A Fast Nonnegative Autoencoder-based Approach to Latent Feature Analysis on High-Dimensional and Incomplete Data Supplementary File

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This is the supplementary file for the paper entitled "A Fast Nonnegative Autoencoder-based Approach to Latent Feature Analysis on High-Dimensional and Incomplete Data". Additional tables and figures are put into this file and cited by this paper.

1. Supplementary Tables

1.1 Results of Ablation and Effectiveness Analysis (RQ1)

- Table **S1** (discussed in Section 4.2(1)) summarizes the RMSE, MAE, and training epochs in RMSE and MAE of FNAE and its variant labeled FANE-w/o-NC that removes the nonnegativity constraints on D1-8;
- Table **S2** (discussed in Section 4.2(2)) summarizes the RMSE, MAE, and training epochs in RMSE and MAE of FNAE with different hidden layers on D1-8.

TABLE S1
THE RMSE, MAE, AND TRAINING EPOCHS IN RMSE AND MAE OF FNAE AND FNAE-W/O-NC ON D1-8.

No.	RMSE		Epochs (RMSE)		MAE		Epochs (MAE)	
	FNAE	FNAE-w/o-NC	FNAE	FNAE-w/o-NC	FNAE	FNAE-w/o-NC	FNAE	FNAE-w/o-NC
D1	**0.7811**±7.9E-5	0.7853±1.6E-4	344±4.26	**187**±1.94	**0.5941**±1.1E-4	0.5970±1.3E-4	345±11.01	**193**±5.71
D2	**0.7052**±1.1E-4	0.7076±1.4E-4	125±3.31	**98**±2.32	**0.5506**±1.5E-4	0.5522±1.9E-4	129±8.11	**107**±3.93
D3	**0.7904**±2.5E-4	0.7936±2.0E-4	183±4.78	**103**±4.45	**0.6052**±1.7E-4	0.6072±1.7E-4	180±3.68	**105**±2.94
D4	**0.1088**±1.1E-4	0.1101±6.0E-5	528±4.21	**269**±5.85	**0.0743**±8.6E-5	0.0760±4.2E-5	555±6.24	**263**±4.27
D5	**0.1110**±7.0E-5	0.1135±3.9E-5	392±7.39	**244**±7.00	**0.0743**±5.4E-5	0.0766±6.5E-5	445±4.50	**244**±6.15
D6	**0.1203**±4.0E-5	0.1233±2.4E-5	511±6.69	**238**±6.01	**0.0822**±1.8E-4	0.0853±4.4E-5	597±8.59	**254**±9.88
D7	**0.0979**±1.2E-4	0.0993±8.5E-5	798±13.85	**371**±5.82	**0.0638**±6.8E-5	0.0640±6.1E-5	810±2.83	**360**±6.90
D8	**0.3791**±1.3E-3	0.3906±7.9E-4	353±12.53	**169**±6.36	**0.1431**±1.9E-3	0.1499±2.7E-4	679±25.28	**167**±6.79

w/o stands for the removal operation; and NC stands for the nonnegativity constraints.

TABLE S2
THE RMSE, MAE, AND TRAINING EPOCHS IN RMSE AND MAE OF FNAE WITH DIFFERENT HIDDEN LAYERS ON D1-8.

No.	RMSE			Epochs (RMSE)			MAE			Epochs (MAE)		
	FNAE-1	FNAE-2	FNAE-3*	FNAE-1	FNAE-2	FNAE-3*	FNAE-1	FNAE-2	FNAE-3*	FNAE-1	FNAE-2	FNAE-3*
D1	0.7811±7.9E-5	**0.7784**±1.9E-4	0.8005±4.5E-4	**344**±4.26	482±12.04	676±172.29	0.5941±1.1E-4	**0.5908**±1.5E-4	0.6079±5.1E-4	**345**±11.01	487±21.47	677±164.90
D2	0.7052±1.1E-4	**0.7039**±1.7E-4	0.7146±1.1E-4	**125**±3.31	385±37.84	435±30.19	0.5506±1.5E-4	**0.5502**±1.5E-4	0.5598±2.0E-4	**129**±8.11	429±49.74	435±30.19
D3	0.7904±2.5E-4	**0.7855**±6.9E-4	0.8122±5.6E-4	**183**±4.78	610±109.36	644±76.31	0.6052±1.7E-4	**0.6015**±6.0E-4	0.6204±5.0E-4	**180**±3.68	608±115.54	638±73.99
D4	**0.1088**±1.1E-4	0.1097±1.7E-4	0.1238±2.0E-4	528±4.21	558±4.38	**463**±4.59	**0.0743**±8.6E-5	0.0747±7.9E-5	0.0850±8.2E-5	555±6.24	566±8.96	**467**±12.06
D5	**0.1110**±7.0E-5	0.1122±1.7E-4	0.1221±4.6E-5	392±7.39	353±3.20	**296**±5.78	**0.0743**±5.4E-5	0.0751±1.1E-4	0.0822±1.4E-4	445±4.50	359±7.81	**307**±16.11
D6	**0.1203**±4.0E-5	0.1216±2.7E-4	0.1321±1.5E-4	511±6.69	508±6.05	**433**±7.58	**0.0822**±1.8E-4	0.0834±1.8E-4	0.0911±1.5E-4	597±8.59	540±14.39	**456**±21.33
D7	0.0979±1.2E-4	**0.0970**±8.7E-5	0.1085±4.5E-5	798±13.85	865±4.34	**661**±16.59	0.0638±6.8E-5	**0.0625**±9.6E-5	0.0704±3.3E-4	810±2.83	834±2.71	**599**±46.05
D8	0.3791±1.3E-3	0.3354±4.7E-4	**0.3351**±8.5E-4	353±12.53	**160**±8.66	268±15.45	0.1431±1.9E-3	0.1087±2.9E-4	**0.1062**±2.6E-4	679±25.28	**157**±7.83	280±10.07

** FNAE-3 represents the model with three hidden layers, and similar symbols for others.*

1.2 Results of Comparison Experiments (RQ3)

- Tables **S3-4** (discussed in Section 4.4(1)) record the RMSE, MAE, and win/loss counts of M1-16 on D1-8;
- Tables **S5-6** (discussed in Section 4.4(2)) record the time cost in RMSE and MAE, and win/loss counts of M1-16 on D1-8;
- Table **S7** (discussed in Section 4.4(1-2)) records the results of Friedman test in accuracy and efficiency regarding M1-16;
- Tables **S8-9** (discussed in Section 4.4(1-2)) record the Wilcoxon singed-ranks test results regarding M1-16.

TABLE S3
THE RMSE AND WIN/LOSS COUNTS OF M1-16 ON ALL TESTING CASES.

