

# Package ‘snreg’

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**Description** Models with a skew-normally distributed error term: Badunenko & Henderson (2023). Production analysis with asymmetric noise. Journal of Productivity Analysis, <https://doi.org/10.1007/s11123-023-00680-5>.

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banks07

*U.S. Commercial Banks Data*

## Description

banks07 is a data frame containing selected variables for 500 U.S. commercial banks, randomly sampled from approximately 5000 banks, based on the dataset of Koetter et al. (2012) for year 2007. The dataset is provided solely for illustration and pedagogical purposes and is not suitable for empirical research.

## Usage

```
data(banks07)
```

## Format

A data frame with the following variables:

year Year (2007).  
 id Entity (bank) identifier.  
 TA Gross total assets.  
 LLP Loan loss provisions.  
 Y1 Total securities (thousands of USD).  
 Y2 Total loans and leases (thousands of USD).  
 W1 Cost of fixed assets divided by the cost of borrowed funds.  
 W2 Cost of labor (thousands of USD) divided by the cost of borrowed funds.  
 ER Equity-to-assets ratio (gross).  
 TC Total operating cost.  
 LA Ratio of total loans and leases to gross total assets.

## Details

U.S. Commercial Banks Data (2007)

The dataset was created by sampling and transforming variables as shown in the section **Examples**. It is intended to illustrate the usage of functions from this package (e.g. stochastic frontier models with skew-normal noise).

## Source

<http://qed.econ.queensu.ca/jae/2014-v29.2/restrepo-tobon-kumbhakar/>

## References

- Koetter, M., Kolari, J., & Spierdijk, L. (2012). *Enjoying the quiet life under deregulation? Evidence from adjusted Lerner indices for U.S. banks*. Review of Economics and Statistics, **94**(2), 462–480.
- Restrepo-Tobon, D. & Kumbhakar, S. (2014). *Enjoying the quiet life under deregulation? Not Quite*. Journal of Applied Econometrics, **29**(2), 333–343.

## Examples

```
## Not run:

## -----
## Construct sample panel dataset (banks00_07)
## -----


# Download data from the link in "Source"
banks00_07 <- read.delim("2b_QLH.txt")

# rename 'entity' to 'id'
colnames(banks00_07)[colnames(banks00_07) == "entity"] <- "id"

# keep only years 2000–2007
banks00_07 <- banks00_07[
  banks00_07$year >= 2000 & banks00_07$year <= 2007, ]

# restrict sample to interquartile range of total assets
q1q3 <- quantile(banks00_07$TA, probs = c(.25, .75))
banks00_07 <- banks00_07[
  banks00_07$TA >= q1q3[1] & banks00_07$TA <= q1q3[2], ]

# generate required variables
banks00_07$TC <- banks00_07$TOC
banks00_07$ER <- banks00_07$Z / banks00_07$TA    # Equity ratio
banks00_07$LA <- banks00_07$Y2 / banks00_07$TA    # Loans-to-assets ratio

# keep only needed variables
keep.vars <- c("id", "year", "Ti", "TC", "Y1", "Y2", "W1", "W2",
              "ER", "LA", "TA", "LLP")
banks00_07 <- banks00_07[, colnames(banks00_07) %in% keep.vars]

# number of periods per id
t0 <- as.vector( by(banks00_07$id, banks00_07$id,
                      FUN = function(qq) length(qq)) )
banks00_07$Ti <- rep(t0, times = t0)

# keep if Ti > 4
banks00_07 <- banks00_07[banks00_07$Ti > 4, ]

# complete observations only
banks00_07 <- banks00_07[complete.cases(banks00_07), ]

# sample 500 banks at random
set.seed(816376586)
id_names <- unique(banks00_07$id)
ids2choose <- sample(id_names, 500)
banks00_07 <- banks00_07[banks00_07$id %in% ids2choose, ]

# recompute Ti
t0 <- as.vector( by(banks00_07$id, banks00_07$id,
                      FUN = function(qq) length(qq)) )
banks00_07$Ti <- rep(t0, times = t0)
banks00_07 <- banks00_07[banks00_07$Ti > 4, ]

# sort
```

```

banks00_07 <- banks00_07[order(banks00_07$id, banks00_07$year), ]

banks07 <- banks00_07[banks00_07$year == 2007, ]

## End(Not run)

```

**coef.snreg***Extract Model Coefficients***Description**

`coef.snreg` is the S3 method for extracting the estimated regression coefficients from an object of class "snreg".

**Usage**

```
## S3 method for class 'snreg'
coef(obj, ...)
```

**Arguments**

<code>obj</code>	an object of class "snreg", typically returned by <a href="#">snreg</a> .
<code>...</code>	additional arguments (currently unused).

