TP1 : Robust support vector classification with Ellipsoidal Uncertainty in $\mathbb{R}^2$

1. Input the training set by Generating a sample i.i.d according to a uniform distribution, then assume that each point $\overline{x_i}$ has a random value of perturbation δ_i , where the resulting input data should be represented by circles with center $\overline{x_i}$ and radius δ_i and respective class labels $y_i \in \{+1, -1\}$.
2. Choose an appropriate penalty parameter $C > 0$.
3. Construct and solve the problem obtaining $\left(\alpha^{*T}, \gamma^*\right) = \left((\alpha_1^*, \dots, \alpha_m^*), \gamma^*\right)$

$$\max_{\alpha,\, \beta,\, \gamma, z_u, z_v} \beta + \sum_{i=1}^{m} \alpha_i,$$

$$s.t.\ \gamma \le \beta + \sum_{i=1}^{m} \delta_i \alpha_i - \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y_i y_j (x_i \cdot x_j)},$$

$$\beta + z_u = \frac{1}{2}, \quad \beta + z_v = \frac{1}{2},$$

$$\sum_{i=1}^{m} y_i \alpha_i = 0,$$

$$0 \le \alpha_i \le C,\ i = 1,\ \cdots,\ m,$$

$$\sqrt{\gamma^2 + z_v^2} \le z_u.$$

4. Compute

$$w^* = \frac{\gamma^*}{\left(\gamma^* - \sum_{i=1}^{m} \delta_i \alpha_i^*\right)} \sum_{i=1}^{m} \alpha_i^* y_i x_i$$

5. Choose a positive component of α^*, $\alpha_j^* \in (0,\ C)$, and compute

$$b^* = y_j - \frac{\gamma^*}{\left(\gamma^* - \sum_{i=1}^{m} \delta_i \alpha_i^*\right)} \sum_{i=1}^{m} \alpha_i^* y_i (x_i . x_j) - y_j \delta_j \gamma^*$$

6. Construct the decision function $f(x) = \mathrm{sign}\left(w^{*T} x + b^*\right)$ and visualize the separator.