No.	D1	D2	D3	D4	D5	D6	D7	D8	Win/Loss
M1	0.7796±2.7E-4	0.7042±1.1E-4	0.8066±3.1E-4	0.1135±8.8E-5	0.1164±7.3E-5	0.1276±1.2E-4	0.1062±9.5E-5	0.3483±5.1E-4	8/0
M2	0.8207±8.4E-4	0.7324±2.2E-4	0.8287±9.4E-4	0.1238±2.8E-4	0.1278±6.6E-5	0.1380±3.7E-5	0.1103±1.9E-4	0.4067±2.1E-3	8/0
M3	0.7946±1.3E-3	0.7045±8.8E-4	0.8777±3.0E-3	0.1880±9.1E-2	0.1352±3.1E-4	0.1544±1.1E-4	0.1185±9.5E-5	0.4358±1.5E-1	8/0
M4	0.8176±1.8E-3	0.7146±1.5E-3	0.8612±1.2E-3	0.1438±2.4E-3	0.1413±5.8E-4	0.1512±2.8E-4	0.1383±6.3E-3	0.3662±3.1E-3	8/0
M5	0.8197±3.0E-4	0.7187±7.6E-4	0.8267±1.2E-3	0.1280±5.4E-4	0.1276±2.4E-4	0.1374±5.3E-4	0.1112±1.1E-3	0.5722±1.3E-2	8/0
M6	0.7922±3.2E-4	0.7142±9.6E-5	0.8021±3.7E-4	0.1271±3.5E-4	0.1267±3.4E-4	0.1468±1.1E-3	0.1217±3.5E-4	0.3464±7.9E-4	8/0
M7	0.8133±3.0E-4	0.7722±4.3E-4	0.8084±3.6E-4	0.1187±3.3E-4	0.1183±3.6E-4	0.1276±8.8E-5	0.1015±1.3E-4	**0.3310±4.4E-4**	7/1
M8	0.7893±4.2E-4	0.7033±2.0E-4	0.8003±3.1E-4	0.1265±2.5E-4	0.1241±2.9E-4	0.1360±2.3E-4	0.1142±5.4E-4	0.3378±4.8E-4	8/0
M9	0.8034±8.7E-4	0.7510±3.5E-3	0.8055±6.6E-4	0.1232±4.1E-4	0.1222±2.7E-4	0.1314±6.4E-5	0.1066±1.7E-4	0.3419±5.1E-4	8/0
M10	0.8114±1.5E-4	0.7747±1.4E-4	0.8055±2.9E-4	0.1495±3.9E-4	0.1437±4.2E-4	0.1491±2.3E-4	0.1292±1.5E-4	0.3394±7.6E-4	8/0
M11	0.7867±1.8E-4	0.7214±1.6E-4	0.7943±2.4E-4	0.1185±3.4E-4	0.1179±3.2E-4	0.1273±9.2E-5	0.1013±1.1E-4	0.3381±3.4E-4	8/0
M12	0.8352±3.8E-3	0.8040±3.5E-3	0.8305±4.2E-3	0.1202±2.8E-4	0.1200±2.9E-4	0.1294±3.9E-5	0.1038±2.1E-4	0.3390±6.2E-4	8/0
M13	0.9254±2.9E-2	0.8140±1.0E-2	0.8567±3.5E-3	0.1299±2.1E-4	0.1329±5.4E-4	0.1353±1.6E-2	0.1188±8.9E-4	0.3851±9.9E-3	8/0
M14	0.8612±6.5E-4	0.7897±4.7E-4	0.8592±6.1E-4	0.1271±3.5E-4	0.1267±3.4E-4	0.1369±9.0E-5	0.1073±1.3E-4	0.3754±1.2E-3	8/0
M15	0.9189±2.0E-2	0.8428±3.2E-2	0.9653±1.7E-2	0.1467±2.5E-3	0.1544±3.3E-3	0.1603±3.9E-3	0.1220±5.3E-4	0.5684±1.4E-2	8/0
M16	**0.7757±1.4E-4**	**0.7027±1.3E-4**	**0.7898±9.0E-5**	**0.1105±1.1E-4**	**0.1125±9.7E-5**	**0.1215±7.4E-5**	**0.0997±2.0E-4**	0.3343±4.7E-4	—

TABLE S4
THE MAE AND WIN/LOSS COUNTS OF M1-16 ON ALL TESTING CASES.

No.	D1	D2	D3	D4	D5	D6	D7	D8	Win/Loss
M1	0.5929±2.0E-4	0.5504±1.2E-4	0.6167±1.3E-4	0.0777±5.7E-5	0.0786±7.4E-5	0.0893±8.9E-5	0.0700±5.5E-5	0.1184±6.5E-4	8/0
M2	0.6232±7.3E-4	0.5707±2.0E-4	0.6340±6.9E-4	0.0880±1.6E-4	0.0894±6.4E-5	0.0988±2.1E-4	0.0742±1.6E-4	0.1720±2.1E-3	8/0
M3	0.6045±7.8E-4	0.5558±1.1E-3	0.6784±2.4E-3	0.1429±8.7E-2	0.0949±2.3E-4	0.1071±7.6E-5	0.0722±1.2E-4	0.1486±5.1E-2	8/0
M4	0.6223±1.6E-3	0.5633±1.3E-3	0.6640±1.5E-3	0.0918±2.2E-3	0.0917±5.4E-4	0.0944±7.1E-4	0.0883±4.5E-3	0.1099±6.2E-4	8/0
M5	0.6313±2.0E-3	0.5683±6.7E-4	0.6434±3.0E-2	0.0954±1.6E-3	0.0907±4.4E-4	0.1004±6.6E-4	0.0765±1.0E-3	0.3520±1.2E-2	8/0
M6	0.6044±3.1E-4	0.5608±1.6E-4	0.6160±3.1E-4	0.0906±2.0E-4	0.0895±1.6E-4	0.1077±1.1E-3	0.0848±3.8E-4	0.1351±5.7E-4	8/0
M7	0.6166±2.3E-4	0.5873±2.0E-4	0.6186±2.8E-4	0.0839±1.9E-4	0.0818±1.7E-4	0.0909±6.0E-5	0.0673±2.1E-4	0.1145±1.3E-4	8/0
M8	0.6000±3.8E-4	0.5530±1.9E-4	0.6123±2.7E-4	0.0905±2.8E-4	0.0870±2.8E-4	0.0978±9.3E-5	0.0757±2.9E-4	0.1228±4.5E-4	8/0
M9	0.6098±4.7E-4	0.5736±1.3E-3	0.6174±4.5E-4	0.0876±2.1E-4	0.0851±1.4E-4	0.0942±7.2E-5	0.0706±1.1E-4	0.1260±3.8E-4	8/0
M10	0.6168±1.3E-4	0.5908±1.7E-4	0.6182±2.5E-4	0.1088±3.0E-4	0.1029±2.7E-4	0.1080±2.3E-4	0.0896±9.9E-5	0.1260±2.7E-4	8/0
M11	0.5986±2.6E-4	0.5598±1.1E-4	0.6089±1.6E-4	0.0837±2.1E-4	0.0813±1.3E-4	0.0907±1.1E-4	0.0669±6.3E-5	0.1195±3.2E-4	8/0
M12	0.6358±3.0E-3	0.6125±2.2E-3	0.6381±3.4E-3	0.0849±1.8E-4	0.0831±1.4E-4	0.0925±1.3E-4	0.0682±1.3E-4	0.1130±6.0E-4	8/0
M13	0.7195±3.1E-2	0.6464±9.0E-3	0.6777±1.5E-2	0.0940±2.4E-4	0.0939±3.2E-4	0.1023±2.9E-4	0.0801±6.8E-4	0.1342±4.2E-3	8/0
M14	0.6622±4.1E-4	0.6115±2.5E-4	0.6657±4.6E-4	0.0906±2.0E-4	0.0895±1.6E-4	0.0992±4.8E-5	0.0720±8.2E-5	0.1412±2.0E-4	8/0
M15	0.7011±1.5E-2	0.6560±2.0E-2	0.7411±1.2E-2	0.1041±1.9E-3	0.1076±2.3E-3	0.1161±2.5E-3	0.0814±6.7E-4	0.2393±6.8E-3	8/0
M16	**0.5897±1.5E-4**	**0.5494±1.3E-4**	**0.6034±8.3E-5**	**0.0762±9.4E-5**	**0.0764±9.7E-5**	**0.0848±1.0E-4**	**0.0647±1.2E-4**	**0.1071±3.4E-4**	—

TABLE S5
THE TIME COST TO CONVERGE IN RMSE AND WIN/LOSS COUNTS OF M1-16 ON ALL TESTING CASES.

