43             continue;
44         }
45         while (j < r && p[j].y < p[i].y && (p[j].y-p[i].
46         y)*(p[j].y-p[i].y) < delta2) {
47             if ((p[j].x-xmid)*(p[j].x-xmid) < delta2) {
48                 Q.push(p[j]);
49             }
50             ++j;
51         }
52         while (!Q.empty() && Q.front().y < p[i].y && (Q.
53         front().y-p[i].y)*(Q.front().y-p[i].y) > delta2
54         ) {
55             Q.pop();
56         }
57         while (!Q.empty()) {
58             delta2 = min(delta2, dist2(p[i], Q.front()));
59             Q.pop();
60         }
61     }
62     return delta2;
63 }
64 ll find_distance(int l, int r) {

```

```

59 if (r - l <= 3000) {
60     ll ans = 0x3f3f3f3f3f3f3f3f;
61     for (int i = l; i < r; ++i)
62         for (int j = i+1; j < r; ++j)
63             ans = min(ans, dist2(p[i], p[j]));
64     return ans;
65 }
66 int m = (l+r)/2;
67 ll delta2 = min(find_distance(l, m), find_distance(m, r));
68 return min(delta2, closet_point(l, m, r, delta2));
69 }
70 int main() {
71     ios_base::sync_with_stdio(false);
72     cin.tie(NULL);
73     int n;
74     cin >> n >> k;
75     F(n) cin >> p[i];
76     sort(p, p+n);
77     find_distance(0, n);
78     cout << PQ.top() << '\n';
79 }

```

#### 4.10 MCC

```

1 struct Mcc{
2     // return pair of center and r^2
3     static const int MAXN = 1000100;
4     int n;
5     pdd p[MAXN], cen;
6     double r2;
7     void init(int _n, pdd _p[]){
8         n = _n;
9         memcpy(p, _p, sizeof(pdd)*n);
10    }
11    double sqr(double a){ return a*a; }
12    double abs2(pdd a){ return a*a; }
13    pdd center(pdd p0, pdd p1, pdd p2) {
14        pdd a = p1-p0;
15        pdd b = p2-p0;
16        double c1=abs2(a)*0.5;
17        double c2=abs2(b)*0.5;
18        double d = a.x*b.y-b.x*a.y;
19        double x = p0.x + (c1 * b.y - c2 * a.y) / d;
20        double y = p0.y + (a.x * c2 - b.x * c1) / d;
21        return pdd(x,y);
22    }
23    pair<pdd,double> solve(){
24        random_shuffle(p,p+n);
25        r2=0;
26        for (int i=0; i<n; i++){
27            if (abs2(cen-p[i]) <= r2) continue;
28            cen = p[i];
29            r2 = 0;
30            for (int j=0; j<i; j++){
31                if (abs2(cen-p[j]) <= r2) continue;
32                cen = 0.5 * (p[i]+p[j]);
33                r2 = abs2(cen-p[j]);
34                for (int k=0; k<j; k++){
35                    if (abs2(cen-p[k]) <= r2)
36                        continue;
37                    cen = center(p[i],p[j],p[k]);
38                    r2 = abs2(cen-p[k]);
39                }
40            }
41            return {cen,r2};
42        }
43    }mcc;

```

#### 4.11 LineIntersection

```

1 pdd interPnt(pdd p1, pdd p2, pdd q1, pdd q2, bool &
2 res)
3 {
4     double f1 = cross(p2, q1, p1);
5     double f2 = -cross(p2, q2, p1);
6     double f = (f1 + f2);
7     if(fabs(f) < EPS) {
8         res = false;
9         return {};
10    }
11    res = true;
12    return (f2 / f) * q1 + (f1 / f) * q2;

```

## 5 Graph

### 5.1 Planar

```

1 //skydog
2 #include <iostream>
3 #include <cstdio>
4 #include <cstdlib>
5 #include <iomanip>
6
7 #include <vector>
8 #include <cstring>
9 #include <string>
10 #include <queue>
11 #include <deque>
12 #include <stack>
13 #include <map>
14 #include <set>
15
16 #include <utility>
17 #include <list>
18
19 #include <cmath>
20 #include <algorithm>
21 #include <cassert>
22 #include <bitset>
23 #include <complex>
24 #include <climits>
25 #include <functional>
26 using namespace std;
27
28 typedef long long ll;
29 typedef pair<int, int> ii;
30 typedef pair<ll, ll> ll;
31
32 #define mp make_pair
33 #define pb push_back
34
35 #define debug(x) cerr << #x << " = " << x << " "
36
37 const int N=400+1;
38
39 struct Planar
40 {
41     int n,m,hash[N],fa[N],deep[N],low[N],ecp[N];
42     vector<int> g[N],son[N];
43     set< pair<int,int> > SDlist[N],proots[N];
44     int nxt[N][2],back[N],rev[N];
45     deque<int> q;
46     void dfs(int u)
47     {
48         hash[u]=1; q.pb(u);
49         ecp[u]=low[u]=deep[u];
50         int v;
51         for (int i = 0; i < g[u].size(); ++i)
52             if(!hash[v=g[u][i]])
53             {
54                 fa[v]=u;
55                 deep[v]=deep[u]+1;
56                 dfs(v);
57                 low[u]=min(low[u],low[v]);
58                 SDlist[u].insert(mp(low[v],v));
59             }
60             else ecp[u]=min(ecp[u],deep[v]);
61             low[u]=min(low[u],ecp[u]);
62     }
63
64     int visited[N];
65
66     void addtree(int u,int t1,int v,int t2)
67     {
68         nxt[u][t1]=v; nxt[v][t2]=u;
69     }
70
71     void findnxt(int u,int v,int& u1,int& v1)
72     {
73         u1=nxt[u][v^1];
74         if(nxt[u1][0]==u) v1=0;
75         else v1=1;
76     }
77
78     void walkup(int u,int v)
79     {

```

```

80     back[v]=u;
81     int v1=v, v2=v, u1=1, u2=0, z;
82     for (;;)
83     {
84         if(hash[v1]==u || hash[v2]==u) break;
85         hash[v1]=u; hash[v2]=u; z=max(v1, v2);
86         if(z>n)
87         {
88             int p=fa[z-n];
89             if(p!=u)
90             {
91                 proots[p].insert(mp(-low[z-n], z
92             ));
93             v1=p, v2=p, u1=0, u2=1;
94             }
95             else break;
96         }
97         else
98         {
99             findnxt(v1, u1, v1, u1);
100            findnxt(v2, u2, v2, u2);
101        }
102    }
103
104    int topstack;
105    pair<int, int> stack[N];
106
107    int outer(int u, int v)
108    {
109        return ecp[v]<deep[u] || (SDlist[v].size()
110        && SDlist[v].begin()->first<deep[u]);
111    }
112
113    int inside(int u, int v)
114    {
115        return proots[v].size()>0 || back[v]==u;
116    }
117
118    int active(int u, int v)
119    {
120        return inside(u, v) || outer(u, v);
121    }
122
123    void push(int a, int b)
124    {
125        stack[++topstack]=mp(a, b);
126    }
127
128    void mergestack()
129    {
130        int v1, t1, v2, t2, s, s1;
131        v1=stack[topstack].first; t1=stack[topstack].
132        second;
133        topstack--;
134        v2=stack[topstack].first; t2=stack[topstack].
135        second;
136        topstack--;
137
138        s=nxt[v1][t1^1];
139        s1=(nxt[s][1]==v1);
140        nxt[s][s1]=v2;
141        nxt[v2][t2]=s;
142
143        SDlist[v2].erase( make_pair(low[v1-n], v1-n)
144        );
145        proots[v2].erase( make_pair(-low[v1-n], v1) )
146        ;
147    }
148
149    void findnxtActive(int u, int t, int& v, int& w1,
150    int S)
151    {
152        findnxt(u, t, v, w1);
153        while(u!=v && !active(S, v))
154            findnxt(v, w1, v, w1);
155    }
156
157    void walkdown(int S, int u)
158    {
159        topstack=0;
160        int t1, v=S, w1, x2, y2, x1, y1, p;

```

```

155        for(t1=0; t1<2; ++t1)
156        {
157            findnxt(S, t1^1, v, w1);
158            while(v!=S)
159            {
160                if(back[v]==u)
161                {
162                    while(topstack>0) mergestack();
163                    addtree(S, t1, v, w1); back[v]=0;
164                }
165                if(proots[v].size())
166                {
167                    push(v, w1);
168                    p=proots[v].begin()->second;
169                    findnxtActive(p, 1, x1, y1, u);
170                    findnxtActive(p, 0, x2, y2, u);
171                    if(active(u, x1) && !outer(u, x1))
172                        v=x1, w1=y1;
173                    else if(active(u, x2) && !outer(u
174                    , x2))
175                        v=x2, w1=y2;
176                    else if(inside(u, x1) || back[x1
177                    ]==u)
178                        v=x1, w1=y1;
179                    else v=x2, w1=y2;
180                    push(p, v==x2);
181                }
182                else if(v>n || ( ecp[v]>=deep[u] &&
183                !outer(u, v) ))
184                    findnxt(v, w1, v, w1);
185                else if(v<=n && outer(u, v) && !
186                topstack)
187                {
188                    addtree(S, t1, v, w1); break;
189                }
190                else break;
191            }
192        }
193    }
194
195    int work(int u)
196    {
197        int v;
198        for (int i = 0; i < g[u].size(); ++i)
199            if(fa[v=g[u][i]]==u)
200            {
201                son[u].push_back(n+v);
202                proots[n+v].clear();
203                addtree(n+v, 1, v, 0);
204                addtree(n+v, 0, v, 1);
205            }
206        for (int i = 0; i < g[u].size(); ++i)
207            if(deep[v=g[u][i]]>deep[u]+1)
208                walkup(u, v);
209        topstack=0;
210        for (int i = 0; i < son[u].size(); ++i)
211            walkdown(son[u][i], u);
212        for (int i = 0; i < g[u].size(); ++i)
213            if(deep[v=g[u][i]]>deep[u]+1 && back[v])
214                return 0;
215        return 1;
216    }
217
218    void init(int _n)
219    {
220        n = _n;
221        m = 0;
222        for(int i=1; i<=2*n; ++i)
223        {
224            g[i].clear();
225            SDlist[i].clear();
226            son[i].clear();
227            proots[i].clear();
228            nxt[i][0]=nxt[i][1]=0;
229            fa[i]=0;
230            hash[i]=0; low[i]=ecp[i]=deep[i]=back[i]
231            ]=0;
232            q.clear();
233        }
234    }
235
236    void add(int u, int v)
237    {

```

