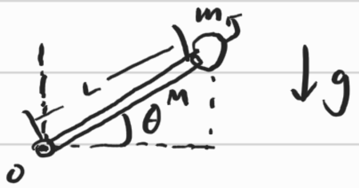
1 DoF Robot Arm Lagrangian Analysis



$$\frac{2\pi}{\omega} = T$$

1

2π

$$x_M = \frac{L}{2}\cos(\theta) \quad \dot{x}_M = -\frac{L}{2}\dot{\theta}\sin(\theta)$$

$$y_M = \frac{L}{2}\sin(\theta) \quad \dot{y}_M = \frac{L}{2}\dot{\theta}\cos(\theta)$$

$$x_m = L\cos(\theta) \quad \dot{x}_m = -L\dot{\theta}\sin(\theta)$$

$$y_m = L\sin(\theta) \quad \dot{y}_m = L\dot{\theta}\cos(\theta)$$

$$T = \underbrace{\frac{1}{2}M\left(\frac{L^2}{4}\dot{\theta}^2\right)}_{\text{Linear Energy of } M} + \underbrace{\frac{1}{2}\left(\frac{1}{12}ML^2\right)\dot{\theta}^2}_{\text{Angular Energy of } M} + \underbrace{\frac{1}{2}m\left(L^2\dot{\theta}^2\right)}_{\text{Linear Energy of } m} + \underbrace{0}_{\text{Angular Energy of } m}$$

$$U = Mg\frac{L}{2}\sin(\theta) + mgL\sin(\theta)$$

$$L = \frac{1}{6}ML^2\dot{\theta}^2 + \frac{1}{2}mL^2\dot{\theta}^2 - (M+2m)g\frac{L}{2}\sin(\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \tau$$

$$\left(\frac{ML^2\ddot{\theta}}{3} + mL^2\ddot{\theta}\right) - \left(-(M+2m)g\frac{L}{2}\cos(\theta)\right) = \tau$$

$$\left[\frac{M}{3} + m\right]L^2\ddot{\theta} + \left[\frac{M}{2} + m\right]gL\cos(\theta) = \tau$$