**Details**

Coefficients from an snreg Model

This method simply returns the `coef` component stored inside the fitted "snreg" object. If the object does not contain coefficient estimates (e.g., if estimation was not completed in a scaffold), an informative error is raised.

**Value**

A numeric vector containing the model coefficients.

**Examples**

```

## Not run:
m <- snreg(y ~ x1 + x2, data = df)
coef(m)

## End(Not run)

```

---

<code>lm.mle</code>	<i>Linear Regression via MLE</i>
---------------------	----------------------------------

---

## Description

`lm.mle` fits a linear regression model by maximum likelihood, allowing for optional multiplicative heteroskedasticity in the disturbance variance via a log-linear specification provided through `ln.var.v`.

## Usage

```
lm.mle(
  formula,
  data,
  subset,
  ln.var.v = NULL,
  technique = c("bfgs"),
  lmtol = 1e-05,
  reltol = 1e-12,
  maxit = 199,
  optim.report = 1,
  optim.trace = 10,
  print.level = 3,
  digits = 4,
  only.data = FALSE,
  ...
)
```

## Arguments

<code>formula</code>	an object of class <code>formula</code> specifying the regression: typically $y \sim x_1 + \dots$ , where $y$ is the dependent variable and the $x$ 's are regressors.
<code>data</code>	an optional <code>data.frame</code> containing the variables referenced in <code>formula</code> . If not found in <code>data</code> , variables are taken from <code>environment(formula)</code> .
<code>subset</code>	an optional logical or numeric vector specifying the subset of observations to be used in estimation.
<code>ln.var.v</code>	optional one-sided formula; e.g. <code>ln.var.v ~ z1 + z2</code> . When provided, the error variance is modeled as $\log(\sigma_i^2) = w_i^\top \gamma_v$ . If <code>NULL</code> , the variance is homoskedastic.
<code>technique</code>	character vector specifying the preferred optimization routine(s) in order of preference. Recognized keywords (for future implementation) include "bfgs", "bhhh", "nm" (Nelder–Mead), "bfgs", and "cg". Default is "bfgs". This scaffold records but does not execute the chosen routine.
<code>lmtol</code>	numeric. Convergence tolerance based on scaled gradient (when applicable). Default <code>1e-5</code> .
<code>reldtol</code>	numeric. Relative convergence tolerance for likelihood maximization. Default <code>1e-12</code> .
<code>maxit</code>	integer. Maximum number of iterations for the optimizer. Default 199.
<code>optim.report</code>	integer. Verbosity level for reporting progress (if implemented). Default 1.

optim.trace	integer. Trace level for optimization (if implemented). Default 1.
print.level	integer. Printing level for summaries. Default 3.
digits	integer. Number of digits for printing. Default 4.
only.data	logical. If TRUE, returns only constructed data/matrices without estimation. Default FALSE.
...	additional arguments reserved for future methods (e.g., bounds, penalties).

## Details

Linear Model by Maximum Likelihood (with optional heteroskedasticity)

The model is

$$y_i = x_i^\top \beta + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \sigma_i^2).$$

When `ln.var.v` is supplied, the variance follows

$$\log(\sigma_i^2) = w_i^\top \gamma_v,$$

otherwise  $\sigma_i^2 = \sigma^2$  is constant (homoskedastic).

This function:

- Builds the model frame and `X, y`.
- Builds `Zv` for the log-variance index when `ln.var.v` is provided.
- Returns a structured object with placeholders for `coef, vcov, loglik`.

Insert your MLE engine to estimate  $\beta$ , and (optionally)  $\sigma^2$  or  $\gamma_v$ ; compute standard errors via AIM/OPG as required by `vcetype`.

## Value

A list of class "snreg" containing (and extending) the fields returned by `optim`:

- `par` — numeric vector of the MLE parameter estimates.
- `value` — numeric scalar: maximized log-likelihood value.
- `ll` — numeric scalar: maximized log-likelihood value.
- `counts, convergence, message` — standard `optim` outputs.
- `hessian` — the observed Hessian at the solution (as returned by `optim(hessian=TRUE)`).
- `coef` — named numeric vector equal to `par` (estimates).
- `vcov` — variance-covariance matrix, computed as `solve(-hessian)`.
- `sds` — standard errors, `sqrt(diag(vcov))`.
- `ctab` — coefficient table with columns: Estimate, Std.Err, Z value, Pr(>z).
- `esample` — logical vector indicating the observations used in estimation.
- `n` — scalar, number of observations in the estimation sample.

The object inherits the default `optim` components and is assigned class "snreg".