```

231     ++m;
232     g[u].pb(v); g[v].pb(u);
233 }
234 bool check_planar()
235 {
236     if(m>3*n-5)
237         return false;
238     // memset(hash,0,sizeof hash);
239     for(int i=1;i<=n;++i)
240         if(!hash[i])
241             {
242                 deep[i]=1;
243                 dfs(i);
244             }
245     memset(hash,0,sizeof hash);
246     //memset(hash, 0, (2*n+1)*sizeof(hash[0]));
247     // originally only looks at last n element
248     assert(q.size() == n);
249     while (!q.empty())
250     {
251         if (!work(q.back()))
252             return false;
253         q.pop_back();
254     }
255     return true;
256 }
257 } base, _new;
258 vector<ii> edges;
259 int n, m;
260 inline void build(int n, Planar &_new)
261 {
262     _new.init(n);
263     for (auto e : edges)
264         _new.add(e.first, e.second);
265 }
266 void end()
267 {
268     puts("-1");
269     exit(0);
270 }
271 bool vis[N];
272 const int maxp = 5;
273 int path[maxp], tp=0;
274 void dfs(int cur)
275 {
276     vis[cur] = true;
277     path[tp++] = cur;
278     if (tp == maxp)
279     {
280         auto it = lower_bound(base.g[cur].begin(), base.g[cur].end(), path[0]);
281         if (it != base.g[cur].end() && *it == path[0])
282             {
283                 //a cycle
284                 int x = n+1;
285                 for (int i = 0; i < 5; ++i) edges.pb(mp(x, path[i]));
286                 build(x, _new);
287                 if (_new.check_planar())
288                     {
289                         for (int i = 0; i < maxp; ++i)
290                             printf("%d%c", path[i], i==maxp-1?'\\n':' ');
291                         exit(0);
292                     }
293                 for (int i = 0; i < 5; ++i) edges.pop_back();
294             }
295         else
296             {
297                 for (auto e : base.g[cur]) if (!vis[e]) dfs(e);
298             }
299         vis[cur] = false;
300         --tp;
301     }
302 }
303 int main()
304 {
305     scanf("%d %d", &n, &m);
306     if (n <= 4)

```

```

307     assert(false);
308     puts("0"); return 0;
309 }
310 for (int i = 0; i < m; ++i)
311 {
312     int u, v; scanf("%d %d", &u, &v);
313     edges.pb(mp(u, v));
314 }
315 build(n, base);
316 if (!base.check_planar()) end();
317 for (int i = 1; i <= n; ++i)
318     sort(base.g[i].begin(), base.g[i].end());
319 for (int i = 1; i <= n; ++i)
320     dfs(i);
321 end();
322 }
323 }

```

## 5.2 MMC

```

1 /* minimum mean cycle 最小平均值環*/
2 const int MXN = 16004;
3 const int MAXE = 1805;
4 const int MAXN = 35;
5 const double inf = 1029384756;
6 const double eps = 1e-6;
7 struct Edge {
8     int v,u;
9     double c;
10 };
11 int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
12 Edge e[MAXE];
13 vector<int> edgeID, cycle, rho;
14 double d[MAXN][MAXN];
15 inline void bellman_ford() {
16     for(int i=0; i<n; i++) d[0][i]=0;
17     for(int i=0; i<n; i++) {
18         fill(d[i+1], d[i+1]+n, inf);
19         for(int j=0; j<m; j++) {
20             int v = e[j].v, u = e[j].u;
21             if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
22                 d[i+1][u] = d[i][v]+e[j].c;
23                 prv[i+1][u] = v;
24                 prve[i+1][u] = j;
25             }
26         }
27     }
28 }
29 double karp_mmc() {
30     // returns inf if no cycle, mmc otherwise
31     double mmc=inf;
32     int st = -1;
33     bellman_ford();
34     for(int i=0; i<n; i++) {
35         double avg=-inf;
36         for(int k=0; k<n; k++) {
37             if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
38             else avg=max(avg,inf);
39         }
40         if (avg < mmc) tie(mmc, st) = tie(avg, i);
41     }
42     MEM(vst); edgeID.clear(); cycle.clear(); rho.clear();
43     for (int i=n; !vst[st]; st=prv[i--][st]) {
44         vst[st]++;
45         edgeID.pb(prve[i][st]);
46         rho.pb(st);
47     }
48     while (vst[st] != 2) {
49         int v = rho.back(); rho.pop_back();
50         cycle.pb(v);
51         vst[v]++;
52     }
53     reverse(edgeID.begin(),edgeID.end());
54     edgeID.resize(cycle.size());
55     return mmc;
56 }
57 }

```

### 5.3 SomeTheroem

```

1 /*
2 General graph
3 |maximum independent set|+|minimum vertex cover|=|V|
4 |maximum independent edge|+|minimum edge cover|=|V|
5 ||
6 Max_match
7 Bipartite graph
8 |Maximun independent set|=|Minimun edge cover|
9 |Maximun independent edge|=|Minimun vertex cover|
10 |Maximun Independent set|+|Minimun vertex cover|=|V|
11 + +
12 |Maximun Independent edge|+|Minimun edge cover|=|V|
13 || ||
14 |V| |V|
15 */

```

### 5.4 Dominator

```

1 struct DominatorTree{
2     static const int MAXN = 200010;
3     int n,s;
4     vector<int> g[MAXN],pred[MAXN];
5     vector<int> cov[MAXN];
6     int dfn[MAXN],nfd[MAXN],ts;
7     int par[MAXN];
8     int sdom[MAXN],idom[MAXN];
9     int mom[MAXN],mn[MAXN];
10
11     inline bool cmp(int u,int v) { return dfn[u] < dfn[v]; }
12
13     int eval(int u) {
14         if(mom[u] == u) return u;
15         int res = eval(mom[u]);
16         if(cmp(sdom[mn[mom[u]]],sdom[mn[u]]))
17             mn[u] = mn[mom[u]];
18         return mom[u] = res;
19     }
20
21     void init(int _n, int _s) {
22         n = _n;
23         s = _s;
24         REP1(i,1,n) {
25             g[i].clear();
26             pred[i].clear();
27             idom[i] = 0;
28         }
29     }
30     void add_edge(int u, int v) {
31         g[u].push_back(v);
32         pred[v].push_back(u);
33     }
34     void DFS(int u) {
35         ts++;
36         dfn[u] = ts;
37         nfd[ts] = u;
38         for(int v:g[u]) if(dfn[v] == 0) {
39             par[v] = u;
40             DFS(v);
41         }
42     }
43     void build() {
44         ts = 0;
45         REP1(i,1,n) {
46             dfn[i] = nfd[i] = 0;
47             cov[i].clear();
48             mom[i] = mn[i] = sdom[i] = i;
49         }
50         DFS(s);
51         for (int i=ts; i>=2; i--) {
52             int u = nfd[i];
53             if(u == 0) continue;
54             for(int v:pred[u]) if(dfn[v]) {
55                 eval(v);
56                 if(cmp(sdom[mn[v]],sdom[u])) sdom[u] = sdom[mn[v]];
57             }
58             cov[sdom[u]].push_back(u);
59             mom[u] = par[u];
60             for(int w:cov[par[u]]) {
61                 eval(w);
62                 if(cmp(sdom[mn[w]],par[u])) idom[w] = mn[w];

```

```

63         else idom[w] = par[u];
64     }
65     cov[par[u]].clear();
66 }
67 REP1(i,2,ts) {
68     int u = nfd[i];
69     if(u == 0) continue;
70     if(idom[u] != sdom[u]) idom[u] = idom[idom[u]];
71 }
72 }
73 }dom;

```

### 5.5 DMST

```

1 struct zhu_liu{
2     static const int MAXN=1100,MAXM=1005005;
3     struct node{
4         int u,v;
5         LL w,tag;
6         node *l,*r;
7         node(int u=0,int v=0,LL w=0):u(u),v(v),w(w),tag(0),l(0),r(0){}
8         void down(){
9             w+=tag;
10            if(l)l->tag+=tag;
11            if(r)r->tag+=tag;
12            tag=0;
13        }
14    }mem[MAXN];
15    node *pq[MAXN*2],*E[MAXN*2];
16    int st[MAXN*2],id[MAXN*2],m,from[MAXN*2];
17    void init(int n){
18        for(int i=1;i<=n;++i){
19            pq[i]=E[i]=0;
20            st[i]=id[i]=i;
21            from[i]=0;
22        }m=0;
23    }
24    node *merge(node *a,node *b){//skew heap
25        if(!a||!b)return a?a:b;
26        a->down(),b->down();
27        if(b->w<a->w)return merge(b,a);
28        if(b->w==a->w&&b->v<a->v)return merge(b,a);//
29        swap(a->l,a->r);
30        a->l=merge(b,a->l);
31        return a;
32    }
33    void add_edge(int u,int v,LL w){
34        if(u!=v)pq[v]=merge(pq[v],&(mem[m++]=node(u,v,w)));
35    }
36    int find(int x,int *st){
37        return st[x]==x?x:st[x]=find(st[x],st);
38    }
39    LL build(int root,int n){
40        LL ans=0;int N=n,all=n;
41        for(int i=1;i<=N;++i){
42            if(i==root||!pq[i])continue;
43            while(pq[i]){
44                pq[i]->down(),E[i]=pq[i];
45                pq[i]=merge(pq[i]->l,pq[i]->r);
46                if(find(E[i]->u,id)!=find(i,id))break;
47            }
48            if(find(E[i]->u,id)==find(i,id))continue;
49            from[E[i]->v]=E[i]->u;
50            ans+=E[i]->w;
51            if(find(E[i]->u,st)==find(i,st)){
52                if(pq[i])pq[i]->tag-=E[i]->w;
53                pq[++N]=pq[i],id[N]=N;
54                for(int u=find(E[i]->u,id);u!=i;u=find(E[u]->u,id)){
55                    if(pq[u])pq[u]->tag-=E[u]->w;
56                    id[find(u,id)]=N;
57                    pq[N]=merge(pq[N],pq[u]);
58                }
59                st[N]=find(i,st);
60                id[find(i,id)]=N;
61            }else st[find(i,st)]=find(E[i]->u,st),--all;
62        }
63        return all==1?ans:-1;//圖不連通就無解
64    }
65 }MST;

```



## 5.6 SCC

