TP 4 : ϵ-band SVRNR with kernels

Objective:

The aim of this laboratory exercise is to gain an understanding of the functioning of Kernels in regression to handle non-linear data, and to train the model using the Sequential Minimal Optimization (SMO) optimization algorithm. To implement ϵ-band SVRNR with kernels and SMO, we will utilize a NumPy and Matplotlib for visualization.

ϵ-band SVRNR with kernels:

We give the mathematical models of the dual forms of ϵ-band SVRNR with some kernels:

ϵ-band SVRNR:

- Input: training set : $\{(x_i, y_i\}_{i=1}^n$ where $x_i \in \mathbb{R}^d, y_i \in \mathbb{R}, \varepsilon_{\text{inf}} \geq 0$
- Choose an approriate parameter $\varepsilon > \varepsilon_{\text{inf}}$, map ϕ_Q and penalty parameter $C > 0$
- Contruct and solve the optimzation problem DLS $-\varepsilon$-band - SVR obtaining $\lambda^* = (\lambda_1^*, \ldots, \lambda_n^*)$ and

  $\mu^* = (\mu_1^*, \ldots, \mu_n^*)\, k(x_j, x_i) = \phi_Q(x_j)^T \phi_Q(x_i)$

DLS - ε-band -SVRNR $\begin{cases} \min \frac{1}{2} \sum_{i,j=1}^n (\mu_i - \lambda_i)(\mu_j - \lambda_j)\left(\phi_Q(x_j)^T \phi_Q(x_i)\right) - \varepsilon \sum_{i=1}^n (\mu_i + \lambda_i) + \sum_{i=1}^n y_i (\mu_i - \lambda_i) \\ \text{s. à} \quad \sum_{i=1}^n (\mu_i - \lambda_i) = 0 \\ C \geq \lambda_i \geq 0, C \geq \mu_i \geq 0, \quad i = 1, \ldots, n \end{cases}$

if $\lambda^* \neq 0$ and $\mu^* \neq 0$, the solution to the problem, (w^*, b^*), can be obtained in the following way

- $w^* = \sum_{i=1}^n (\mu_i - \lambda_i) \phi_Q(x_i)$,
- for any component $\lambda_j^* \in]\,0, C\,[$ of $\lambda^*, b^* = y_j - (w^*)^T \phi_Q(x_j) + \varepsilon$
- Or for any component $\mu_j^* \in]\,0, C\,[$ of $\mu^*, b^* = y_j - (w^*)^T \phi_Q(x_j) - \varepsilon$

$$h(x) = (w^*)^T \phi_Q(x) + b^*$$

Kernels:

We give the following kernels:

- Polynomial Kernel: $k_{\text{poly}}(x, z) = \left(x^T z + R\right)^d$
- Gaussian Radial Basis function RBF Kernel: $k_{RBF}(x, z) = \exp\left(\frac{-||x - z\|_2^2}{2\sigma^2}\right)$

Method:

1. Choose a simple dataset of simple regression (one x and one y), dataset must be purely nonlinear.
2. Perform exploratory analysis of the data and preprocess the data if necessary.
3. Split data into train and test
4. Implement the SMO optimization algorithm.
5. Implement Epsilon -band SVRNR with the two kernels given above
6. Train the two models, and compare the error mean squared errors (MSE) of the two kernels, what can we conclude?
7. Visualize the regression results(the approximation hperplane in $2\,\mathrm{D}$)