No.	D1	D2	D3	D4	D5	D6	D7	D8	Win/Loss
M1	7078±853.14	7475±1015.36	2388±170.60	175±3.33	124±1.23	86±2.81	61±2.56	**7±0.55**	7/1
M2	6616±289.89	5940±77.15	2402±96.09	**24±2.12**	298±28.12	183±16.52	62±1.58	154±11.95	7/1
M3	104682±31354.44	250091±89575.46	30971±11771.14	3784±1328.82	2847±888.49	2201±401.66	1341±181.53	4437±1003.92	8/0
M4	71767±17433.29	45523±5966.34	6585±1026.60	16636±5717.27	21886±775.19	4123±886.96	683±296.22	13723±4029.15	8/0
M5	79311±7034.76	22307±2245.37	26559±2711.56	7264±2205.15	3598±2076.02	3433±354.06	1212±330.47	10894±2117.14	8/0
M6	21863±684.73	7994±253.44	8650±380.04	5418±537.70	218±315.74	2005±152.49	998±80.46	966±200.24	8/0
M7	33007±1137.05	35737±491.00	8104±197.44	6504±91.71	3641±79.94	398±4.38	146±2.73	2404±31.84	8/0
M8	15933±882.56	8847±333.80	6945±568.32	812±12.78	522±12.12	620±16.24	359±7.82	2600±54.49	8/0
M9	76680±2769.13	48136±1107.52	30568±2269.40	1909±36.29	1024±17.55	1204±19.56	426±10.76	10319±91.76	8/0
M10	35451±2864.30	24142±624.52	14832±166.83	4109±152.29	2698±41.52	2616±154.62	883±37.12	1358±43.52	8/0
M11	24995±661.42	18767±195.75	9465±82.54	6386±68.83	3507±47.87	4329±50.16	1511±7.98	3715±95.13	8/0
M12	83442±4160.54	66710±599.93	28419±1317.36	2706±74.28	1274±3.51	1667±50.70	657±17.94	19744±1310.35	8/0
M13	72382±13711.38	12674±3094.18	11400±2851.86	2148±458.26	1307±119.87	2286±516.97	741±126.90	4053±814.72	8/0
M14	80798±2247.66	59081±1389.49	30321±619.38	5418±537.70	218±315.74	2924±263.32	982±104.19	10829±890.48	8/0
M15	14838±4847.60	7476±2297.27	8069±4781.40	129±32.41	224±37.42	102±40.91	76±20.79	62±25.16	8/0
M16	**5346±598.34**	**5136±401.65**	**2193±64.34**	41±1.04	**52±3.08**	**83±4.38**	**43±2.25**	21±2.83	—

TABLE S6
THE TIME COST TO CONVERGE IN MAE AND WIN/LOSS COUNTS OF M1-16 ON ALL TESTING CASES.

No.	D1	D2	D3	D4	D5	D6	D7	D8	Win/Loss
M1	7065±800.43	7151±892.81	2558±247.14	170±5.06	120±3.02	**88±1.75**	52±1.77	**9±1.00**	6/2
M2	6749±275.16	6150±145.92	2514±154.52	**22±2.75**	297±22.37	184±15.99	56±0.83	154±12.15	7/1
M3	111027±32555.86	250260±68249.72	13180±6587.40	3546±1292.37	2993±970.69	2240±469.73	1308±198.85	5487±880.73	8/0
M4	69098±18074.34	48651±11212.19	7284±1746.86	17226±4821.14	21905±790.42	4905±415.94	530±378.30	21167±3763.01	8/0
M5	43963±11211.60	29673±3861.08	12866±2737.54	2543±242.30	2248±876.67	2899±749.96	459±126.06	1050±382.93	8/0
M6	22087±652.82	10107±728.89	8710±328.57	4944±899.33	2100±256.52	1999±59.22	898±69.64	736±171.61	8/0
M7	33371±1266.95	36575±631.52	8520±220.18	5763±122.28	3500±78.80	391±5.77	125±3.08	1580±61.15	8/0
M8	16017±1033.00	9086±591.60	7022±717.66	712±41.90	491±20.89	593±29.02	317±14.62	1812±101.06	8/0
M9	79211±5091.89	49478±1986.16	31300±515.13	1681±32.21	973±22.12	1172±14.08	336±7.73	6374±268.47	8/0
M10	35800±2657.46	25700±526.76	15409±469.16	2487±84.92	2433±38.51	2616±154.62	358±14.24	805±66.86	8/0
M11	25278±667.66	19505±242.15	9640±72.67	5758±92.65	3394±34.17	4214±43.86	1284±19.23	3117±234.68	8/0
M12	84303±3390.46	71883±6582.28	28554±1430.92	2285±71.03	1200±42.06	1641±72.74	510±12.54	13524±2257.22	8/0
M13	72382±13711.38	12674±3094.18	8572±3041.32	1801±294.71	1317±137.64	1500±102.35	451±162.16	4468±1136.64	8/0
M14	81438±1702.52	59148±1721.32	30321±619.38	4944±899.33	2100±256.52	2897±305.57	781±40.16	9283±732.94	8/0
M15	14838±4847.60	7476±2297.27	9623±4614.71	127±32.03	222±35.91	102±40.91	74±22.64	53±19.57	8/0
M16	**5285±626.31**	**5361±404.83**	**2319±86.96**	50±1.43	**57±3.13**	91±5.10	**38±2.51**	21±4.11	—

TABLE S7
RESULTS OF THE FRIEDMAN TEST IN ESTIMATION ACCURACY (RMSE AND MAE) AND EFFICIENCY (CONVERGING TIME IN RMSE AND MAE).

No.	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13	M14	M15	M16
Accuracy*	3.66	9.81	11.44	10.81	11.00	8.63	5.59	5.94	6.81	11.69	3.38	7.56	12.63	10.81	15.19	**1.06**
Efficiency**	2.38	2.94	13.25	12.56	11.75	8.25	9.06	5.81	10.06	9.88	11.50	11.63	9.06	12.50	4.06	**1.31**

** High F-rank denotes low RMSE/MAE; ** high F-rank denotes low time cost to converge.*

TABLE S8
RESULTS OF THE WILCOXON SIGNED-RANKS TEST IN RMSE AND MAE CORRESPONDING TO TABLES S3, S4, AND S7.