```

1 struct Scc{
2     int n, nScc, vst[MXN], bln[MXN];
3     vector<int> E[MXN], rE[MXN], vec;
4     void init(int _n){
5         n = _n;
6         for (int i=0; i<MXN; i++){
7             E[i].clear();
8             rE[i].clear();
9         }
10    }
11    void add_edge(int u, int v){
12        E[u].pb(v);
13        rE[v].pb(u);
14    }
15    void DFS(int u){
16        vst[u]=1;
17        for (auto v : E[u])
18            if (!vst[v]) DFS(v);
19        vec.pb(u);
20    }
21    void rDFS(int u){
22        vst[u] = 1;
23        bln[u] = nScc;
24        for (auto v : rE[u])
25            if (!vst[v]) rDFS(v);
26    }
27    void solve(){
28        nScc = 0;
29        vec.clear();
30        MEM(vst);
31        for (int i=0; i<n; i++)
32            if (!vst[i]) DFS(i);
33        reverse(vec.begin(), vec.end());
34        FZ(vst);
35        for (auto v : vec){
36            if (!vst[v]){
37                rDFS(v);
38                nScc++;
39            }
40        }
41    }
42 };

```

## 5.7 General Graph Maximum Value Match

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 //from vfleaking
4 //自己進行一些進行一些小修改
5 #define INF INT_MAX
6 #define MAXN 400
7 struct edge{
8     int u,v,w;
9     edge(){}
10    edge(int u,int v,int w):u(u),v(v),w(w){}
11 };
12 int n,n_x;
13 edge g[MAXN*2+1][MAXN*2+1];
14 int lab[MAXN*2+1];
15 int match[MAXN*2+1],slack[MAXN*2+1],st[MAXN*2+1],pa[
    MAXN*2+1];
16 int flower_from[MAXN*2+1][MAXN+1],S[MAXN*2+1],vis[
    MAXN*2+1];
17 vector<int> flower[MAXN*2+1];
18 queue<int> q;
19 inline int e_delta(const edge &e){ // does not work
    inside blossoms
20    return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
21 }
22 inline void update_slack(int u,int x){
23    if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]
    ][x]))slack[x]=u;
24 }
25 inline void set_slack(int x){
26    slack[x]=0;
27    for(int u=1;u<=n;u++)
28        if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
29            update_slack(u,x);
30 }
31 void q_push(int x){
32    if(x<=n)q.push(x);
33    else for(size_t i=0;i<flower[x].size();i++)q.push(
    flower[x][i]);

```

```

33 }
34 inline void set_st(int x,int b){
35    st[x]=b;
36    if(x>n)for(size_t i=0;i<flower[x].size();i++)
37        set_st(flower[x][i],b);
38 }
39 inline int get_pr(int b,int xr){
40    int pr=find(flower[b].begin(),flower[b].end(),xr)-
    flower[b].begin();
41    if(pr%2==1){//檢查他在前一層圖是奇點還是偶點
42        reverse(flower[b].begin()+1,flower[b].end());
43        return (int)flower[b].size()-pr;
44    }else return pr;
45 }
46 inline void set_match(int u,int v){
47    match[u]=g[u][v].v;
48    if(u>n){
49        edge e=g[u][v];
50        int xr=flower_from[u][e.u],pr=get_pr(u,xr);
51        for(int i=0;i<pr;i++)set_match(flower[u][i],
    flower[u][i^1]);
52        set_match(xr,v);
53        rotate(flower[u].begin(),flower[u].begin()+pr,
    flower[u].end());
54    }
55 }
56 inline void augment(int u,int v){
57    for(;;){
58        int xnv=st[match[u]];
59        set_match(u,v);
60        if(!xnv)return;
61        set_match(xnv,st[pa[xnv]]);
62        u=st[pa[xnv]],v=xnv;
63    }
64 }
65 inline int get_lca(int u,int v){
66    static int t=0;
67    for(++t;u||v;swap(u,v)){
68        if(u==0)continue;
69        if(vis[u]==t)return u;
70        vis[u]=t;//這種方法可以不用清空v陣列
71        u=st[match[u]];
72        if(u)u=st[pa[u]];
73    }
74    return 0;
75 }
76 inline void add_blossom(int u,int lca,int v){
77    int b=n+1;
78    while(b<=n_x&&st[b])++b;
79    if(b>n_x)++n_x;
80    lab[b]=0,S[b]=0;
81    match[b]=match[lca];
82    flower[b].clear();
83    flower[b].push_back(lca);
84    for(int x=u,y;x!=lca;x=st[pa[y]])
85        flower[b].push_back(x),flower[b].push_back(y=st[
    match[x]]),q.push(y);
86    reverse(flower[b].begin()+1,flower[b].end());
87    for(int x=v,y;x!=lca;x=st[pa[y]])
88        flower[b].push_back(x),flower[b].push_back(y=st[
    match[x]]),q.push(y);
89    set_st(b,b);
90    for(int x=1;x<=n_x;xx++)g[b][x].w=g[x][b].w=0;
91    for(int x=1;x<=n;xx++)flower_from[b][x]=0;
92    for(size_t i=0;i<flower[b].size();i++){
93        int xs=flower[b][i];
94        for(int x=1;x<=n_x;xx++)
95            if(g[b][x].w==0||e_delta(g[xs][x])<e_delta(g[b]
    ][x]))
96                g[b][x]=g[xs][x],g[x][b]=g[x][xs];
97        for(int x=1;x<=n;xx++)
98            if(flower_from[xs][x])flower_from[b][x]=xs;
99    }
100    set_slack(b);
101 }
102 inline void expand_blossom(int b){ // S[b] == 1
103    for(size_t i=0;i<flower[b].size();i++)
104        set_st(flower[b][i],flower[b][i]);
105    int xr=flower_from[b][g[b][pa[b]].u],pr=get_pr(b,
    xr);
106    for(int i=0;i<pr;i+=2){
107        int xs=flower[b][i],xns=flower[b][i+1];

```

```

108 pa[xs]=g[xns][xs].u;
109 S[xs]=1,S[xns]=0;
110 slack[xs]=0,set_slack(xns);
111 q_push(xns);
112 }
113 S[xr]=1,pa[xr]=pa[b];
114 for(size_t i=pr+1;i<flower[b].size();++i){
115     int xs=flower[b][i];
116     S[xs]=-1,set_slack(xs);
117 }
118 st[b]=0;
119 }
120 inline bool on_found_edge(const edge &e){
121     int u=st[e.u],v=st[e.v];
122     if(S[v]==-1){
123         pa[v]=e.u,S[v]=1;
124         int nu=st[match[v]];
125         slack[v]=slack[nu]=0;
126         S[nu]=0,q_push(nu);
127     }else if(S[v]==0){
128         int lca=get_lca(u,v);
129         if(!lca)return augment(u,v),augment(v,u),true;
130         else add_blossom(u,lca,v);
131     }
132     return false;
133 }
134 inline bool matching(){
135     memset(S+1,-1,sizeof(int)*n_x);
136     memset(slack+1,0,sizeof(int)*n_x);
137     q=queue<int>();
138     for(int x=1;x<=n_x;++x)
139         if(st[x]==x&&!match[x])pa[x]=0,S[x]=0,q_push(x);
140     if(q.empty())return false;
141     for(;;){
142         while(q.size()){
143             int u=q.front();q.pop();
144             if(S[st[u]]==1)continue;
145             for(int v=1;v<=n_x;++v)
146                 if(g[u][v].w>0&&st[u]!=st[v]){
147                     if(e_delta(g[u][v])==0){
148                         if(on_found_edge(g[u][v]))return true;
149                     }else update_slack(u,st[v]);
150                 }
151         }
152         int d=INF;
153         for(int b=n+1;b<=n_x;++b)
154             if(st[b]==b&&S[b]==1)d=min(d,lab[b]/2);
155         for(int x=1;x<=n_x;++x)
156             if(st[x]==x&&slack[x]){
157                 if(S[x]==-1)d=min(d,e_delta(g[slack[x]][x]));
158             }else if(S[x]==0)d=min(d,e_delta(g[slack[x]][x])/2);
159         for(int u=1;u<=n;++u){
160             if(S[st[u]]==0){
161                 if(lab[u]<=d)return 0;
162                 lab[u]-=d;
163             }else if(S[st[u]]==1)lab[u]+=d;
164         }
165         for(int b=n+1;b<=n_x;++b)
166             if(st[b]==b){
167                 if(S[st[b]]==0)lab[b]+=d*2;
168                 else if(S[st[b]]==1)lab[b]-=d*2;
169             }
170         q=queue<int>();
171         for(int x=1;x<=n_x;++x)
172             if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&
173                e_delta(g[slack[x]][x])==0)
174                 if(on_found_edge(g[slack[x]][x]))return true;
175         for(int b=n+1;b<=n_x;++b)
176             if(st[b]==b&&S[b]==1&&lab[b]==0)expand_blossom(b);
177     }
178     return false;
179 }
180 inline pair<long long,int> weight_blossom(){
181     memset(match+1,0,sizeof(int)*n);
182     n_x=n;
183     int n_matches=0;
184     long long tot_weight=0;

```

```

185     for(int u=0;u<=n;++u)st[u]=u,flower[u].clear();
186     int w_max=0;
187     for(int u=1;u<=n;++u)
188         for(int v=1;v<=n;++v){
189             flower_from[u][v]=(u==v?u:0);
190             w_max=max(w_max,g[u][v].w);
191         }
192     for(int u=1;u<=n;++u)lab[u]=w_max;
193     while(matching())n_matches++;
194     for(int u=1;u<=n;++u)
195         if(match[u]&&match[u]<u)
196             tot_weight+=g[u][match[u]].w;
197     return make_pair(tot_weight,n_matches);
198 }
199 inline void init_weight_graph(){
200     for(int u=1;u<=n;++u)
201         for(int v=1;v<=n;++v)
202             g[u][v]=edge(u,v,0);
203 }
204 int main(){
205     int m;
206     scanf("%d",&n,&m);
207     init_weight_graph();
208     for(int i=0;i<m;++i){
209         int u,v,w;
210         scanf("%d%d%d",&u,&v,&w);
211         g[u][v].w=g[v][u].w=w;
212     }
213     printf("%lld\n",weight_blossom().first);
214     for(int u=1;u<=n;++u)printf("%d ",match[u]);puts("
");
215     return 0;
216 }

```

## 5.8 Stable Marriage

```

1 #define F(n) Fi(i, n)
2 #define Fi(i, n) Fl(i, 0, n)
3 #define Fl(i, l, n) for(int i = l ; i < n ; ++i)
4 #include <bits/stdc++.h>
5 using namespace std;
6 int D, quota[205], weight[205][5];
7 int S, scoretodep[12005][205], score[5];
8 int P, prefer[12005][85], iter[12005];
9 int ans[12005];
10 typedef pair<int, int> PII;
11 map<int, int> samescore[205];
12 typedef priority_queue<PII, vector<PII>, greater<PII>
13 >> QQQ;
14 QQQ pri[205];
15 void check(int d) {
16     PII t = pri[d].top();
17     if (pri[d].size() - samescore[d][t.first] + 1 <=
18         quota[d]) return;
19     while (pri[d].top().first == t.first) {
20         v = pri[d].top().second;
21         ans[v] = -1;
22         --samescore[d][t.first];
23         pri[d].pop();
24     }
25 }
26 void push(int s, int d) {
27     if (pri[d].size() < quota[d]) {
28         pri[d].push(PII(scoretodep[s][d], s));
29         ans[s] = d;
30         ++samescore[s][scoretodep[s][d]];
31     } else if (scoretodep[s][d] >= pri[d].top().first) {
32         pri[d].push(PII(scoretodep[s][d], s));
33         ans[s] = d;
34         ++samescore[s][scoretodep[s][d]];
35         check(d);
36     }
37 }
38 void f() {
39     int over;
40     while (true) {
41         over = 1;
42         for (int q, S) {
43             if (ans[q] != -1 || iter[q] >= P) continue;
44             push(q, prefer[q][iter[q]++]);
45             over = 0;
46         }
47     }