## Examples

```
## Not run:

library(snreg)

data("banks07")
head(banks07)

# Translog cost function specification
spe.tl <- log(TC) ~ log(Y1) + log(Y2) + log(W1) + log(W2))^2 +
  I(0.5 * log(Y1)^2) + I(0.5 * log(Y2)^2) +
  I(0.5 * log(W1)^2) + I(0.5 * log(W2)^2)

# -----
# Specification 1: homoskedastic noise (ln.var.v = NULL)
# -----
formSV <- NULL

m1 <- lm.mle(
  formula = spe.tl,
  data = banks07,
  ln.var.v = formSV
)
coef(m1)

# -----
# Specification 2: heteroskedastic noise (variance depends on TA)
# -----
formSV <- ~ log(TA)

m2 <- lm.mle(
  formula = spe.tl,
  data = banks07,
  ln.var.v = formSV
)
coef(m2)

## End(Not run)
```

`print.summary.snreg`    *Print Summary of snreg Results*

## Description

Prints the contents of a "summary.snreg" object in a structured format. The method reports convergence status (based on gradient-Hessian scaling), log-likelihood, estimation results, and—when present—summaries for technical/cost efficiencies and marginal effects.

## Usage

```
## S3 method for class 'summary.snreg'
print(obj, digits = NULL, ...)
```

## Arguments

<code>obj</code>	an object of class "summary.snreg" (produced by <a href="#">summary.snreg</a> ).
<code>digits</code>	integer indicating the number of digits to print; default NULL (internally set to 4).
<code>...</code>	additional arguments (currently unused).

## Details

Print Method for Summary of snreg Objects

This method expects a fitted "snreg" object.

## Value

The input `obj` is returned (invisibly) after printing.

## See Also

[summary.snreg](#)

`residuals.snreg`

*Residuals for snreg Objects*

## Description

`residuals.snreg` is the S3 method for extracting residuals from a fitted `snreg` model. Residuals may be returned either for the full data or only for the estimation sample.

## Usage

```
## S3 method for class 'snreg'
residuals(obj, esample = TRUE, ...)
```

## Arguments

<code>obj</code>	an object of class "snreg", typically produced by <a href="#">snreg</a> .
<code>esample</code>	logical. If TRUE (default), residuals are returned only for observations used in estimation (others are NA). If FALSE, the raw vector of residuals ( <code>obj\$resid</code> ) is returned.
<code>...</code>	additional arguments (currently unused).

## Details

Extract Residuals from an `snreg` Model

This method simply accesses the `obj$resid` component of a fitted "snreg" object. An informative error is produced if residuals are not available.

## Value

A numeric vector of residuals. If `esample = TRUE`, the vector matches the length of the original data and contains NA for non-estimation observations. If `esample = FALSE`, only the computed residuals are returned.

## See Also

`snreg`, `fitted.snreg`, `coef.snreg`

## Examples

```
## Not run:
m <- snreg(y ~ x1 + x2, data = df)

# Residuals for estimation sample only
residuals(m)

# Residuals for all observations
residuals(m, esample = FALSE)

## End(Not run)
```

## Description

`snreg` fits a linear regression model where the disturbance term follows a skew-normal distribution. The function supports multiplicative heteroskedasticity of the noise variance via a log-linear specification (`ln.var.v`) and allows the skewness parameter to vary linearly with exogenous variables (`skew.v`).

## Usage

```
snreg(
  formula,
  data,
  subset,
  init.sk = NULL,
  ln.var.v = NULL,
  skew.v = NULL,
  start.val = NULL,
  technique = c("nr"),
  vcetype = c("aim"),
  lmtol = 1e-05,
  reltol = 1e-12,
  maxit = 199,
  optim.report = 1,
  optim.trace = 1,
  print.level = 3,
  digits = 4,
```

```
only.data = FALSE,
...
)
```