Comparison	*R*+*	*R*−	*p*-value**
M16 vs M1	136	0	**2.41E-04**
M16 vs M2	136	0	**2.41E-04**
M16 vs M3	136	0	**2.41E-04**
M16 vs M4	136	0	**2.41E-04**
M16 vs M5	136	0	**2.41E-04**
M16 vs M6	136	0	**2.41E-04**
M16 vs M7	133	3	**4.25E-04**
M16 vs M8	136	0	**2.41E-04**
M16 vs M9	136	0	**2.41E-04**
M16 vs M10	136	0	**2.41E-04**
M16 vs M11	136	0	**2.41E-04**
M16 vs M12	136	0	**2.41E-04**
M16 vs M13	136	0	**2.41E-04**
M16 vs M14	136	0	**2.41E-04**
M16 vs M15	136	0	**2.41E-04**

** For M16, higher R+ values indicate higher estimation accuracy; ** The accepted hypotheses are highlighted as significance level=0.1.*

TABLE S9

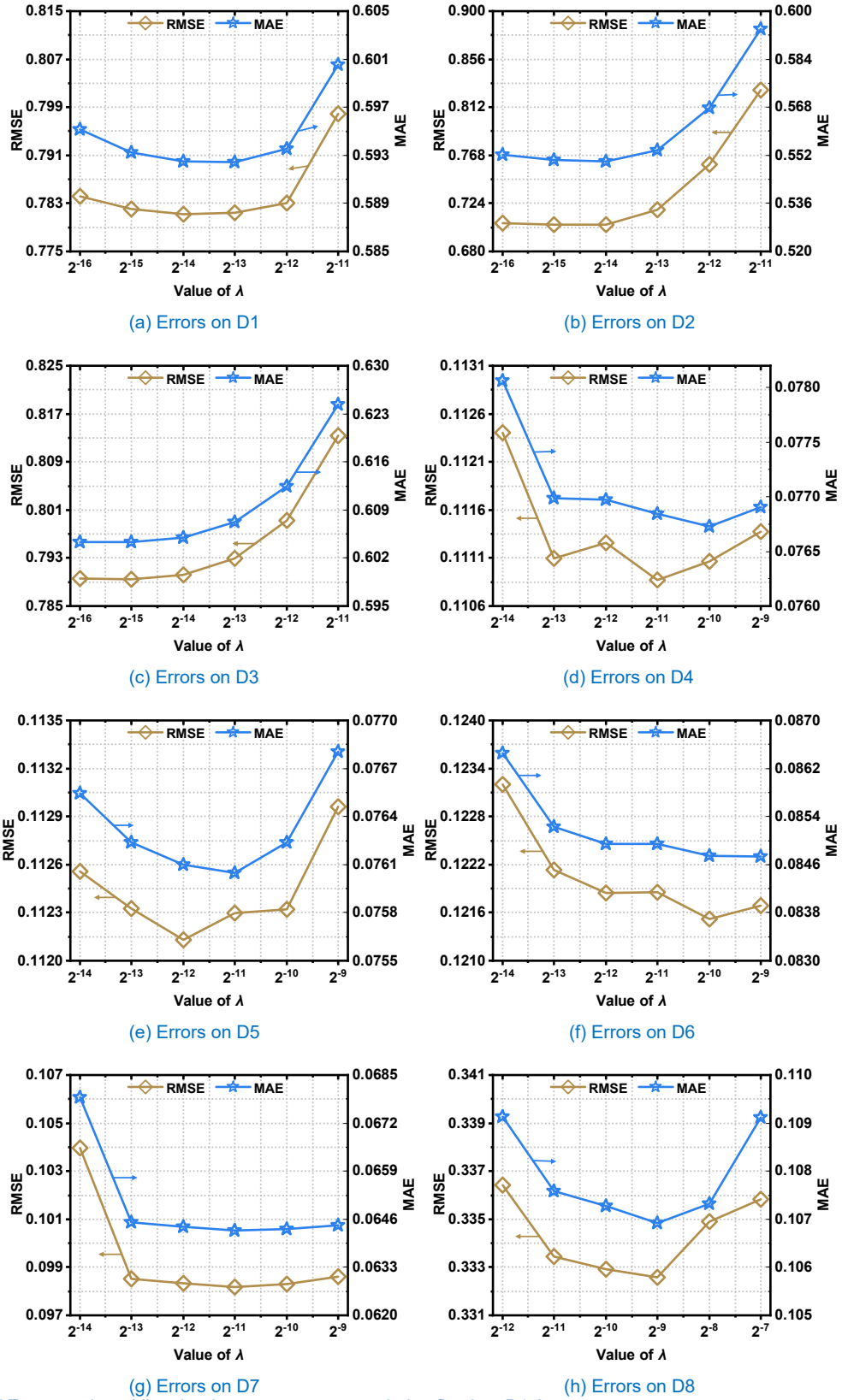
RESULTS OF THE WILCOXON SIGNED-RANKS TEST ON CONVERGING TIME IN RMSE AND MAE CORRESPONDING TO TABLES S5, S6, AND S7.

Comparison	*R*+*	*R*–	*p*-value**
M16 vs M1	127	9	**1.24E-03**
M16 vs M2	131	5	**6.14E-04**
M16 vs M3	136	0	**2.41E-04**
M16 vs M4	136	0	**2.41E-04**
M16 vs M5	136	0	**2.41E-04**
M16 vs M6	136	0	**2.41E-04**
M16 vs M7	136	0	**2.41E-04**
M16 vs M8	136	0	**2.41E-04**
M16 vs M9	136	0	**2.41E-04**
M16 vs M10	136	0	**2.41E-04**
M16 vs M11	136	0	**2.41E-04**
M16 vs M12	136	0	**2.41E-04**
M16 vs M13	136	0	**2.41E-04**
M16 vs M14	136	0	**2.41E-04**
M16 vs M15	136	0	**2.41E-04**

** For M16, higher R+ values indicate higher computational efficiency; ** The accepted hypotheses are highlighted as significance level=0.1.*

2. SUPPLEMENTARY FIGURES

- Figs. **S1-2** plot the lowest errors and training epochs of FNAE as λ varies;
- Figs. **S3-4** plot the lowest errors and training epochs of FNAE as η varies;
- Figs. **S5-6** plot the lowest errors and training epochs of FNAE as D varies.