```

```

45     }
46     if (over) break;
47 }
48 }
49 main() {
50     ios::sync_with_stdio(false);
51     cin.tie(NULL);
52     int sadmit, stof, dexceed, dfew;
53     while (cin >> D, D) { // Beware of the input
54         format or judge may troll us.
55         sadmit = stof = dexceed = dfew = 0;
56         memset(iter, 0, sizeof(iter));
57         memset(ans, 0, sizeof(ans));
58         Fi(q, 205) {
59             pri[q] = QQQ();
60             samescore[q].clear();
61         }
62         cin >> S >> P;
63         Fi(q, D) {
64             cin >> quota[q];
65             Fi(w, 5) cin >> weight[q][w];
66         }
67         Fi(q, S) {
68             Fi(w, 5) cin >> score[w];
69             Fi(w, D) {
70                 scoretoDep[q][w] = 0;
71                 F(5) scoretoDep[q][w] += weight[w][i] *
72                 score[i];
73             }
74             Fi(q, S) Fi(w, P) {
75                 cin >> prefer[q][w];
76                 --prefer[q][w];
77             }
78             f();
79             Fi(q, D) sadmit += pri[q].size();
80             Fi(q, S) if (ans[q] == prefer[q][0]) ++stof;
81             Fi(q, D) if (pri[q].size() > quota[q]) ++
82             dexceed;
83             Fi(q, D) if (pri[q].size() < quota[q]) ++dfew;
84             cout << sadmit << ' ' << stof << ' ' << dexceed
85             << ' ' << dfew << '\n';
86         }
87     }
88 }

```

## 5.9 BCCvertex

```

1 const int MXN = 16004;
2 struct BccVertex {
3     int n, nScc, step, dfn[MXN], low[MXN];
4     vector<int> E[MXN], sccv[MXN];
5     int top, stk[MXN];
6     void init(int _n) {
7         n = _n;
8         nScc = step = 0;
9         for (int i=0; i<n; i++) E[i].clear();
10    }
11    void add_edge(int u, int v) {
12        E[u].pb(v);
13        E[v].pb(u);
14    }
15    void DFS(int u, int f) {
16        dfn[u] = low[u] = step++;
17        stk[top++] = u;
18        for (auto v:E[u]) {
19            if (v == f) continue;
20            if (dfn[v] == -1) {
21                DFS(v, u);
22                low[u] = min(low[u], low[v]);
23                if (low[v] >= dfn[u]) {
24                    int z;
25                    sccv[nScc].clear();
26                    do {
27                        z = stk[--top];
28                        sccv[nScc].pb(z);
29                    } while (z != v);
30                    sccv[nScc].pb(u);
31                    nScc++;
32                }
33            } else {
34                low[u] = min(low[u], dfn[v]);
35            }
36        }
37    }

```

```

37    }
38    vector<vector<int>> solve() {
39        vector<vector<int>> res;
40        for (int i=0; i<n; i++) {
41            dfn[i] = low[i] = -1;
42        }
43        for (int i=0; i<n; i++) {
44            if (dfn[i] == -1) {
45                top = 0;
46                DFS(i, i);
47            }
48        }
49        for (int i=0; i<nScc; i++) res.pb(sccv[i]);
50        return res;
51    }
52 }graph;

```

## 5.10 MaxClique

```

1 class MaxClique {
2 public:
3     static const int MV = 210;
4     int V;
5     int el[MV][MV/30+1];
6     int dp[MV];
7     int ans;
8     int s[MV][MV/30+1];
9     vector<int> sol;
10    void init(int v) {
11        V = v; ans = 0;
12        MEMS(el); MEMS(dp);
13    }
14    /* Zero Base */
15    void addEdge(int u, int v) {
16        if (u > v) swap(u, v);
17        if (u == v) return;
18        el[u][v/32] |= (1<<(v%32));
19    }
20    bool dfs(int v, int k) {
21        int c = 0, d = 0;
22        for (int i=0; i<(V+31)/32; i++) {
23            s[k][i] = el[v][i];
24            if (k != 1) s[k][i] &= s[k-1][i];
25            c += __builtin_popcount(s[k][i]);
26        }
27        if (c == 0) {
28            if (k > ans) {
29                ans = k;
30                sol.clear();
31                sol.push_back(v);
32                return 1;
33            }
34            return 0;
35        }
36        for (int i=0; i<(V+31)/32; i++) {
37            for (int a = s[k][i]; a; a += dp[i]) {
38                if (k + (c-d) <= ans) return 0;
39                int lb = a & (-a), lg = 0;
40                a ^= lb;
41                while (lb != 1) {
42                    lb = (unsigned int)(lb) >> 1;
43                    lg++;
44                }
45                int u = i*32 + lg;
46                if (k + dp[u] <= ans) return 0;
47                if (dfs(u, k+1)) {
48                    sol.push_back(v);
49                    return 1;
50                }
51            }
52        }
53        return 0;
54    }
55    int solve() {
56        for (int i=V-1; i>=0; i--) {
57            dfs(i, 1);
58            dp[i] = ans;
59        }
60        return ans;
61    }
62 };

```

## 5.11 BCCedge

```

1 vector<vector<int>> > v;
2 int vis[100005], lwn[100005];
3 vector<int> stk;
4 int f[100005];
5 int bln[100005];
6 int Find(int a){
7     if(bln[a]==a) return a;
8     return bln[a]=Find(bln[a]);
9 }
10 int t;
11 void dfs(int a, int p){
12     stk.pb(a);
13     bln[a]=a;
14     vis[a]=lwn[a]++;
15     int cnt=0;
16     for(int i=0; i<v[a].size(); i++){
17         int x=v[a][i];
18         if(x!=p || cnt==1){
19             if(vis[x]==0){
20                 dfs(x, a);
21                 if(lwn[x]>vis[a]){
22                     int fa=Find(x);
23                     f[x]=Find(a);
24                     while(stk.back()!=x){
25                         bln[stk.back()]=fa;
26                         stk.pop_back();
27                     }
28                     bln[stk.back()]=fa;
29                     stk.pop_back();
30                 }
31                 lwn[a]=min(lwn[a], lwn[x]);
32             }
33             else{
34                 lwn[a]=min(lwn[a], vis[x]);
35             }
36         }
37         else{
38             cnt++;
39         }
40     }
41 }

```

## 5.12 MinimumSteinerTree

```

1 // Minimum Steiner Tree
2 //  $O(V^3 \log T + V^2 \log T)$ 
3 struct SteinerTree{
4     #define V 33
5     #define T 8
6     #define INF 1023456789
7     int n, dst[V][V], dp[1<<T][V], tdst[V];
8     void init(int _n){
9         n = _n;
10        for(int i = 0; i < n; i++){
11            for(int j = 0; j < n; j++){
12                dst[i][j] = INF;
13                dst[i][i] = 0;
14            }
15        }
16        void add_edge(int ui, int vi, int wi){
17            dst[ui][vi] = min(dst[ui][vi], wi);
18            dst[vi][ui] = min(dst[vi][ui], wi);
19        }
20        void shortest_path(){
21            for(int k = 0; k < n; k++){
22                for(int i = 0; i < n; i++){
23                    for(int j = 0; j < n; j++){
24                        dst[i][j] = min(dst[i][j],
25                            dst[i][k] + dst[k][j]);
26                    }
27                }
28            }
29            int solve(const vector<int>& ter){
30                int t = (int)ter.size();
31                for(int i = 0; i < (1<<t); i++){
32                    for(int j = 0; j < n; j++){
33                        dp[i][j] = INF;
34                    }
35                    for(int msk = 1; msk < (1<<t); msk++){
36                        if(msk == (msk & (-msk))){
37                            int who = __lg(msk);
38                            for(int i = 0; i < n; i++){
39                                dp[msk][i] = dst[ter[who]][i];
40                            }
41                        }
42                    }
43                }
44            }
45        }
46    };

```

```

39        continue;
40    }
41    for(int i = 0; i < n; i++){
42        for(int submsk = (msk - 1) & msk; submsk
43            ;
44            submsk = (submsk - 1) & msk){
45            dp[msk][i] = min(dp[msk][i],
46                dp[submsk][i] +
47                dp[msk ^ submsk][i]);
48        }
49        for(int i = 0; i < n; i++){
50            tdst[i] = INF;
51            for(int j = 0; j < n; j++){
52                tdst[i] = min(tdst[i],
53                    dp[msk][j] + dst[j][i]);
54            }
55        }
56        for(int i = 0; i < n; i++){
57            dp[msk][i] = tdst[i];
58        }
59        int ans = INF;
60        for(int i = 0; i < n; i++){
61            ans = min(ans, dp[(1<<t) - 1][i]);
62        }
63        return ans;
64    }
65    } solver;

```

## 6 JAVAAndPy

### 6.1 Big Integer

```

1 import java.math.*;
2 import java.io.*;
3 import java.util.*;
4 public class Main{
5     public static void main(String []argv){
6         c[0][0]=BigInteger.ONE;
7         for(int i=1; i<3001; i++){
8             c[i][0]=BigInteger.ONE;
9             c[i][i]=BigInteger.ONE;
10            for(int j=1; j<i; j++){
11                c[i][j]=c[i-1][j].
12                add(c[i-1][j-1]);
13            }
14            Scanner scanner = new Scanner(System.in);
15            int T = scanner.nextInt();
16            BigInteger x;
17            BigInteger ans;
18            while(T-- > 0){
19                ans = BigInteger.ZERO;
20                int n = scanner.nextInt();
21                for(int i=0; i<n; i++){
22                    x = new BigInteger(scanner.next());
23                    if(i%2 == 1) ans=ans.subtract(c[n-1][i].multiply(x));
24                    else ans=ans.add(c[n-1][i].multiply(x));
25                }
26                if(n%2 == 0) ans=BigInteger.ZERO.subtract(ans);
27                System.out.println(ans);
28            }
29        }
30    }

