$$
\begin{cases}
\min t_f \\
\dot q(t) = \begin{pmatrix} w_x + \cos(\theta(t)) \\ w_y + \sin(\theta(t)) \\ u(t) \end{pmatrix} \quad \text{où } q = (x, y, \theta) \\
|u(t)| \leq 1 \\
q(0) = q_0, \quad q(t_f) = q_f
\end{cases}
$$

② $H(p, q, u) = p_x(w_x + \cos(\theta)) + p_y(w_y + \sin(\theta)) + p_\theta u$

③ $\dot p(t) = -\dfrac{\partial H}{\partial q}[t] = \begin{pmatrix} 0 \\ 0 \\ p_x \sin(\theta) - p_y \cos(\theta) \end{pmatrix}$

$[t] = (q(t), p(t), u(t))$

④ $H[t_f] = -p^0 = 1$

(5) $\mu(p_\theta) \in \begin{cases} \{1\} & \text{si } p_\theta > 0 \\ \{-1\} & \text{si } p_\theta < 0 \\ [-1,1] & \text{si } p_\theta = 0 \end{cases}$

(6) On suppose $p_\theta(t) = 0$

$$\Rightarrow \dot{p}_\theta(t) = p_x \sin(\theta(t)) - p_y \cos(\theta(t))$$
$$= 0$$

$$\Rightarrow \ddot{p}_\theta(t) = \underbrace{\left(p_x \cos(\theta(t)) + p_y \sin(\theta(t))\right)}_{\neq 0} \underbrace{\times \dot{\theta}(t)}_{= u(t)}$$

Il faut montrer que :

$$p_x \cos(\theta(t)) + p_y \sin(\theta(t)) \neq 0$$

Supposons que ce terme $= 0$.

$$\underbrace{\begin{bmatrix} \sin(\theta(t)) & -\cos(\theta(t)) \\ \cos(\theta(t)) & \sin(\theta(t)) \end{bmatrix}}_{A} \begin{pmatrix} p_x \\ p_y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\det(A) = \cos(\theta(r))^2 + \sin(\theta(r))^2$$

$$\det(A) = 1$$

donc $p_x = p_y = 0$.

donc $H[r] = 0$ or

H est constant et $H[r_f] = 1$

donc $0 = 1 \Rightarrow$ <u>Contradiction!</u>