## Arguments

formula	an object of class <code>formula</code> specifying the regression: typically $y \sim x_1 + \dots$ , where $y$ is the dependent variable and $x$ 's are regressors.
data	an optional <code>data.frame</code> containing the variables in <code>formula</code> . If not found in <code>data</code> , variables are taken from <code>environment(formula)</code> .
subset	an optional logical or numeric vector specifying the subset of observations to be used in estimation.
init.sk	numeric. Initial value for the (global) skewness parameter of the noise; can be <code>NULL</code> if <code>skew.v</code> is supplied with its own coefficients to initialize.
ln.var.v	optional one-sided formula; e.g. <code>ln.var.v ~ z1 + z2</code> . Specifies exogenous variables entering the (log) variance of the random noise component. If <code>NULL</code> , the noise variance is homoskedastic.
skew.v	optional one-sided formula; e.g. <code>skew.v ~ z3 + z4</code> . Specifies exogenous variables determining the skewness of the noise via a linear index; if <code>NULL</code> , the skewness is constant (scalar).
start.val	optional numeric vector of starting values for all free parameters (regression coefficients, variance/heteroskedasticity parameters, skewness parameters).
technique	character vector giving the preferred maximization routine(s) in order of preference. Currently recognized keywords include "nr" (Newton–Raphson), "bhhh", "nm" (Nelder–Mead), "bfgs", "cg". This scaffold does not implement them yet, but records the choice.
vcetype	character specifying the variance-covariance estimator type: "aim" for the approximated information matrix or "opg" for the outer product of gradients. Default is "aim".
lmtol	numeric. Convergence tolerance based on the scaled gradient (if applicable). Default is <code>1e-5</code> .
reltol	numeric. Relative convergence tolerance for likelihood maximization. Default is <code>1e-12</code> .
maxit	integer. Maximum number of iterations for the optimizer. Default is 199.
optim.report	integer. Verbosity for reporting progress (if implemented). Default is 1.
optim.trace	integer. If positive, tracing information is printed (if implemented). Default is 1.
print.level	integer. Printing level for summaries: 1—print estimation results; 2—print optimization details; 3—print compact summary. Default 3.
digits	integer. Number of digits for printing. Default 4.
only.data	logical. If <code>TRUE</code> , the function returns only the constructed model matrices and design sets (no estimation). Default <code>FALSE</code> .
...	additional arguments reserved for future methods (e.g., box constraints).

## Details

Linear Regression with Skew-Normal Errors

The model is

$$y_i = x_i^\top \beta + \varepsilon_i, \quad \varepsilon_i \sim SN(0, \sigma_i^2, \alpha_i),$$

where  $SN$  denotes the skew-normal distribution (Azzalini).

Heteroskedasticity in the noise variance (if specified via `ln.var.v`) is modeled as

$$\log(\sigma_i^2) = w_i^\top \gamma_v,$$

and the (optional) covariate-driven skewness (if specified via `skew.v`) as

$$\alpha_i = s_i^\top \delta.$$

This function constructs the model frame and design matrices for  $\beta$ ,  $\gamma_v$ , and  $\delta$ , and is designed to be paired with a maximum likelihood routine to estimate parameters and (optionally) their asymptotic covariance via either AIM or OPG.

### Value

An object of class "snreg" containing the maximum-likelihood results and, depending on the optimization routine, additional diagnostics:

`par` Numeric vector of parameter estimates at the optimum.  
`coef` Named numeric vector equal to `par`.  
`vcov` Variance–covariance matrix of the estimates.  
`sds` Standard errors, computed as `sqrt(diag(vcov))`.  
`ctab` Coefficient table with columns: Estimate, Std.Err, Z value, Pr(>z).  
`RSS` Residual sum of squares.  
`esample` Logical vector indicating which observations were used in estimation.  
`n` Number of observations used in the estimation sample.  
`skewness` Vector of the fitted skewness index.  
`hessian` (BFGS only) Observed Hessian at the optimum. If `vcetype == "opg"`, this is set to the negative outer product of the individual gradients; otherwise a numerical Hessian is computed.  
`value` (BFGS only) Objective value returned by `optim`. With `control$fnscale = -1`, this equals the maximized log-likelihood.  
`counts` (BFGS only) Number of iterations / function evaluations returned by `optim`.  
`convergence` (BFGS only) Convergence code from `optim`.  
`message` (BFGS only) Additional `optim` message, if any.  
`ll` Maximized log-likelihood value.  
`gradient` (NR only) Gradient at the solution.  
`gg` (NR only) Optional gradient-related diagnostic.  
`ghg` (NR only) Optional Newton-step diagnostic.  
`theta_rel_ch` (NR only) Relative parameter change metric across iterations.

The returned object has class "snreg".

### References

- Azzalini, A. (1985). *A Class of Distributions Which Includes the Normal Ones*. Scandinavian Journal of Statistics, 12(2), 171–178.
- Azzalini, A., & Capitanio, A. (2014). *The Skew-Normal and Related Families*. Cambridge University Press.