```

### 6.2 Fraction Limit

```

1 from fractions import Fraction
2 Fraction.limit_denominator(max_denominator=1000)

```

## 7 Other

### 7.1 Annealing

```

1 double distForAllPoints(double x, double y,
2     vector<pair<int, int>> &D) {
3     double sum = 0;
4     for(int i = D.size()-1; i >= 0; i--) {
5         sum += hypot(D[i].first - x, D[i].second - y);
6     }
7     return sum;
8 }
9 double randDouble() {
10    return (rand() % 32767) / 32767.0;
11 }

```

```

12 double annealing(vector< pair<int, int> > &D) {
13     #define S_MUL 0.6f
14     #define S_LEN 1000
15     #define T_CNT 10
16     #define E_CNT 10
17     double step = S_LEN;
18     double x[E_CNT], y[E_CNT], val[E_CNT];
19     double Lx, Ly, Rx, Ry, tx, ty, tcost;
20     Lx = Rx = D[0].first;
21     Ly = Ry = D[0].second;
22     for(int i = 0; i < D.size(); i++) {
23         Lx = min(Lx, (double)D[i].first);
24         Rx = max(Rx, (double)D[i].first);
25         Ly = min(Ly, (double)D[i].second);
26         Ry = max(Ry, (double)D[i].second);
27     }
28     for(int i = 0; i < E_CNT; i++) {
29         x[i] = randDouble() * (Rx - Lx) + Lx;
30         y[i] = randDouble() * (Ry - Ly) + Ly;
31         val[i] = distForAllPoints(x[i], y[i], D);
32     }
33     while(step > 0.1) {
34         for(int i = 0; i < E_CNT; i++) {
35             for(int j = 0; j < T_CNT; j++) {
36                 tx = x[i] + randDouble() * 2 * step - step;
37                 ty = y[i] + randDouble() * 2 * step - step;
38                 tcost = distForAllPoints(tx, ty, D);
39                 if(tcost < val[i]) {
40                     val[i] = tcost, x[i] = tx, y[i] = ty;
41                 }
42             }
43         }
44         step *= S_MUL;
45     }
46     double ret = val[0];
47     for(int i = 0; i < E_CNT; i++) {
48         ret = min(ret, val[i]);
49     }
50     printf("%.0lf\n", ret);
51 }
52 int main() {
53     int testcase, N;
54     scanf("%d", &testcase);
55     while(testcase--) {
56         scanf("%d", &N);
57         vector< pair<int, int> > D;
58         int x, y;
59         for(int i = 0; i < N; i++) {
60             scanf("%d %d", &x, &y);
61             D.push_back(make_pair(x, y));
62         }
63         annealing(D);
64         if(testcase)
65             puts("");
66     }
67     return 0;
68 }

```

## 7.2 MahattanMST

```

1 #include<bits/stdc++.h>
2 #define REP(i,n) for(int i=0;i<n;i++)
3 using namespace std;
4 typedef long long LL;
5 const int N=200100;
6 int n,m;
7 struct PT {int x,y,z,w,id;}p[N];
8 inline int dis(const PT &a,const PT &b){return abs(a.x-b.x)+abs(a.y-b.y);}
9 inline bool cpx(const PT &a,const PT &b){return a.x!=b.x;}
10 inline bool cpy(const PT &a,const PT &b){return a.y!=b.y;}
11 inline bool cpz(const PT &a,const PT &b){return a.z<b.z;}
12 ;}
13 struct E{int a,b,c;}e[8*N];
14 bool operator<(const E&a,const E&b){return a.c<b.c;}
15 struct Node{
16     int L,R,key;
17 }node[4*N];
18 int s[N];
19 int F(int x){return s[x]==x?s[x]:F(s[x]);}
20 void U(int a,int b){s[F(b)]=F(a);}

```

```

21 void init(int id,int L,int R) {
22     node[id]=(Node){L,R,-1};
23     if(L==R)return;
24     ;
25     init(id*2,L,(L+R)/2);
26     init(id*2+1,(L+R)/2+1,R);
27 }
28 void ins(int id,int x) {
29     if(node[id].key==-1 || p[node[id].key].w>p[x].w)
30         node[id].key=x;
31     if(node[id].L==node[id].R)return;
32     ;
33     if(p[x].z<=(node[id].L+node[id].R)/2)ins(id*2,x);
34     else ins(id*2+1,x);
35 }
36 int Q(int id,int L,int R){
37     if(R<node[id].L || L>node[id].R)return -1;
38     if(L<=node[id].L && node[id].R<=R)return node[id].key;
39     int a=Q(id*2,L,R),b=Q(id*2+1,L,R);
40     if(b==-1 || (a!=-1 && p[a].w<p[b].w)) return a;
41     else return b;
42 }
43 void calc() {
44     REP(i,n) {
45         p[i].z=p[i].y-p[i].x;
46         p[i].w=p[i].x+p[i].y;
47     }
48     sort(p,p+n,cpz);
49     int cnt=0,j,k;
50     for
51     (int i=0;i<n;i=j){
52         for(j=i+1;p[j].z==p[i].z && j<n;j++);
53         for(k=i,cnt++;k<j;k++)p[k].z=cnt;
54     }
55     init(1,1,cnt);
56     sort(p,p+n,cpx);
57     REP(i,n) {
58         j=Q(1,p[i].z,cnt);
59         if(j!=-1)e[m++]=(E){p[i].id,p[j].id,dis(p[i],p[j])};
60     }
61     ins(1,i);
62 }
63 }
64 LL MST() {
65     LL r=0;
66     sort(e,e+m);
67     REP(i,m) {
68         if(F(e[i].a)==F(e[i].b))continue;
69         U(e[i].a,e[i].b);
70         r+=e[i].c;
71     }
72     return r;
73 }
74 int main(){
75     int ts;
76     scanf("%d", &ts);
77     while (ts--) {
78         m = 0;
79         scanf("%d",&n);
80         REP(i,n) {scanf("%d%d",&p[i].x,&p[i].y);p[i].id=s[i]=i;}
81         calc();
82         REP(i,n)p[i].y=-p[i].y;
83         calc();
84         REP(i,n)swap(p[i].x,p[i].y);
85         calc();
86         REP(i,n)p[i].x=-p[i].x;
87         calc();
88         printf("%lld\n",MST()*2);
89     }
90     return 0;
91 }

```

## 7.3 MoOnTree

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define IOS ios_base::sync_with_stdio(0); cin.tie(0);

```



```

4 #define SZ(x) ((int)((x).size()))
5 const int MX = 500005;
6 const int SQ = 1400;
7 const int LOG = 17;
8 struct BIT {
9     int bit[MX];
10    int lb(int x) { return x & -x; }
11    void add(int p, int v) {
12        p++;
13        for (int i=p; i<MX; i+=lb(i)) bit[i] += v;
14    }
15    int qry() {
16        int v = 0;
17        for (int i=1<<LOG; i>0; i>=>=1) {
18            if ((v|i) < MX and bit[v|i]==i) v |= i;
19        }
20        return v;
21    }
22 }bit;
23 struct Query {
24     int l,r,qid;
25 }qry[MX];
26 struct Edge {
27     int v,x;
28 };
29 int N,Q,timestamp[MX],ans[MX];
30 int in[MX],cnt[MX];
31 vector<Edge> E[MX];
32 vector<Edge> seq;
33 void DFS(int u, int f) {
34     timestamp[u] = SZ(seq);
35     for (auto it:E[u]) {
36         if (it.v == f) continue;
37         seq.push_back(it);
38         DFS(it.v,u);
39         seq.push_back(it);
40     }
41 }
42 void poke(int id) {
43     int v = seq[id].v;
44     int x = seq[id].x;
45     in[v] ^= 1;
46     cnt[x] += in[v] ? 1 : -1;
47     if (in[v] and cnt[x] == 1) bit.add(x, 1);
48     if (!in[v] and cnt[x] == 0) bit.add(x, -1);
49 }
50 int main() {
51     IOS;
52     cin >> N >> Q;
53     for (int i=0; i<N-1; i++) {
54         int u,v,x;
55         cin >> u >> v >> x;
56         x = min(x,N);
57         E[u].push_back({v,x});
58         E[v].push_back({u,x});
59     }
60     DFS(1,1);
61     for (int i=1; i<=Q; i++) {
62         int u,v;
63         cin >> u >> v;
64         int l = timestamp[u], r = timestamp[v];
65         if (l > r) swap(l,r);
66         r--;
67         qry[i] = {l,r,i};
68     }
69     sort(qry+1,qry+1+Q, [](Query a, Query b) {
70         return make_pair(a.l/SQ,a.r) < make_pair(b.l/SQ,b.r);
71     });
72     int curL = 1, curR = 0;
73     for (int i=1; i<=Q; i++) {
74         int ql=qry[i].l,qr=qry[i].r;
75         while (curL > ql) poke(--curL);
76         while (curR < qr) poke(++curR);
77         while (curL < ql) poke(curL++);
78         while (curR > qr) poke(curR--);
79         ans[qry[i].qid] = bit.qry();
80     }
81     for (int i=1; i<=Q; i++) cout << ans[i] << "\n";
82     return 0;
83 }
84 }

```

## 7.4 Dp Optimizer

```

1 list<int> mylist;
2 vector<list<int>::iterator> v;
3 vector<int> vis;
4 int search(list<int>::iterator x,int i,int n){
5     int Max=n+1,Min=*x;
6     list<int>::iterator last=x;
7     last--;
8     while(Max>Min+1){
9         int mid=(Max+Min)/2;
10        int a=*last,b=*x;
11        int val1=dp[a][i-1]+cost[a+1][mid],val2=dp[b][i-1]+cost[b+1][mid];
12        if(val1>=val2)Max=mid;
13        else Min=mid;
14    }
15    return Max;
16 }
17 priority_queue<pii,vector<pii>,greater<pii> > pq;
18 for(int i=2;i<=m;i++){
19     mylist.clear();mylist.pb(i-1);
20     v.clear();v.resize(n);
21     vis.clear();vis.resize(n+1,0);
22     for(int j=i;j<=n;j++){
23         while(!pq.empty()&&pq.top().x<=j){
24             pii p=pq.top();pq.pop();
25             if(vis[p.y])continue;
26             auto it=v[p.y];it--;
27             vis[*it]=1;mylist.erase(it);
28             if(v[p.y]!=mylist.begin())
29                 pq.push(mp(search(v[p.y],i,n),p.y));
30         }
31         int opt=mylist.front();
32         dp[j][i]=dp[opt][i-1]+cost[opt+1][j];
33         mylist.push_back(j);
34         v[j]=mylist.end();v[j]--;
35         pq.push(mp(search(v[j],i,n),j));
36     }
37     while(!pq.empty())pq.pop();
38 }

```

## 7.5 Det

