Here is a list of problems selected from the textbook to help you better prepare for the midterm exam.

HW1:

2.2. Question 16

16. Let A and B be sets. Show that

a) $(A \cap B) \subseteq A$.
b) $A \subseteq (A \cup B)$.
c) $A - B \subseteq A$.
d) $A \cap (B - A) = \emptyset$.
e) $A \cup (B - A) = A \cup B$.

2.2. Question 41
2.2. Question 42

41. Show that $A \oplus B = (A \cup B) - (A \cap B)$.

42. Show that $A \oplus B = (A - B) \cup (B - A)$.

2.2. Question 56

56. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,

a) $A_i = \{i, i+1, i+2, \ldots\}$.
b) $A_i = \{0, i\}$.
c) $A_i = (0, i)$, that is, the set of real numbers x with $0 < x < i$.
d) $A_i = (i, \infty)$, that is, the set of real numbers x with $x > i$.

2.3. Question 15

15. Determine whether the function $f: \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ is onto if

a) $f(m, n) = m + n$.
b) $f(m, n) = m^2 + n^2$.
c) $f(m, n) = m$.
d) $f(m, n) = |n|$.
e) $f(m, n) = m - n$.

2.3. Question 17

7. Find the domain and range of these functions.

a) the function that assigns to each pair of positive integers the maximum of these two integers

b) the function that assigns to each positive integer the number of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 that do not appear as decimal digits of the integer

c) the function that assigns to a bit string the number of times the block 11 appears

d) the function that assigns to a bit string the numerical position of the first 1 in the string and that assigns the value 0 to a bit string consisting of all 0s

HW2:

6.1.17
6.1.22
6.1.30
6.1.41 (this is fun)
6.1.53
6.2.3 Example11
6.2.23

HW3:

6.3 Q13

A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

6.3 Q24

How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other? [*Hint:* First position the women and then consider possible positions for the men.]

6.4 Q22

22. How many permutations of the letters *ABCDEFGH* contain

a) the string *ED*?
b) the string *CDE*?
c) the strings *BA* and *FGH*?
d) the strings *AB*, *DE*, and *GH*?
e) the strings *CAB* and *BED*?
f) the strings *BCA* and *ABF*?

6.4 Q15

Give a formula for the coefficient of x^k in the expansion of $(x^2 - 1/x)^{100}$, where k is an integer.

6.4 Q22

Show that if n and k are integers with $1 \le k \le n$, then $\binom{n}{k} \le n^k/2^{k-1}$.

6.4 Q29

Let n be a positive integer. Show that

$$\binom{2n}{n+1} + \binom{2n}{n} = \binom{2n+2}{n+1}/2.$$

HW4:

Section 3.2 Question 5

5. Show that $(x^2 + 1)/(x + 1)$ is $O(x)$.

Section 3.2 Question 9

9. Show that $x^2 + 4x + 17$ is $O(x^3)$ but that x^3 is not $O(x^2 + 4x + 17)$.

Section 3.2 Question 14

14. Determine whether x^3 is $O(g(x))$ for each of these functions $g(x)$.

a) $g(x) = x^2$
b) $g(x) = x^3$
c) $g(x) = x^2 + x^3$
d) $g(x) = x^2 + x^4$
e) $g(x) = 3^x$
f) $g(x) = x^3/2$

Section 3.2 Question 31

31. Show that $f(x)$ is $\Theta(g(x))$ if and only if $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$.

HW5:

Section 1.7 Question 16

16. Prove that if x, y, and z are integers and $x + y + z$ is odd, then at least one of x, y, and z is odd.

(Prove by contradiction)

Section 1.8 Question 6

6. Use a proof by cases to show that $\min(a, \min(b, c)) = \min(\min(a, b), c)$ whenever a, b, and c are real numbers.

Section 1.8 Question 19

19. Suppose that a and b are odd integers with $a \neq b$. Show there is a unique integer c such that $|a - c| = |b - c|$.

Section 5.1 Question 12

12. Prove that

$$\sum_{j=0}^{n} \left(-\frac{1}{2}\right)^j = \frac{2^{n+1} + (-1)^n}{3 \cdot 2^n}$$

whenever n is a nonnegative integer.

Section 5.1 Question 19

19. Let $P(n)$ be the statement that

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n},$$

where n is an integer greater than 1.

a) What is the statement $P(2)$?

b) Show that $P(2)$ is true, completing the basis step of a proof by mathematical induction that $P(n)$ is true for all integers n greater than 1.

c) What is the inductive hypothesis of a proof by mathematical induction that $P(n)$ is true for all integers n greater than 1?

d) What do you need to prove in the inductive step of a proof by mathematical induction that $P(n)$ is true for all integers n greater than 1?

e) Complete the inductive step of a proof by mathematical induction that $P(n)$ is true for all integers n greater than 1.

f) Explain why these steps show that this inequality is true whenever n is an integer greater than 1.

Section 5.1 Question 37

37. Prove that if n is a positive integer, then 133 divides $11^{n+1} + 12^{2n-1}$.