## Examples

```
## Not run:

library(snreg)

data("banks07")
head(banks07)

# Translog cost function
spe.tl <- log(TC) ~ log(Y1) + log(Y2) + log(W1) + log(W2))^2 +
  I(0.5 * log(Y1)^2) + I(0.5 * log(Y2)^2) +
  I(0.5 * log(W1)^2) + I(0.5 * log(W2)^2)

# -----
# Specification 1: homoskedastic & symmetric noise
# -----
formSV <- NULL      # variance equation
formSK <- NULL      # skewness equation

m1 <- snreg(
  formula  = spe.tl,
  data     = banks07,
  ln.var.v = formSV,
  skew.v   = formSK
)
coef(m1)

# -----
# Specification 2: heteroskedastic + skewed noise
# -----
formSV <- ~ log(TA)  # heteroskedasticity in v
formSK <- ~ ER        # skewness driven by equity ratio

m2 <- snreg(
  formula  = spe.tl,
  data     = banks07,
  ln.var.v = formSV,
  skew.v   = formSK
)
coef(m2)

## End(Not run)
```

## Description

`snsf` performs maximum likelihood estimation of the parameters and technical or cost efficiencies in a Stochastic Frontier Model with a skew-normally distributed error term.

## Usage

```
snsf(
  formula,
  data,
  subset,
  distribution = "e",
  prod = TRUE,
  start.val = NULL,
  init.sk = NULL,
  ln.var.u = NULL,
  ln.var.v = NULL,
  skew.v = NULL,
  mean.u = NULL,
  technique = c("nr"),
  vcetype = c("aim"),
  optim.report = 1,
  optim.trace = 1,
  reltol = 1e-12,
  lmtol = 1e-05,
  maxit = 199,
  print.level = 3,
  report = 1,
  trace = 1,
  threads = 1,
  only.data = FALSE,
  digits = 4,
  ...
)
```

## Arguments

<code>formula</code>	an object of class <code>formula</code> specifying the frontier: a typical model is $y \sim x_1 + \dots$ , where $y$ is the log of output (or total cost), and $x$ 's are inputs (or outputs and input prices, in logs). See <b>Details</b> .
<code>data</code>	an optional <code>data.frame</code> containing the variables in <code>formula</code> . If not found in <code>data</code> , variables are taken from <code>environment(formula)</code> .
<code>subset</code>	an optional logical or numeric vector specifying a subset of observations for which the model is estimated and efficiencies are computed.
<code>distribution</code>	character scalar specifying the distribution of the inefficiency term: default "e" (exponential). "h" (half-normal) and "t" (truncated normal) to be implemented.
<code>prod</code>	logical. If <code>TRUE</code> , estimates correspond to a stochastic <i>production</i> frontier and technical efficiencies are returned; if <code>FALSE</code> , estimates correspond to a stochastic <i>cost</i> frontier and cost efficiencies are returned. Default is <code>TRUE</code> .
<code>start.val</code>	optional numeric vector of starting values for the optimizer.
<code>init.sk</code>	numeric. Initial value for the skewness parameter of the noise component; default is 0.5.

<code>ln.var.u</code>	optional one-sided formula; e.g. <code>ln.var.u = ~ z3 + z4</code> . Specifies exogenous variables entering the (log) variance of the inefficiency component. If <code>NULL</code> , the inefficiency variance is homoskedastic, i.e., $\sigma_{u0}^2 = \exp(\gamma_{u0}[0])$ .
<code>ln.var.v</code>	optional one-sided formula; e.g. <code>ln.var.v = ~ z1 + z2</code> . Specifies exogenous variables entering the (log) variance of the random noise component. If <code>NULL</code> , the noise variance is homoskedastic, i.e., $\sigma_{v0}^2 = \exp(\gamma_{v0}[0])$ .
<code>skew.v</code>	optional one-sided formula; e.g. <code>skew.v = ~ z5 + z6</code> . Allows the skewness of the noise to depend linearly on exogenous variables. If <code>NULL</code> , the skewness is constant across units.
<code>mean.u</code>	optional one-sided formula; e.g. <code>mean.u = ~ z7 + z8</code> . Specifies whether the mean of the pre-truncated normal distribution of the inefficiency term is a linear function of exogenous variables. In cross-sectional models, used only when <code>distribution = "t"</code> . If <code>NULL</code> , the mean is constant across units. To be implemented.
<code>optim.report</code>	integer. Verbosity level for reporting during optimization (if implemented). Default is 1.
<code>optim.trace</code>	integer. Trace level for optimization (if implemented). Default is 1.
<code>reltol</code>	numeric. Relative convergence tolerance used when maximizing the log-likelihood with <code>optim</code> . The algorithm stops if it cannot reduce the objective by a factor of <code>reltol * (abs(val) + reltol)</code> at a step. Default is <code>sqrt(.Machine\$double.eps)</code> .
<code>lmtol</code>	numeric. Convergence tolerance based on the scaled gradient (when applicable). Default is <code>1e-5</code> .
<code>maxit</code>	numeric. Maximum number of iterations for the optimizer. Default is 199.
<code>print.level</code>	integer. Printing level: 1—estimation results; 2—optimization details; 3—summary of (cost/technical) efficiencies; 4—unit-specific point and interval estimates of efficiencies. Default is 3.
<code>digits</code>	integer. Number of digits for displaying estimates and efficiencies. Default is 4.
<code>...</code>	additional arguments passed to internal methods or to <code>optim</code> , as relevant (e.g., <code>cost.eff.less.one = TRUE</code> for cost-frontier conventions).
<code>optim</code>	logical. If <code>TRUE</code> , estimation proceeds via <code>stats:::optim</code> ; if <code>FALSE</code> , an internal routine (if provided) would be used. Default is <code>FALSE</code> .
<code>optim.method</code>	character. Method passed to <code>stats:::optim</code> when <code>optim = TRUE</code> . Default is <code>"bfgs"</code> .
<code>optim.reltol</code>	numeric. Relative tolerance specifically for <code>optim</code> ; default <code>1e-8</code> .