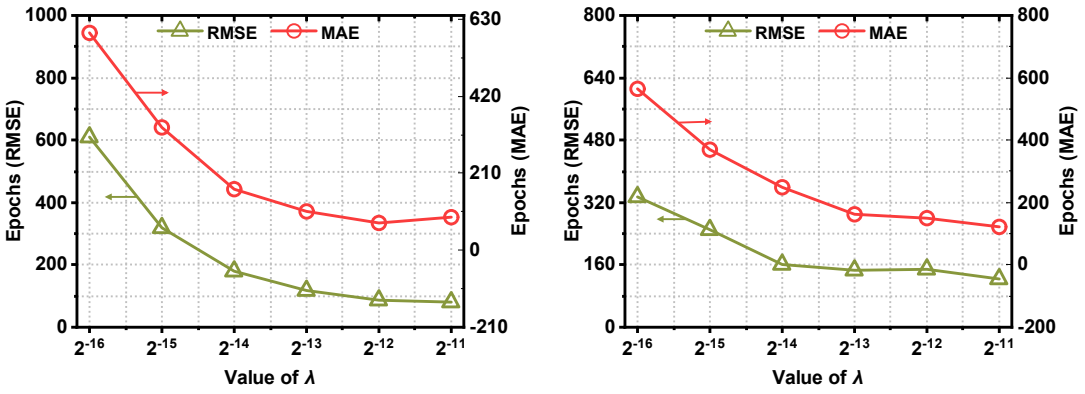
(a) Errors on D1 (b) Errors on D2

(c) Errors on D3 (d) Errors on D4

(e) Errors on D5 (f) Errors on D6

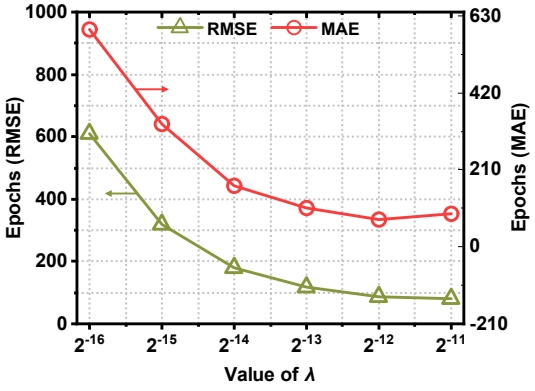
(g) Errors on D7 (h) Errors on D8

Fig. S1. Errors of FNAE as λ varies while other hyperparameters are being fixed on D1-8.

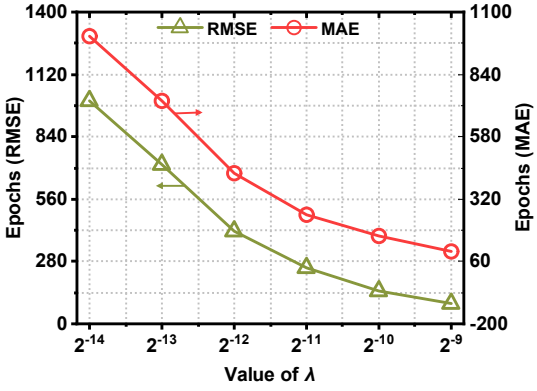


(a) Epochs on D1

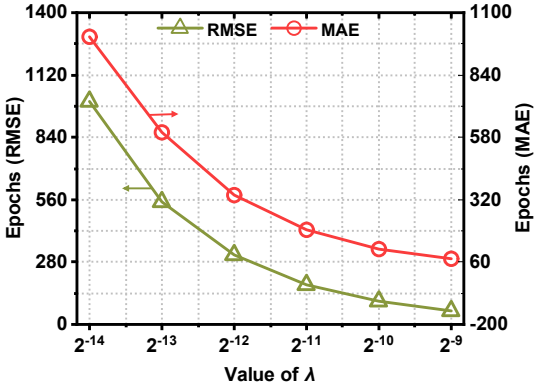
(b) Epochs on D2



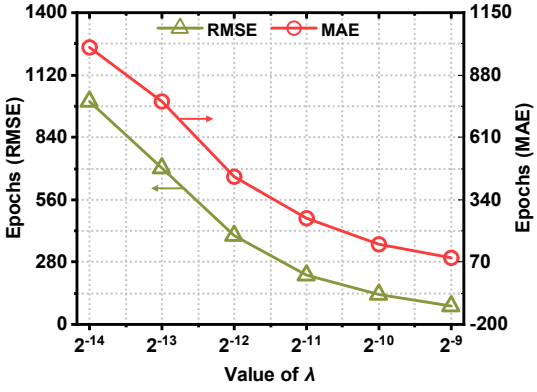
(c) Epochs on D3



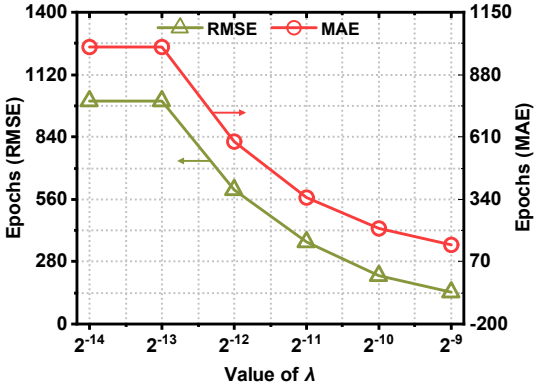
(d) Epochs on D4



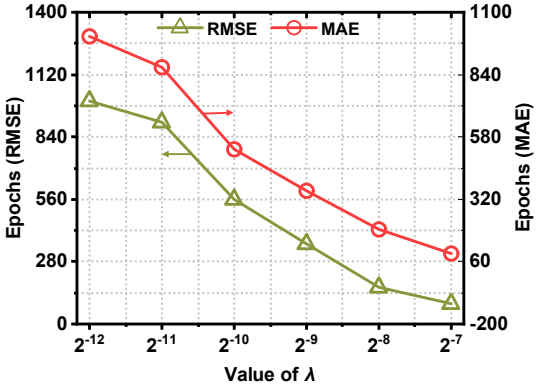
(e) Epochs on D5



(f) Epochs on D6

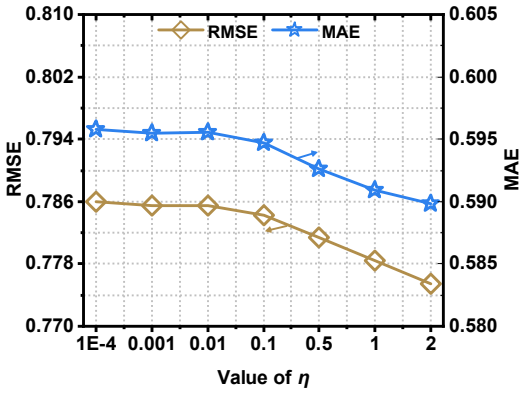


(g) Epochs on D7

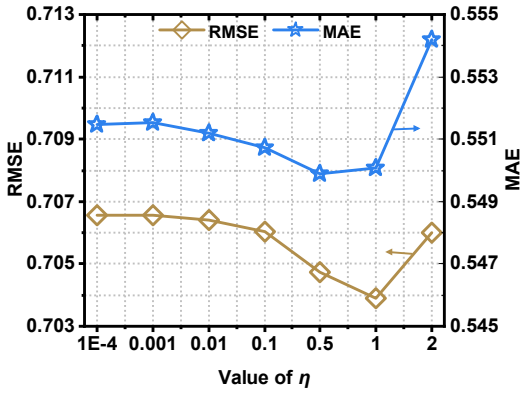


(h) Epochs on D8

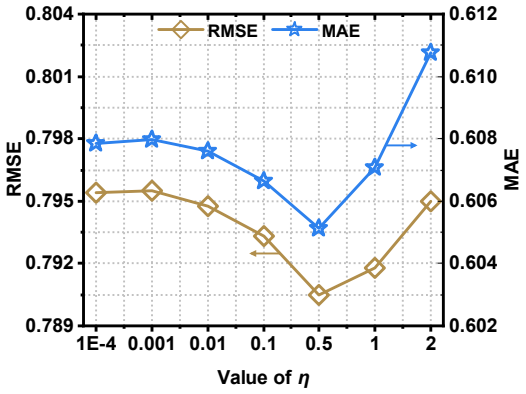
Fig. S2. Epochs of FNAE as λ varies while other hyperparameters are being fixed on D1-8.