```

1 LL det(LL a[][20],int n)
2 {
3     LL ret=1;
4     for(int i=1;i<n;i++)
5     {
6         for(int j=i+1;j<n;j++)
7             while(a[j][i])
8             {
9                 LL t=a[i][i]/a[j][i];
10                for(int k=i;k<n;k++)
11                    a[i][k]=a[i][k]-a[j][k]*t;
12                for(int k=i;k<n;k++)
13                    swap(a[i][k],a[j][k]);
14                ret=-ret;
15            }
16        if(a[i][i]==0)return 0;
17        ret=ret*a[i][i];
18    }
19    return ret;
20 }
21 }

```

## 8 String

### 8.1 AC

```

1 struct Node{
2     Node *index[30];
3     Node *fail;
4     int word;
5     int num;
6     Node(){
7         for(int i=0;i<30;i++)
8             index[i]=NULL;
9         fail=NULL;
10        word=0;
11        num=-1;
12    }

```

```

13 }*root=new Node();
14 void add(char c[]){
15     Node *n=root;
16     for(int i=0;c[i]!='\0';i++){
17         if(!n->index[c[i]-'a'])
18             n->index[c[i]-'a']=new Node();
19         n=n->index[c[i]-'a'];
20     }
21     n->word=1;
22     n->num=t++;
23 }
24 void ac(){
25     queue<Node*> q;
26     q.push(root);
27     root->fail=NULL;
28     while(!q.empty()){
29         Node *n=q.front();
30         q.pop();
31         for(int i=0;i<30;i++){
32             if(n->index[i]){
33                 q.push(n->index[i]);
34                 Node* p=n->fail;
35                 while(p!=NULL&&!p->index[i])
36                     p=p->fail;
37                 if(p)
38                     n->index[i]->fail=p->index[i];
39                 else
40                     n->index[i]->fail=root;
41             }
42         }
43     }
44 }
45 void search(char c[]){
46     Node *n=root;
47     for(int i=0;c[i]!='\0';i++){
48         while(!n->index[c[i]-'a']&&n!=root){
49             n=n->fail;
50         }
51         if(n->index[c[i]-'a'])
52             n=n->index[c[i]-'a'];
53         Node *p=n;
54         while(p){
55             if(p->num!=-1)
56                 ans[p->num]++;
57             p=p->fail;
58         }
59     }
60 }
61 void del(Node *n=root){
62     for(int i=0;i<30;i++){
63         if(n->index[i])
64             del(n->index[i]);
65         free(n);
66     }
67 }

```

## 8.2 SuffixAutomata

```

1 // BZOJ 3998
2 const int MAX_N = 500000 + 10;
3 struct Node {
4     static Node mem[MAX_N<<1] , *pmem;
5     Node *ch[26] , *fail;
6     int mx , val;
7     ll dp;
8     int tag , deg;
9     Node():mx(0),fail(0),dp(0),val(0),tag(0),deg(0){
10         MS(ch , 0);
11     }
12 }
13 Node::mem[MAX_N<<1] , *Node::pmem = Node::mem , *
14     root
15     , *last;
16 int T , N;
17 char s[MAX_N];
18 inline void init() {
19     last = root = new (Node::pmem++)Node();
20 }
21 inline int idx(char c) {
22     return c - 'a';
23 }

```

```

22 }
23 inline void insert(char c) {
24     c = idx(c);
25     Node *p = last;
26     Node *np = new (Node::pmem++)Node();
27     np->mx = p->mx + 1;
28     np->val = 1;
29     while(p && !p->ch[c]) {
30         p->ch[c] = np;
31         np->deg++;
32         p = p->fail;
33     }
34     if(!p) np->fail = root;
35     else
36     {
37         Node *q = p->ch[c];
38         if(q->mx == p->mx + 1) np->fail = q;
39         else
40         {
41             Node *nq = new (Node::pmem++)Node();
42             nq->mx = p->mx + 1;
43             nq->val = 0;
44             memcpy(nq->ch , q->ch , sizeof(q->ch));
45             REP(i , 26) {
46                 if(nq->ch[i]) nq->ch[i]->deg++;
47             }
48             nq->fail = q->fail;
49             q->fail = np->fail = nq;
50             while(p && p->ch[c] == q) {
51                 p->ch[c] = nq;
52                 q->deg--;
53                 nq->deg++;
54                 p = p->fail;
55             }
56         }
57     }
58     last = np;
59 }
60 inline void bfs() {
61     static Node* que[MAX_N<<1];
62     int l = 0 , r = 0;
63     que[r++] = root;
64     root->tag = 2;
65     vector<Node*> vec;
66     while(l < r) {
67         Node *u = que[l++];
68         REP(i , 26) {
69             if(u->ch[i]) {
70                 if(--u->ch[i]->deg == 0 && u->ch[i]
71 ]->
72                 tag != 1) {
73                     u->ch[i]->tag = 1;
74                     que[r++] = u->ch[i];
75                     vec.PB(u->ch[i]);
76                 }
77             }
78         }
79     }
80     for(int i = SZ(vec) - 1; i >= 0; i--) {
81         Node *u = vec[i];
82         if(T) {
83             if(u->fail) u->fail->val += u->val;
84             else u->val = 1;
85         }
86     }
87     root->val = 0;
88     for(int i = SZ(vec) - 1; i >= 0; i--) {
89         Node *u = vec[i];
90         u->dp = u->val;
91         REP(j , 26) {
92             if(u->ch[j]) u->dp += u->ch[j]->dp;
93         }
94     }
95     REP(i , 26) {
96         if(root->ch[i]) root->dp += root->ch[i]->dp;
97     }
98 }
99 inline void solve(int k) {
100     Node *p = root;
101     if(k > p->dp || k <= 0) {
102         puts("-1");
103         return;
104     }

```

```

103     }
104     while(k > 0) {
105         int flag = 0;
106         REP(i, 26) {
107             if(!p->ch[i]) continue;
108             if(k <= p->ch[i]->dp) {
109                 putchar('a' + i);
110                 k -= p->ch[i]->val;
111                 p = p->ch[i];
112                 flag = 1;
113                 break;
114             }
115             else k -= p->ch[i]->dp;
116         }
117         if(!flag) break;
118     }
119 }
120 }
121 int main() {
122     scanf("%s", s);
123     int n = strlen(s);
124     N = n;
125     init();
126     REP(i, n) insert(s[i]);
127     int K;
128     scanf("%d", &T, &K);
129     bfs();
130     solve(K);
131     return 0;
132 }

```

### 8.3 Palindromic Tree

```

1 #include<bits/stdc++.h>
2 #include<unistd.h>
3 using namespace std;
4 #define F first
5 #define S second
6 #define MP make_pair
7 #define PB push_back
8 #define IOS ios_base::sync_with_stdio(0); cin.tie(0);
9 #define SZ(x) ((int)((x).size()))
10 #define ALL(x) begin(x), end(x)
11 #define REP(i, x) for (int i=0; i<(x); i++)
12 #define REP1(i, a, b) for (int i=(a); i<=(b); i++)
13
14 struct palindromic_tree{
15     struct node{
16         int next[26], fail, len;
17         int cnt, num, st, ed;
18         node(int l=0):fail(0), len(l), cnt(0), num(0){
19             for(int i=0; i<26; ++i) next[i]=0;
20         }
21     };
22     vector<node> state;
23     vector<char> s;
24     int last, n;
25
26     void init(){
27         state.clear();
28         s.clear();
29         last=1;
30         n=0;
31         state.push_back(0);
32         state.push_back(-1);
33         state[0].fail=1;
34         s.push_back(-1);
35     }
36     int get_fail(int x){
37         while(s[n-state[x].len-1]!=s[n])x=state[x].fail;
38         return x;
39     }
40     void add(int c){
41         s.push_back(c-'a');
42         ++n;
43         int cur=get_fail(last);
44         if(!state[cur].next[c]){
45             int now=state.size();
46             state.push_back(state[cur].len+2);
47             state[now].fail=state[get_fail(state[cur].fail
48             )].next[c];
49             state[cur].next[c]=now;

```

```

49         state[now].num=state[state[now].fail].num+1;
50     }
51     last=state[cur].next[c];
52     ++state[last].cnt;
53 }
54 int size(){
55     return state.size()-2;
56 }
57 }pt;
58
59 int main() {
60     string s;
61     cin >> s;
62     pt.init();
63     for (int i=0; i<SZ(s); i++) {
64         int prvsz = pt.size();
65         pt.add(s[i]);
66         if (prvsz != pt.size()) {
67             int r = i;
68             int l = r - pt.state[pt.last].len + 1;
69             cout << "Find pal @ [" << l << " " << r << "]"
70             : " << s.substr(l, r-l+1) << endl;
71         }
72     }
73     return 0;
74 }

```

### 8.4 MinLexicographicalRotate

```

1 string mcp(string s){
2     int n = s.length();
3     s += s;
4     int i=0, j=1;
5     while (i<n && j<n){
6         int k = 0;
7         while (k < n && s[i+k] == s[j+k]) k++;
8         if (s[i+k] <= s[j+k]) j += k+1;
9         else i += k+1;
10        if (i == j) j++;
11    }
12    int ans = i < n ? i : j;
13    return s.substr(ans, n);
14 }

```

### 8.5 ZvaluePalindromes

```

1 inline void manacher(char *s, int len, int *z){
2     int l=0, r=0;
3     for(int i=1; i<len; ++i){
4         z[i]=r>i?min(z[2*l-i], r-i):1;
5         while(s[i+z[i]]==s[i-z[i]]) ++z[i];
6         if(z[i]+i>r)r=z[i]+i, l=i;
7     }
8 }

```

### 8.6 SuffixArray

```

1 int ss[N];
2 int heigh[N];
3 int sa[N];
4 int rank[N];
5 int length;
6 int val[30];
7 int c[N]; // counting sort array
8 int temp[2][N];
9 void suffix_array()
10 {
11     int A = 250;
12     int* rank = temp[0];
13     int* new_rank = temp[1];
14     for (int i=0; i<A; ++i) c[i] = 0;
15     for (int i=0; i<length; ++i) c[rank[i] = ss[i]]++;
16     for (int i=1; i<A; ++i) c[i] += c[i-1];
17     for (int i=length-1; i>=0; --i) sa[--c[ss[i]]] = i;
18     for (int n=1; n<length; n*=2)
19     {
20         for (int i=0; i<A; ++i) c[i] = 0;
21         for (int i=0; i<length; ++i) c[rank[i]]++;
22         for (int i=1; i<A; ++i) c[i] += c[i-1];
23         int* sa2 = new_rank;
24         int r = 0;

```

