Here is a list of problems selected from the textbook to help you better prepare for the final exam.

HW7:

Sec# . Q#:

5.3.26

26. Let S be the set of positive integers defined by

Basis step: $1 \in S$.

Recursive step: If $n \in S$, then $3n + 2 \in S$ and $n^2 \in S$.

a) Show that if $n \in S$, then $n \equiv 1 \pmod 4$.

b) Show that there exists an integer $m \equiv 1 \pmod 4$ that does not belong to S.

5.3.48

48. Use generalized induction as was done in Example 13 to show that if $a_{m,n}$ is defined recursively by $a_{11} = 5$ and

$$a_{m,n} = \begin{cases} a_{m-1,n} + 2 & \text{if } n = 1 \text{ and } m > 1 \\ a_{m,n-1} + 2 & \text{if } n > 1, \end{cases}$$

then $a_{m,n} = 2(m + n) + 1$ for all $(m, n) \in \mathbf{Z}^+ \times \mathbf{Z}^+$.

5.3 31

31. Give a recursive definition of each of these sets of ordered pairs of positive integers. Use structural induction to prove that the recursive definition you found is correct. [*Hint:* To find a recursive definition, plot the points in the set in the plane and look for patterns.]

a) $S = \{(a, b) \mid a \in \mathbf{Z}^+, b \in \mathbf{Z}^+, \text{and } a + b \text{ is even}\}$

b) $S = \{(a, b) \mid a \in \mathbf{Z}^+, b \in \mathbf{Z}^+, \text{and } a \text{ or } b \text{ is odd}\}$

c) $S = \{(a, b) \mid a \in \mathbf{Z}^+, b \in \mathbf{Z}^+, a + b \text{ is odd, and } 3 \mid b\}$

5.3 35

35. **a)** Give a recursive definition of the function $m(s)$, which equals the smallest digit in a nonempty string of decimal digits.

b) Use structural induction to prove that $m(st) = \min(m(s), m(t))$.

5.4.12

12. Devise a recursive algorithm for finding x^n **mod** m whenever n, x, and m are positive integers based on the fact that x^n **mod** $m = (x^{n-1}$ **mod** $m \cdot x$ **mod** $m)$ **mod** m.

5.4.23

23. Devise a recursive algorithm for computing n^2 where n is a nonnegative integer, using the fact that $(n + 1)^2 = n^2 + 2n + 1$. Then prove that this algorithm is correct.

HW8:

8.2 Q9:

9. A deposit of \$100,000 is made to an investment fund at the beginning of a year. On the last day of each year two dividends are awarded. The first dividend is 20% of the amount in the account during that year. The second dividend is 45% of the amount in the account in the previous year.

a) Find a recurrence relation for $\{P_n\}$, where P_n is the amount in the account at the end of n years if no money is ever withdrawn.

b) How much is in the account after n years if no money has been withdrawn?

8.2 Q29:

29. **a)** Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 3^n$.

b) Find the solution of the recurrence relation in part (a) with initial condition $a_1 = 5$.

HW9:

9.3 Q31:

31. Determine whether the relations represented by the directed graphs shown in Exercises 23–25 are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.

23.

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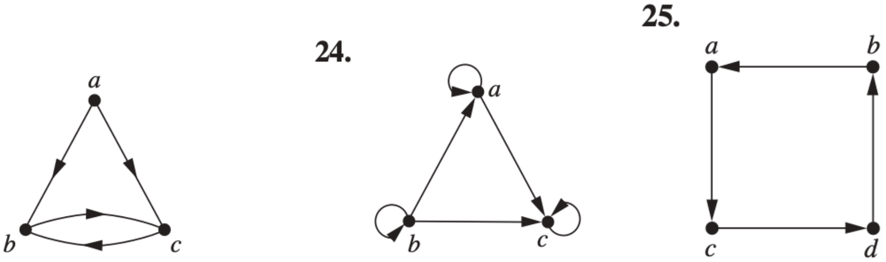
25.



HW10:

10.2 Q45

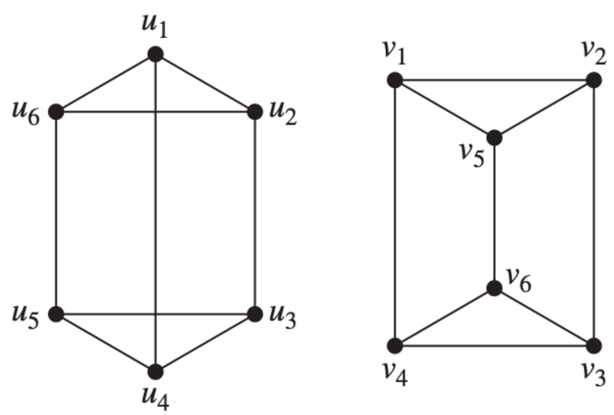
45. Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence.

a) 3, 3, 3, 3, 2 **b)** 5, 4, 3, 2, 1 **c)** 4, 4, 3, 2, 1
d) 4, 4, 3, 3, 3 **e)** 3, 2, 2, 1, 0 **f)** 1, 1, 1, 1, 1

10.3 Q43

In Exercises 38–48 determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists. For additional exercises of this kind, see Exercises 3–5 in the Supplementary Exercises.

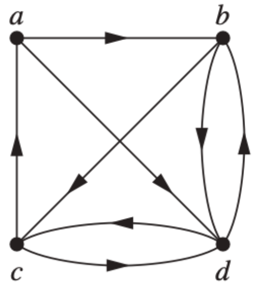
43.



10.5 Q18

In Exercises 18–23 determine whether the directed graph shown has an Euler circuit. Construct an Euler circuit if one exists. If no Euler circuit exists, determine whether the directed graph has an Euler path. Construct an Euler path if one exists.

18.



HW11:

11.1 Q15

☞ * **15.** Let G be a simple graph with n vertices. Show that

a) G is a tree if and only if it is connected and has $n - 1$ edges.

b) G is a tree if and only if G has no simple circuits and has $n - 1$ edges. [*Hint:* To show that G is connected if it has no simple circuits and $n - 1$ edges, show that G cannot have more than one connected component.]

11.1 Q30

☞ **30.** Show that a full m-ary balanced tree of height h has more than m^{h-1} leaves.

FSA:

13.3 Q23

23. Construct a deterministic finite-state automaton that recognizes the set of all bit strings beginning with 01.

13.4 Q8

8. Construct deterministic finite-state automata that recognize each of these sets from I^*, where I is an alphabet.

a) $\emptyset$ **b)** $\{\lambda\}$ **c)** $\{a\}$, where $a \in I$