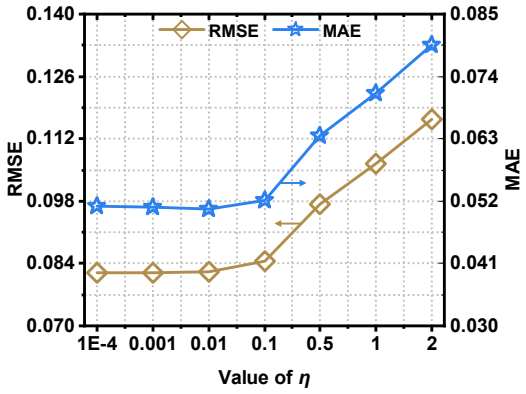
(a) Errors on D1



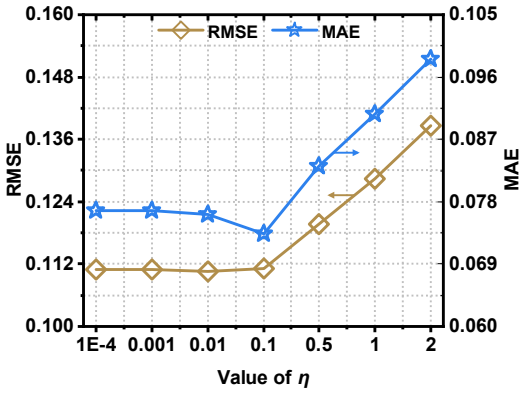
(b) Errors on D2



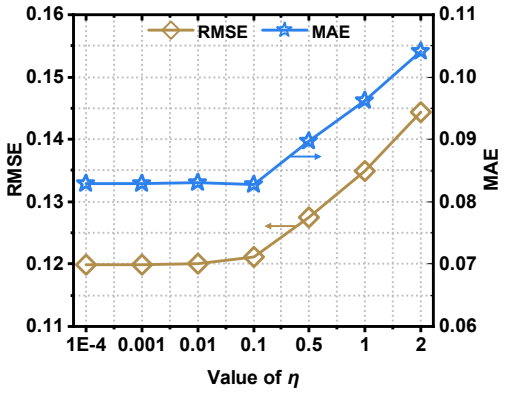
(c) Errors on D3



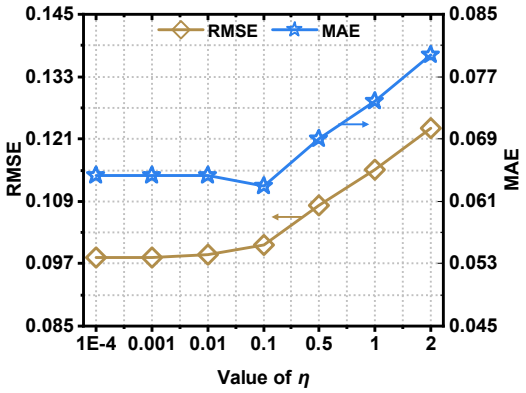
(d) Errors on D4



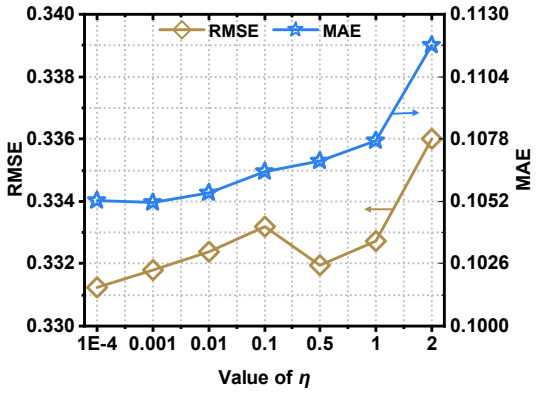
(e) Errors on D5



(f) Errors on D6

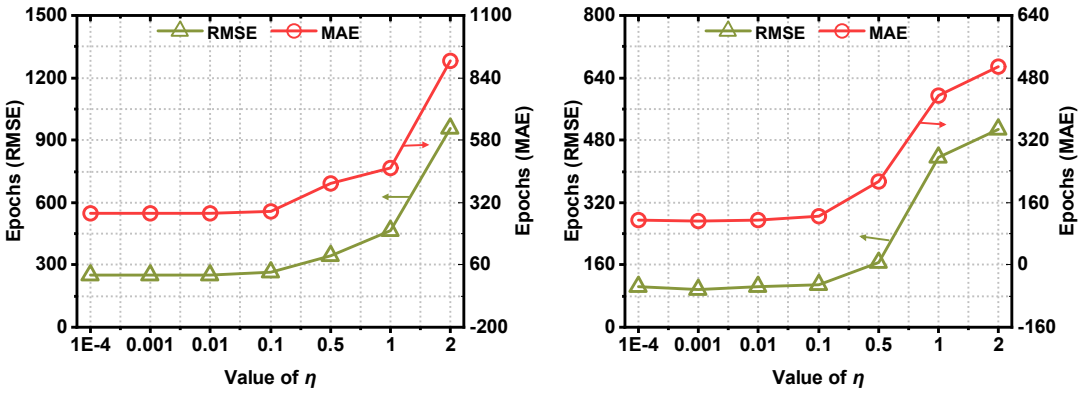


(g) Errors on D7



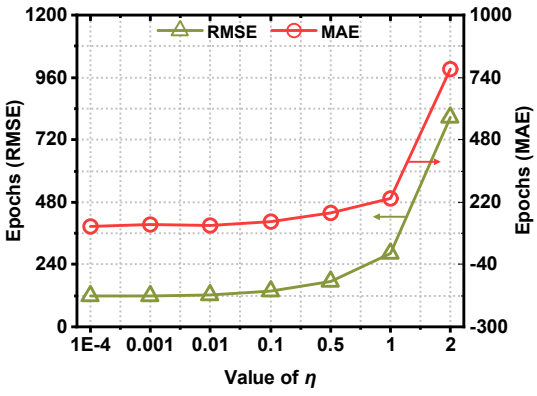
(h) Errors on D8

Fig. S3. Errors of FNAE as η varies while other hyperparameters are being fixed on D1-8.