```

25     for (int i=length-n; i<length; ++i)
26         sa2[r++] = i;
27     for (int i=0; i<length; ++i)
28         if (sa[i] >= n)
29             sa2[r++] = sa[i] - n;
30     for (int i=length-1; i>=0; --i)
31         sa[-c[rank[sa2[i]]]] = sa2[i];
32     new_rank[sa[0]] = r = 0;
33     for (int i=1; i<length; ++i)
34     {
35         if (!(rank[sa[i-1]] == rank[sa[i]] &&
36             sa[i-1]+n < length && // stable
37             rank[sa[i-1]+n] == rank[sa[i]+n]))
38             r++;
39         new_rank[sa[i]] = r;
40     }
41     swap(rank, new_rank);
42     if (r == length-1) break;
43     A = r + 1;
44 }
45 }
46 void lcp_array()
47 {
48     for (int i=0; i<length; ++i)
49         rank[sa[i]] = i;
50
51     for (int i=0, lcp=0, h=0; i<length; i++)
52         if (rank[i] == 0)
53             heigh[0] = 0;
54         else
55         {
56             int j = sa[rank[i]-1];
57             if (lcp > 0) lcp=val[ss[i-1]-'a'], h--;
58             while (ss[i+h] == ss[j+h]) lcp+=val[ss[i+h]-'a'], h++;
59             heigh[rank[i]] = lcp;
60         }
61 }

```

## 8.7 Zvalue

```

1 inline void z_alg1(char *s, int len, int *z){
2     int l=0, r=0;
3     z[0]=len;
4     for(int i=1; i<len; ++i){
5         z[i]=r>i?min(r-i+1, z[z[l]-(r-i+1)]):0;
6         while(i+z[i]<len&&s[z[i]]==s[i+z[i]])++z[i];
7         if(i+z[i]-1>r)r=i+z[i]-1, l=i;
8     }
9 }

```

## 9 Math

### 9.1 MillerRabin

```

1 // 4759123141 2, 7, 61
2 // 2^64 2, 325, 9375, 28178, 450775, 9780504,
3 // 1795265022
4 bool Isprime(LL n)
5 {
6     if (n == 2) return true;
7     if (n < 2 || n % 2 == 0) return false;
8     LL u = n - 1, t = 0;
9     while (u % 2 == 0) {u >>= 1; t++;}
10    LL sprp[7] = {2, 325, 9375, 28178, 450775,
11    9780504, 1795265022};
12    for (int k=0; k<7; ++k)
13    {
14        LL a = sprp[k] % n;
15        if (a == 0 || a == 1 || a == n-1) continue;
16        long long x = f_pow(a, u, n);
17        if (x == 1 || x == n-1) continue;
18        for (int i = 0; i < t-1; i++)
19        {
20            x = f_pow(x, 2, n);
21            if (x == 1) return false;
22            if (x == n-1) break;
23        }
24        if (x == n-1) continue;
25        return false;
26    }
27    return true;
28 }

```

### 9.2 Simplex

```

1 const int maxn = 111;
2 const int maxm = 111;
3 const double eps = 1E-10;
4
5 double a[maxn][maxm], b[maxn], c[maxn], d[maxn][maxm];
6 double x[maxn];
7 int ix[maxn + maxm]; // !!! array all indexed from 0
8 // max{cx} subject to {Ax<=b, x>=0}
9 // n: constraints, m: vars !!!
10 // x[] is the optimal solution vector
11 //
12 // usage :
13 // value = simplex(a, b, c, N, M);
14 double simplex(double a[maxn][maxm], double b[maxn],
15     double c[maxn], int n, int m) {
16     ++m;
17     int r = n, s = m - 1;
18     memset(d, 0, sizeof(d));
19     for (int i = 0; i < n + m; ++i) ix[i] = i;
20     for (int i = 0; i < n; ++i) {
21         for (int j = 0; j < m - 1; ++j)
22             d[i][j] = -a[i][j];
23         d[i][m - 1] = 1;
24         d[i][m] = b[i];
25         if (d[r][m] > d[i][m]) r = i;
26     }
27     for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
28     d[n + 1][m - 1] = -1;
29     for (double dd;; ) {
30         if (r < n) {
31             int t = ix[s];
32             ix[s] = ix[r + m]; ix[r + m] = t;
33             d[r][s] = 1.0 / d[r][s];
34             for (int j = 0; j <= m; ++j)
35                 if (j != s) d[r][j] *= -d[r][s];
36             for (int i = 0; i <= n + 1; ++i)
37                 if (i != r) {
38                     for (int j = 0; j <= m; ++j)
39                         if (j != s)
40                             d[i][j] += d[r][j]*d[i][s];
41                     d[i][s] *= d[r][s];
42                 }
43             r = -1; s = -1;
44             for (int j = 0; j < m; ++j)
45                 if (s < 0 || ix[s] > ix[j]) {
46                     if (d[n + 1][j] > eps || (d[n + 1][j]
47                     ] > -eps && d[n][j] > eps)) s = j;
48                 }
49             if (s < 0) break;
50             for (int i=0; i<n; ++i) if (d[i][s] < -eps)
51             {
52                 if (r < 0 || (dd = d[r][m] / d[r][s] - d
53                 [i][m] / d[i][s]) < -eps || (dd < eps && ix[r +
54                 m] > ix[i + m])) r = i;
55             }
56             if (r < 0) return -1; // not bounded
57         }
58         if (d[n + 1][m] < -eps) return -1; // not
59         executable
60         double ans = 0;
61         for(int i=0; i<m; i++) x[i] = 0;
62         for (int i = m; i < n + m; ++i) { // the missing
63             enumerated x[i] = 0
64             if (ix[i] < m - 1)
65             {
66                 ans += d[i - m][m] * c[ix[i]];
67                 x[ix[i]] = d[i-m][m];
68             }
69         }
70         return ans;
71     }
72 }

```

### 9.3 Theorem

```

1 /*
2 Lucas's Theorem:
3 For non-negative integer n,m and prime P,
4 C(m,n) mod P = C(m/P,n/P) * C(m%P,n%P) mod P
5 -----

```

```

6 Pick's Theorem
7 A = i + b/2 - 1
8 */

```

## 9.4 Rombeg

```

1 // Estimates the definite integral of
2 // \int_a^b f(x) dx
3 template<class T>
4 double romberg( T& f, double a, double b, double eps
5                 =1e-8){
6     vector<double> t; double h=b-a, last, curr; int k=1, i
7     =1;
8     t.push_back(h*(f(a)+f(b))/2);
9     do{ last=t.back(); curr=0; double x=a+h/2;
10        for(int j=0; j<k; j++) curr+=f(x), x+=h;
11        curr=(t[0] + h*curr)/2; double k1=4.0/3.0, k2
12        =1.0/3.0;
13        for(int j=0; j<i; j++){ double temp=k1*curr-k2*t[j]
14        ];
15        t[j]=curr; curr=temp; k2/=4*k1-k2; k1=k2+1;
16        } t.push_back(curr); k*=2; h/=2; i++;
17    }while( fabs(last-curr) > eps);
18    return t.back();
19 }

```

## 9.5 SchreierSims

```

1 namespace SchreierSimsAlgorithm{
2     typedef vector<int> Permu;
3     Permu inv( const Permu& p ){
4         Permu ret( p.size() );
5         for( int i = 0; i < int(p.size()); i ++ )
6             ret[ p[ i ] ] = i;
7         return ret;
8     }
9     Permu operator*( const Permu& a, const Permu& b ){
10        Permu ret( a.size() );
11        for( int i = 0; i < (int)a.size(); i ++ )
12            ret[ i ] = b[ a[ i ] ];
13        return ret;
14    }
15    typedef vector<Permu> Bucket;
16    typedef vector<int> Table;
17    typedef pair<int,int> pii;
18    int n, m;
19    vector<Bucket> bkts, bktsInv;
20    vector<Table> lookup;
21    int fastFilter( const Permu &g, bool addToG = 1 ){
22        n = bkts.size();
23        Permu p;
24        for( int i = 0; i < n; i ++ ){
25            int res = lookup[ i ][ p[ i ] ];
26            if( res == -1 ){
27                if( addToG ){
28                    bkts[ i ].push_back( p );
29                    bktsInv[ i ].push_back( inv( p ) );
30                    lookup[ i ][ p[ i ] ] = (int)bkts[i].size()
31                    -1;
32                }
33                return i;
34            }
35            p = p * bktsInv[i][res];
36        }
37        return -1;
38    }
39    long long calcTotalSize(){
40        long long ret = 1;
41        for( int i = 0; i < n; i ++ )
42            ret *= bkts[i].size();
43        return ret;
44    }
45    bool inGroup( const Permu &g ){
46        return fastFilter( g, false ) == -1;
47    }
48    void solve( const Bucket &gen, int _n ){
49        n = _n, m = gen.size(); // m perm[0..n-1]s
50        //clear all
51        bkts.clear();
52        bktsInv.clear();
53        lookup.clear();
54        for(int i = 0; i < n; i ++ ){

```

```

55            lookup[i].resize(n);
56            fill(lookup[i].begin(), lookup[i].end(), -1);
57        }
58        Permu id( n );
59        for(int i = 0; i < n; i ++ ) id[i] = i;
60        for(int i = 0; i < n; i ++ ){
61            bkts[i].push_back(id);
62            bktsInv[i].push_back(id);
63            lookup[i][i] = 0;
64        }
65        for(int i = 0; i < m; i ++ )
66            fastFilter( gen[i] );
67        queue< pair<pii,pii> > toUpd;
68        for(int i = 0; i < n; i ++ )
69            for(int j = i; j < n; j ++ )
70                for(int k = 0; k < (int)bkts[i].size(); k
71                ++ )
72                    for(int l = 0; l < (int)bkts[j].size(); l
73                    ++ )
74                        toUpd.push( {pii(i,k), pii(j,l)} );
75        while( !toUpd.empty() ){
76            pii a = toUpd.front().first;
77            pii b = toUpd.front().second;
78            toUpd.pop();
79            int res = fastFilter(bkts[a.first][a.second] *
80                                bkts[b.first][b.second]);
81            if(res == -1) continue;
82            pii newPair(res, (int)bkts[res].size() - 1);
83            for(int i = 0; i < n; i ++ )
84                for(int j = 0; j < (int)bkts[i].size(); ++j)
85                    if(i <= res)
86                        toUpd.push(make_pair(pii(i, j), newPair
87                        ));
88                    if(res <= i)
89                        toUpd.push(make_pair(newPair, pii(i, j))
90                    );
91        }
92    }
93 }

```

## 9.6 Prime

```

1 /*
2 * 12721
3 * 13331
4 * 14341
5 * 75577
6 * 123457
7 * 222557
8 * 556679
9 * 999983
10 * 1097774749
11 * 1076767633
12 * 100102021
13 * 999997771
14 * 1001010013
15 * 1000512343
16 * 987654361
17 * 999991231
18 * 999888733
19 * 98789101
20 * 987777733
21 * 999991921
22 * 1010101333
23 * 1010102101
24 * 1000000000039
25 * 100000000000037
26 * 2305843009213693951
27 * 4611686018427387847
28 * 9223372036854775783
29 * 18446744073709551557
30 */

```

## 9.7 FFT