## Details

Stochastic Frontier Model with a Skew-Normally Distributed Error Term

Models for `snsf` are specified symbolically. A typical model has the form  $y \sim x_1 + \dots$ , where  $y$  represents the logarithm of outputs or total costs and  $\{x_1, \dots\}$  is a set of inputs (for production) or outputs and input prices (for cost), all typically in logs.

Options `ln.var.u` and `ln.var.v` allow for multiplicative heteroskedasticity in the inefficiency and/or noise components; i.e., their variances can be modeled as exponential functions of exogenous variables (including an intercept), as in Caudill et al. (1995).

**Value**

An object of class "snreg" with maximum-likelihood estimates and diagnostics:

**par** Numeric vector of ML parameter estimates at the optimum.  
**coef** Named numeric vector equal to **par**.  
**vcov** Variance–covariance matrix of the estimates.  
**sds** Standard errors,  $\sqrt{\text{diag}(\text{vcov})}$ .  
**ctab** Coefficient table with columns Coef., SE, z, P>|z|.  
**ll** Maximized log-likelihood value.  
**hessian** (When computed) Observed Hessian or OPG used to form **vcov**.  
**value** (Optim-only, before aliasing) Objective value from **optim**.  
**counts** (Optim-only) Iteration and evaluation counts from **optim**.  
**convergence** Convergence code).  
**message** (Optim-only) Message returned by **optim**, if any.  
**gradient** (NR-only) Gradient at the solution.  
**gg** (NR-only) Gradient-related diagnostic.  
**gHg** (NR-only) Newton-step diagnostic.  
**theta\_rel\_ch** (NR-only) Relative parameter change metric across iterations.  
**resid** Regression residuals.  
**RSS** Residual sum of squares  $\text{crossprod}(\text{resid})$ .  
**shat2** Residual variance estimate  $\text{var}(\text{resid})$ .  
**shat** Residual standard deviation  $\sqrt{\text{shat2}}$ .  
**aic** Akaike Information Criterion.  
**bic** Bayesian Information Criterion.  
**Mallows** Mallows'  $C_p$ -like statistic.  
**u** Estimated inefficiency term (vector). Returned for models with an inefficiency component (e.g., exponential).  
**eff** Efficiency scores  $\exp(-\text{u})$  (technical or cost, depending on **prod**).  
**sv** Estimated (possibly unit-specific) standard deviation of the noise term.  
**su** Estimated (possibly unit-specific) standard deviation or scale of the inefficiency term. For exponential models.  
**skewness** Estimated skewness index (e.g., from the skewness equation).  
**esample** Logical vector marking observations used in estimation.  
**n** Number of observations used.

The returned object has class "snreg".

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## References

- Badunenko, O., & Henderson, D. J. (2023). *Production analysis with asymmetric noise*. Journal of Productivity Analysis, **61**(1), 1–18. <https://doi.org/10.1007/s11123-023-00680-5>
- Caudill, S. B., Ford, J. M., & Gropper, D. M. (1995). *Frontier estimation and firm-specific inefficiency measures in the presence of heteroskedasticity*. Journal of Business & Economic Statistics, **13**(1), 105–111.