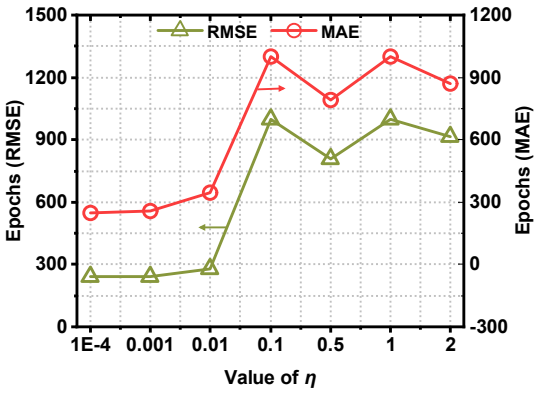


(a) Epochs on D1

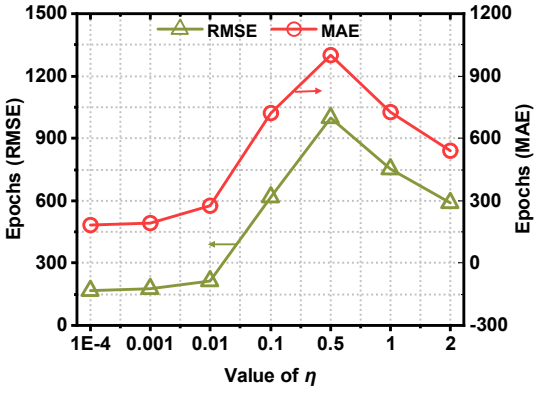
(b) Epochs on D2



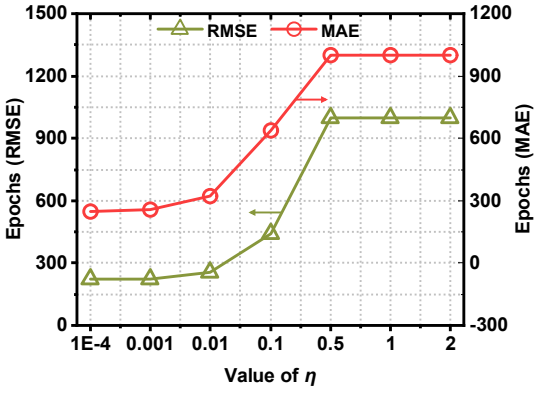
(c) Epochs on D3



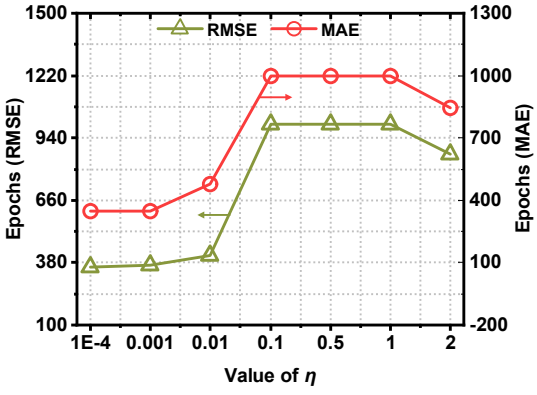
(d) Epochs on D4



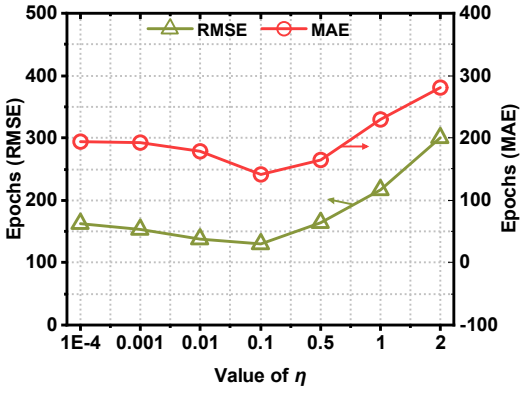
(e) Epochs on D5



(f) Epochs on D6

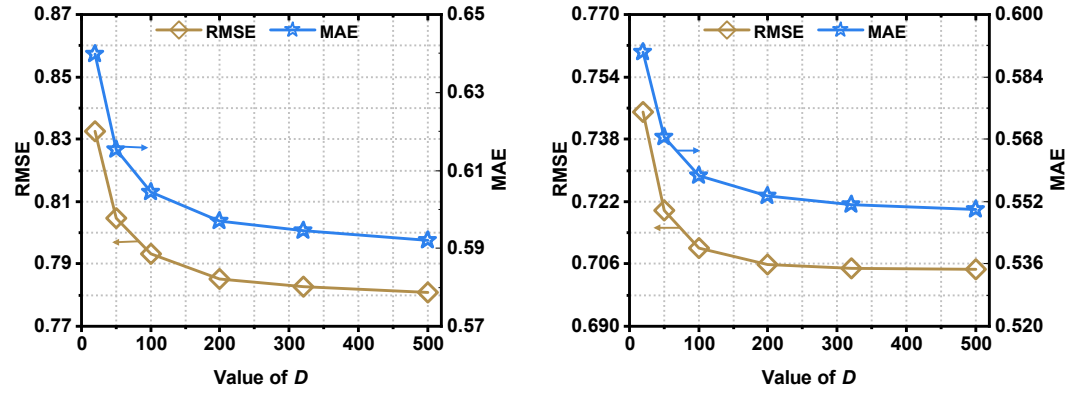


(g) Epochs on D7



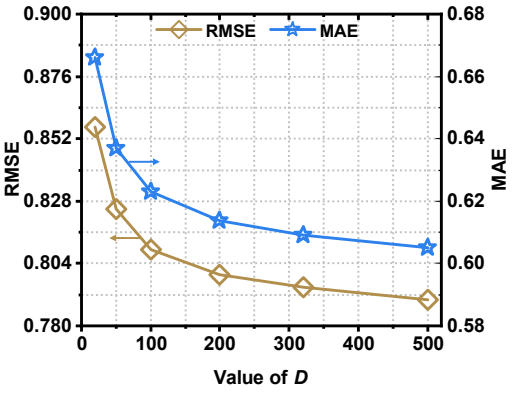
(h) Epochs on D8

Fig. S4. Epochs of FNAE as η varies while other hyperparameters are being fixed on D1-8.

