

1.)

$2, 52!, \ln(n^3), \ln^2(n), \log_2(n), \log_2((4n)^n), n \ln(n), n^3, \left(\frac{3}{2}\right)^n, 2^n, n!$

2 and 52! are constant, same Θ class. $\ln(n^2)$ and $\ln^2(n) = \Theta$.
and $n \ln(n)$ and $\log_2((4n)^n) = \Theta$

2a) $\forall \text{ List } \langle \text{elem} \rangle \rightarrow \text{SortFunction}(A)$

if $A.\text{length}() \leq 1$
return A

$p = A.\text{last}()$

List $\langle \text{elem} \rangle L, R$

for elem e in A (besides last)

if $e \leq p$

$L.\text{Push}(e)$

else

$R.\text{Push}(e)$

$L' = \text{Sort}(L)$

$R' = \text{Sort}(R)$

Return $L' + [p] + R'$

2B.) worst case is $O(n^2)$, if our pivot is an extremely large or small elem, then one list will be way larger, or contain all the elements. Then we would have each call sorting $n-1, n-2, \dots$ elem each call.

$$2C) T(n) = n + T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right)$$

$$T(n) = n + 2T\left(\frac{n}{2}\right)$$

$$T(n) = \Theta(n) + 2T\left(\frac{n}{2}\right)$$

$$2D) T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$$\rightarrow \log_b a \rightarrow \log_2 2 \Rightarrow 1 \rightarrow n^1 \rightarrow n' = f(n) \rightarrow n = n$$

$$\rightarrow \Theta(n \log^K(n)) \rightarrow \Theta(n \log'(n)) \rightarrow \Theta(n \log^2(n))$$

$$= \Theta(n \log(n))$$

3A.) A is the winners essentially Since
"no-one outside this group can reach them."

So IF I've lost to you, you have lost to me!

A. is ^{also} Someone who can push around at
a minimum one player without them winning back,
implies an SCC has an outgoing edge.

B. is the opposite kinda, these are losers
the 12th place Diddy Kong Bike players.

Someone can beat them ($w \rightarrow u$) but
I can't beat them ($u \nrightarrow w$). A SCC
for losers.

C. is the mid tier, They are in some
SCC and can reach other SCC's but
not consistently getting 1st or 12th

3B.) Each Set's size can vary on the
number of players and winners, losers, etc.
we could have set A be extremely large
but set B be relatively small implying a
Dynamic Data Structure necessary

3B.) Triple <int, int, int> Get Set Sizes (G)

Σ

H = Make SCCs (G)

Int ACount, BCount, CCount = 0

for SCC in H

inDeg = size incoming

outDeg = size outgoing

CSize = size SCC

if inDeg = 0 and outDeg > 0
ACount = CSize

else if outDeg = 0 and inDeg > 0
BCount = CSize

else if

CCount = CSize

Return Triple <int, int, int> sizes = (ACount, BCount, CCount)

Return sizes

3C) $\Theta(V + E)$, I assume if we run a False
Kosaraju's then we do some work it will be
dependent on the size of our graph.