## See Also

[sf](#)

## Examples

```
## Not run:

library(snreg)

data("banks07")
head(banks07)

myprod <- FALSE

# Translog cost function
spe.tl <- log(TC) ~ (log(Y1) + log(Y2) + log(W1) + log(W2))^2 +
  I(0.5 * log(Y1)^2) + I(0.5 * log(Y2)^2) +
  I(0.5 * log(W1)^2) + I(0.5 * log(W2)^2)

# -----
# Specification 1: homoskedastic & symmetric
# -----
formSV <- NULL    # variance equation
formSK <- NULL    # skewness equation
formSU <- NULL    # inefficiency equation (unused here)

m1 <- snsf(
  formula  = spe.tl,
  data     = banks07,
  prod     = myprod,
  ln.var.v = formSV,
  skew.v   = formSK
)

coef(m1)

# -----
# Specification 2: heteroskedastic + skewed noise
# -----
formSV <- ~ log(TA)      # heteroskedastic variance
formSK <- ~ ER            # skewness driver
formSU <- ~ LA + ER       # inefficiency

m2 <- snsf(
  formula  = spe.tl,
  data     = banks07,
```

```

prod      = myprod,
ln.var.v = formSV,
skew.v   = formSK
)

coef(m2)

## End(Not run)

```

<b>su</b>	<i>Summary Utility (su)</i>
-----------	-----------------------------

## Description

Computes a compact table of summary statistics for each variable in a vector, matrix, or data frame. The following metrics are returned per variable: number of observations (`Obs`), missing values (`NAs`), mean, standard deviation (`StDev`), interquartile range (`IQR`), minimum (`Min`), user-specified quantiles (`probs`), and maximum (`Max`).

## Usage

```

su(
  x,
  mat.var.in.col = TRUE,
  digits = 4,
  probs = c(0.1, 0.25, 0.5, 0.75, 0.9),
  print = FALSE
)

```

## Arguments

<code>x</code>	a numeric vector, matrix, or data frame. For matrices, variables are assumed to be in columns; set <code>mat.var.in.col = FALSE</code> to treat rows as variables.
<code>mat.var.in.col</code>	logical. If <code>TRUE</code> (default), a matrix is interpreted as variables in columns. If <code>FALSE</code> , the matrix is transposed so that rows are treated as variables.
<code>digits</code>	integer. Number of digits to use when printing (only affects printed output when <code>print = TRUE</code> ). Default is 4.
<code>probs</code>	numeric vector of probabilities in $[0, 1]$ for which quantiles are computed. Default is <code>c(0.1, 0.25, 0.5, 0.75, 0.9)</code> .
<code>print</code>	logical. If <code>TRUE</code> , prints the transposed summary table using the specified number of digits. Default is <code>FALSE</code> .

## Details

Compact Summary Statistics for Vectors, Matrices, and Data Frames

Input handling:

- If `x` is a matrix with a single row or column, it is treated like a vector. Column or row names are used (if available). Otherwise, a default name is created.

- If  $x$  is a matrix with multiple variables, variables are taken as columns. Use `mat.var.in.col = FALSE` to transpose and treat rows as variables.
- If  $x$  is a vector, its deparsed symbol name is used as the variable name.
- If  $x$  is a data frame, each column is summarized.

Missing values are excluded in all summary computations.

## Value

A matrix (coercible to `data.frame`) where each row corresponds to a variable and columns contain the summary statistics: Obs, NAs, Mean, StDev, IQR, Min, the requested probs quantiles (named), and Max. The returned object is given class "snreg" for compatibility with package-specific print/summarization methods.

## Examples

```
## Not run:
# Vector
set.seed(1)
v <- rnorm(100)
su(v, print = TRUE)

# Matrix: variables in columns
M <- cbind(x = rnorm(50), y = runif(50))
su(M)

# Matrix: variables in rows
Mr <- rbind(x = rnorm(50), y = runif(50))
su(Mr, mat.var.in.col = FALSE)

# Data frame
DF <- data.frame(a = rnorm(30), b = rexp(30), c = rbinom(30, 1, 0.3))
out <- su(DF)
head(out)

## End(Not run)
```

## Description

Produces a summary object for objects of class "snreg". The function assigns the class "summary.snreg" to the fitted model object, enabling a dedicated print method (`print.summary.snreg`) to display results in a structured format.

## Usage

```
## S3 method for class 'snreg'
summary(obj, ...)
```

## Arguments

- |     |  |
|-----|--|
| obj | an object of class "snreg", typically returned by <code>snreg</code> . |
| ... | additional arguments (currently not used).                             |

## Details

### Summary Method for snreg Objects

This method expects a fitted "snreg" object.

`summary.snreg` does not modify the contents of the object; it only updates the class attribute to "summary.snreg". The corresponding print method (`print.summary.snreg`) is responsible for formatting and displaying estimation details, such as convergence criteria, log-likelihood, coefficient tables, and (if present) heteroskedastic and skewness components.

## Value

An object of class "summary.snreg", identical to the input obj except for its class attribute.