```

1 #define N 524288
2 #define pi acos(-1)
3 typedef complex<double> C;
4 int n,m,i,t,g[N];
5 C a[N],b[N];
6 void FFTinit(){

```



```

7   for (i=1;i<N;i++) g[i]=g[i>>1]>>1((i&1)<<18);
8   }
9   void FFT(C *a,int f)
10  {
11      int i,j,k,p;
12      for (i=0;i<N;i++)
13          if (g[i]>i) swap(a[i],a[g[i]]);
14      for (i=1;i<N;i<=1)
15      {
16          C e(cos(pi/i),f*sin(pi/i));
17          for (j=0;j<N;j+=i<<1)
18          {
19              C w(1,0);for (k=0;k<i;k++,w*=e)
20              {
21                  C x=a[j+k],y=w*a[j+k+i];
22                  a[j+k]=x+y;a[j+k+i]=x-y;
23              }
24          }
25      }
26  }
27  int res[400005];
28  int main()
29  {
30      FFTinit();
31      FFT(a,1);
32      FFT(b,1);
33      for(i=0;i<N;i++) a[i]=a[i]*b[i];
34      FFT(a,-1);
35      for (i=0;i<n+m;i++)
36          (int)a[i].real()/N+0.5)
37  }

```

## 9.8 NTT

```

1   int P=998244353,root=3,MAXNUM=1<<23;
2   // Remember coefficient are mod P
3   /*
4   p=a*2^n+1 degree(poly) <= 2^n
5   n   2^n   p   a   root
6   16   65536   65537   1   3
7   20   1048576   7340033   7   3
8   23   998244353   119
9   */
10  int bigmod(long long a,int b){
11      if(b==0)return 1;
12      return (bigmod((a*a)%P,b/2)*(b%2?a:11))%P;
13  }
14  int inv(int a,int b){
15      if(a==1)return 1;
16      return (((long long)(a-inv(b%a,a))*b+1)/a)%b;
17  }
18  std::vector<long long> ps(MAXNUM);
19  std::vector<int> rev(MAXNUM);
20  LL f_pow(unsigned int a,LL b){
21      LL res=1,temp=a;
22      while(b){
23          if(b&1)res=res*temp%P;
24          temp=temp*temp%P;
25          b>>=1;
26      }
27      return res;
28  }
29  struct poly{
30      std::vector<unsigned int> co;
31      int n;//polynomial degree = n
32      poly(int d){n=d;co.resize(n+1,0);}
33      void ntt(int NN){
34          int r=0,st,N;
35          unsigned int a,b;
36          while((1<<r)<(NN>>1))++r;//inv:r=0
37          for(N=2;N<=NN;N<=1,--r){
38              //inv for(N=NN;N>1;N>>=1,r++)
39              for(st=0;st<NN;st+=N){
40                  int i,ss=st+(N>>1);
41                  for(i=(N>>1)-1;i>=0;--i){
42                      a=co[st+i]; b=(ps[i<<r]*co[ss+i])%P;
43                      //inv b=co[ss+i];
44                      co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
45                      co[ss+i]=a-P+b; if(co[ss+i]>=P)co[ss+i]-=P;
46                      //inv co[ss+i]=((a+P-b)*ps[i<<r
47                      ])%p;

```

```

47      }
48      }
49      }
50      }
51      void ntt_inv(int NN){
52      }
53      poly operator*(const poly& _b)const{
54      poly a=*this,b=_b;
55      int k=n+b.n,i,N=1;
56      while(N<=k)N*=2;
57      a.co.resize(N,0); b.co.resize(N,0);
58      int r=bigmod(root,(P-1)/N),Ni=inv(N,P);
59      ps[0]=1;
60      for(i=1;i<N;++i)ps[i]=(ps[i-1]*r)%P;
61      a.ntt(N);b.ntt(N);
62      for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*b.co
63      [i])%P;
64      r=inv(r,P);
65      for(i=1;i<N/2;++i)std::swap(ps[i],ps[N-i]);
66      a.ntt_inv(N);
67      for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*Ni)%
68      P;
69      a.n=n+_b.n; return a;
70      }
71      }
72      }

```

## 9.9 Crt Solve2

```

1   LL solve(LL x1, LL m1, LL x2, LL m2){
2       LL g = __gcd(m1, m2);
3       if((x2 - x1) % g)return -1;
4       m1 /= g; m2 /= g;
5       pll p = gcd(m1, m2);
6       LL lcm = m1 * m2 * g;
7       LL res = p.x * (x2 - x1) * m1 + x1;
8       return (res % lcm + lcm) % lcm;
9   }

```

## 9.10 DiscreteSart

```

1   void calCH(int &t, int &h, const int p) {
2       int tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
3   }
4   // solve equation x^2 mod p = a
5   bool solve(int a, int p, int &x, int &y) {
6       if(p == 2) { x = y = 1; return true; }
7       int p2 = p / 2, tmp = mypow(a, p2, p);
8       if (tmp == p - 1) return false;
9       if ((p + 1) % 4 == 0) {
10          x=mypow(a,(p+1)/4,p); y=p-x; return true;
11      } else {
12          int t, h, b, pb; calCH(t, h, p);
13          if (t >= 2) {
14              do {b = rand() % (p - 2) + 2;
15                  } while (mypow(b, p / 2, p) != p - 1);
16              pb = mypow(b, h, p);
17              int s = mypow(a, h / 2, p);
18              for (int step = 2; step <= t; step++) {
19                  int ss = (((LL)(s * s) % p) * a) % p;
20                  for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
21                  if (ss + 1 == p) s = (s * pb) % p;
22                  pb = ((LL)pb * pb) % p;
23              } x = ((LL)s * a) % p; y = p - x;
24          } return true;
25      }

```

## 9.11 FWT

```

1   void FWT(int *x,int inv) {
2       for(int i=1;i<lim;i<=1){
3           for(int j=0;j<lim;j+=i)/* or */
4               if(j&i)
5                   x[j]= inv ? x[j]-x[j^i] : x[j]+x[j^i]
6       ];
7       for(int j=0;j<lim;j+=(i<<1))/* and */
8           for(int k=0;k<i;k++)
9               x[j+k]=inv ? x[j+k]-x[j+k+i] : x[j+k
10      ]+x[j+k+i];
11       for(int j=0;j<lim;j+=(i<<1))/* xor */
12           for(int k=0;k<i;k++) {
13               int y=x[j+k],z=x[j+k+i];
14               x[j+k]=inv ? (y+z)/2 : y+z;
15               x[j+k+i]=inv ? (y-z)/2 : y-z;
16           }
17      }

```

## 9.12 Faulhaber

```

1 /* faulhaber 's formula -
2 * cal power sum formula of all p=1~k in O(k^2) */
3 #define MAXK 2500
4 const int mod = 1000000007;
5 int b[MAXK]; // bernoulli number
6 int inv[MAXK+1]; // inverse
7 int cm[MAXK+1][MAXK+1]; // combinactories
8 int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i
9 inline int getinv(int x) {
10     int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
11     while(b) {
12         int q, t;
13         q=a/b; t=b; b=a-b*q; a=t;
14         t=b0; b0=a0-b0*q; a0=t;
15         t=b1; b1=a1-b1*q; a1=t;
16     }
17     return a0<0?a0+mod:a0;
18 }
19 inline void pre() {
20     /* combinational */
21     for(int i=0; i<=MAXK; i++) {
22         cm[i][0]=cm[i][i]=1;
23         for(int j=1; j<i; j++)
24             cm[i][j]=add(cm[i-1][j-1], cm[i-1][j]);
25     }
26     /* inverse */
27     for(int i=1; i<=MAXK; i++) inv[i]=getinv(i);
28     /* bernoulli */
29     b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
30     for(int i=2; i<=MAXK; i++) {
31         if(i&1) { b[i]=0; continue; }
32         b[i]=1;
33         for(int j=0; j<i; j++)
34             b[i]=sub(b[i], mul(cm[i][j], mul(b[j], inv[i-j+1])));
35     }
36     /* faulhaber */
37     // sigma_x=1~n {x^p} =
38     // 1/(p+1) * sigma_j=0~p {C(p+1, j)*Bj*n^(p-j+1)}
39     for(int i=1; i<=MAXK; i++) {
40         co[i][0]=0;
41         for(int j=0; j<=i; j++)
42             co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
43     }
44 }
45 }
46 /* sample usage: return f(n,p) = sigma_x=1~n (x^p)
47 */
48 inline int solve(int n, int p) {
49     int sol=0, m=n;
50     for(int i=1; i<=p+1; i++) {
51         sol=add(sol, mul(co[p][i], m));
52         m = mul(m, n);
53     }
54     return sol;
55 }

```

## 9.13 Extgcd

```

1 typedef pair<int, int> pii;
2 pii gcd(int a, int b){
3     if(b == 0) return mp(1, 0);
4     else{
5         int p = a / b;
6         pii q = gcd(b, a % b);
7         return make_pair(q.y, q.x - q.y * p);
8     }
9 }

```

## 9.14 Pollard'sRho

```

1 // does not work when n is prime
2 inline LL f(LL x, LL mod) {
3     return (x * x % mod + 1) % mod;
4 }
5 inline LL pollard_rho(LL n) {
6     if(!(n&1)) return 2;
7     while(true) {
8         LL y = 2, x = rand() % (n - 1) + 1, res = 1;
9         for(int sz = 2; res == 1; sz *= 2) {
10             for(int i = 0; i < sz && res <= 1; i++) {
11                 x = f(x, n);

```

```

12         res = __gcd(abs(x - y), n);
13     }
14     y = x;
15 }
16 if (res != 0 && res != n) return res;
17 }
18 }

```

## 10 monge

$$i \leq i' < j \leq j'$$

$$m(i, j) + m(i', j') \leq m(i', j) + m(i, j')$$

$$k(i, j-1) \leq k(i, j) \leq k(i+1, j)$$

## 11 四心

$$\frac{sa \cdot A + sb \cdot B + sc \cdot C}{sa + sb + sc}$$

外心  $\sin 2A : \sin 2B : \sin 2C$

內心  $\sin A : \sin B : \sin C$

垂心  $\tan A : \tan B : \tan C$

重心  $1 : 1 : 1$

## 12 Runge-Kutta

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(t_n, y_n)$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$$

$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

## 13 Householder Matrix

$$I - 2 \frac{vv^T}{v^T v}$$

## 14 Simpson's-rule

$$\int_a^b f(x) dx = \frac{b-a}{6} (f(a) + 4f(\frac{a+b}{2}) + f(b))$$