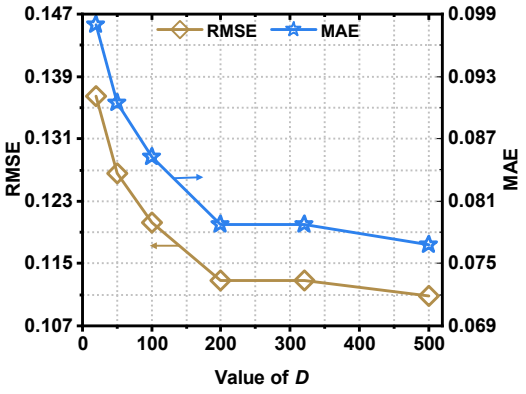


(a) Errors on D1

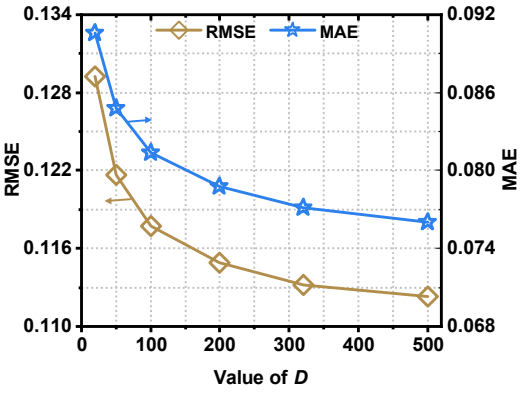
(b) Errors on D2



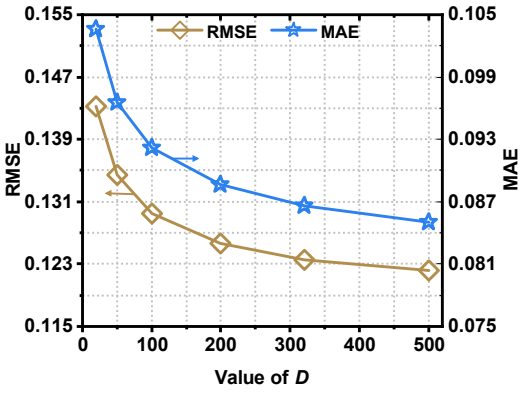
(c) Errors on D3



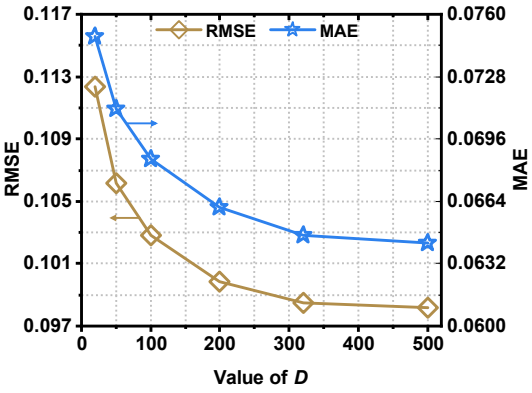
(d) Errors on D4



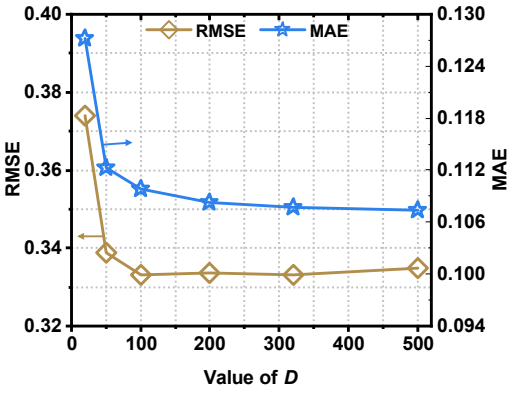
(e) Errors on D5



(f) Errors on D6

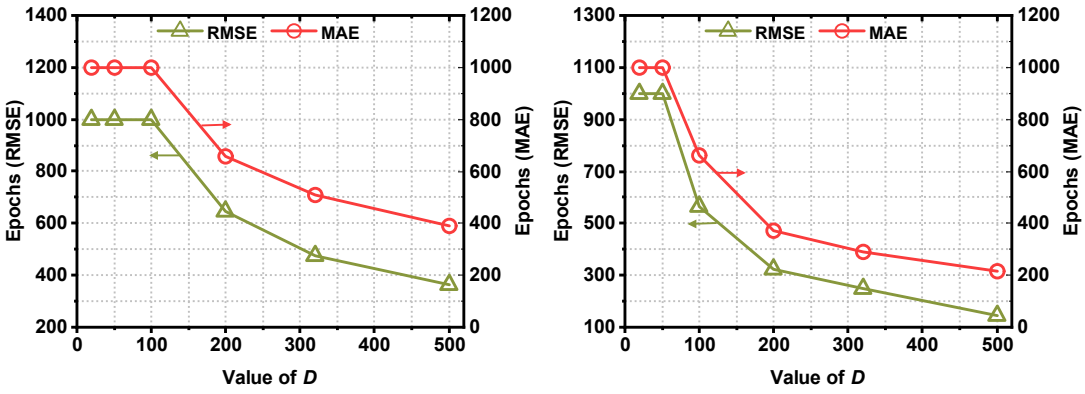


(g) Errors on D7



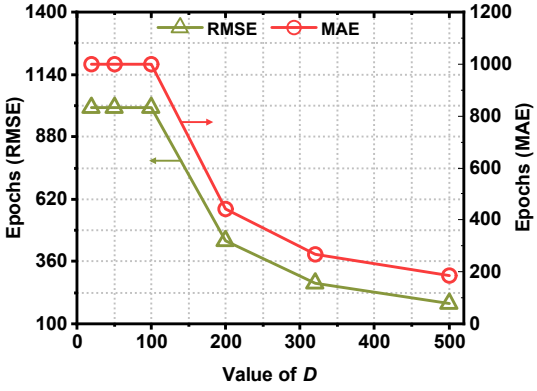
(h) Errors on D8

Fig. S5. Errors of FNAE as *D* varies while other hyperparameters are being fixed on D1-8.

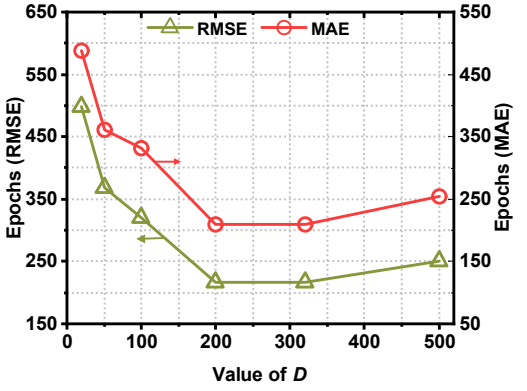


(a) Epochs on D1

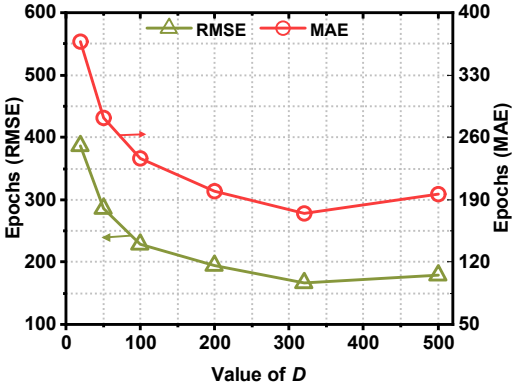
(b) Epochs on D2



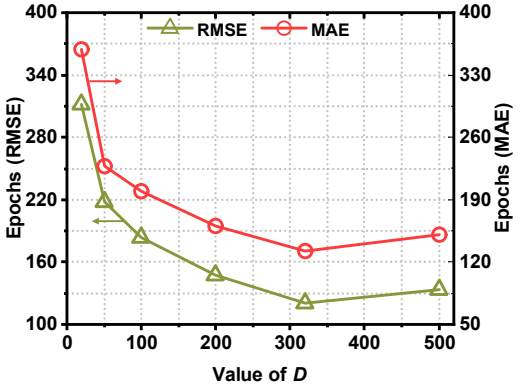
(c) Epochs on D3



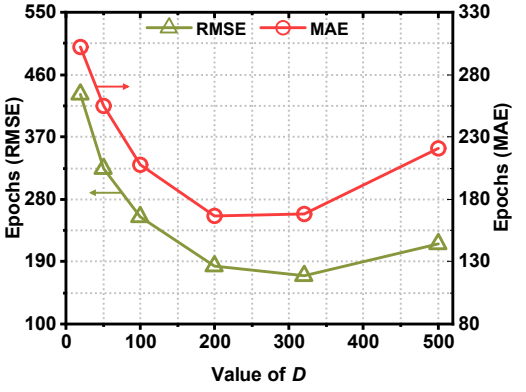
(d) Epochs on D4



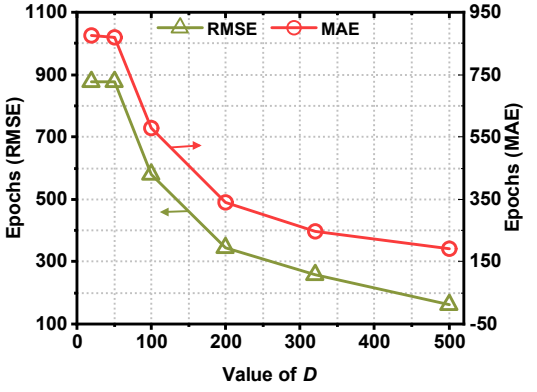
(e) Epochs on D5



(f) Epochs on D6



(g) Epochs on D7



(h) Epochs on D8

Fig. S6. Epochs of FNAE as *D* varies while other hyperparameters are being fixed on D1-8.