## See Also

`snreg`, `print.summary.snreg`

## Examples

```
## Not run:
# m <- snreg(TC ~ Y1 + Y2, data = banks07)
# s <- summary(m)
# print(s)

## End(Not run)
```

## Description

`TOwen1` computes an Owen's  $T$ -function variant (or a related special function) for vectors  $h$  and  $a$  based on the  $t$  function in [https://people.sc.fsu.edu/~jb Burkardt/c\\_src/owen/owen.html](https://people.sc.fsu.edu/~jb Burkardt/c_src/owen/owen.html). Non-finite inputs (in  $h$  or  $a$ ) produce NA at corresponding positions, while finite pairs are computed in C in a vectorized fashion.

## Usage

```
TOwen(h, a, threads = 1)
```

## Arguments

- |                      |   |
|----------------------|---|
| <code>h</code>       | numeric vector of $h$ arguments.  |
| <code>a</code>       | numeric vector of $a$ arguments. Must be either the same length as $h$ or of length 1 (will be recycled by standard R rules). |
| <code>threads</code> | integer. Number of threads to request from the C implementation (if supported). Default is 1.                                 |

## Details

Owen's T Function via C Backend

Owen's  $T$  function is commonly defined as

$$T(h, a) = \frac{1}{2\pi} \int_0^a \frac{\exp\left(-\frac{1}{2}h^2(1+t^2)\right)}{1+t^2} dt,$$

for real  $h$  and  $a$ .

The function accepts vector inputs and:

- Computes results only for entries where both  $h$  and  $a$  are finite.
- Returns NA where either  $h$  or  $a$  is non-finite.
- Optionally passes a `threads` hint to the C backend (ignored if not supported).

## Value

A numeric vector of length `length(h)` containing  $T(h_i, a_i)$ . Elements where either  $h_i$  or  $a_i$  is not finite are NA. The returned object is given class "snreg" for downstream compatibility with your package's print/summary helpers.

## See Also

`pnorm`, `dnorm`

## Examples

```
## Not run:
# Basic usage. Vectorized 'a'
h <- c(-1, 0, 1, 2)
a <- 0.5
TOwen(h, a)

# Vectorized 'a' with non-finite entries; non-finite entries yield NA
a2 <- c(0.2, NA, 1, Inf)
TOwen(h, a2)

## End(Not run)
```

TOwen1

*Compute Owen-like Function* TOwen1( $h, a$ )

## Description

TOwen1 computes an Owen's  $T$ -function variant (or a related special function) for vectors  $h$  and  $a$  based on the `tha` function in [https://people.sc.fsu.edu/~jburkardt/c\\_src/owen/owen.html](https://people.sc.fsu.edu/~jburkardt/c_src/owen/owen.html). Non-finite inputs in  $h$  or  $a$  yield NA at the corresponding positions.

## Usage

`TOwen1(h, a, threads = 1)`

## Arguments

threads	integer. Number of threads to request from the C implementation (if supported). Default is 1.
---------	---

## Details

Owen's T Function Variant via C Backend

This is a thin R wrapper around a native routine with signature:

```
void TOwen1(int *n, double *h, double *a, double *out, int *threads)
```

## Value

A numeric vector of length `length(h)` with the computed values. Elements where either `h` or `a` is non-finite are NA. The returned vector is given class "snreg" for downstream compatibility.

## See Also

[TOwen](#)

## Examples

```
## Not run:
library(snreg)

# Basic usage. Vectorized 'a':
h <- c(-1, 0, 1, 2)
a <- 0.3
TOwen1(h, a)

# Vectorized 'a' with non-finite entries:
a2 <- c(0.2, NA, 1, Inf)
TOwen1(h, a2)

## End(Not run)
```

## Description

`vcov.snreg` is the `vcov` S3 method for objects of class "snreg". It returns the model-based variance–covariance matrix stored in the fitted object.

## Usage

```
## S3 method for class 'snreg'
vcov(obj, ...)
```

## Arguments

- obj                   an object of class "snreg", typically returned by [snreg](#).  
...                   additional arguments (currently unused).

## Details

Variance–Covariance Matrix for snreg Objects

This method expects a fitted "snreg" object.

This method simply returns the vcov component stored in obj. If your estimator did not compute standard errors (e.g., because estimation hasn't been run yet in a scaffold), this field may be NULL, and the method will error accordingly.

## Value

A numeric matrix containing the variance–covariance of the estimated parameters.

## See Also

[snreg](#), [summary.snreg](#)

## Examples

```
## Not run:  
library(snsf)  
  
data("banks07")  
head(banks07)  
# V <- vcov(m)  
# diag(V)  
  
## End(Not run)
```

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