

The index x_k of the first bit belonging to node k with depth d_k can be expressed as the sum of:

$$D + 1 + 2$$

the bits allocated for storing depth and padding,

$$\sum_{i=0}^{d_k-1} (2^i \cdot N_i)$$

the total amount of bits needed to store all nodes with depths lower than k , and

$$(k - 2^{d_k}) \cdot N_{d_k}$$

the amount of bits needed for all nodes stored before k but on the same level as k .
Equation (2) from the CBT Paper can now be derived as:

$$\begin{aligned} x_k &= D + 3 + \sum_{i=0}^{d_k-1} (2^i N_i) + (k - 2^{d_k}) \cdot N_{d_k} \\ &= D + 3 + \sum_{i=0}^{d_k-1} (2^i (D - i + 1)) + (k - 2^{d_k}) \cdot N_{d_k} \\ &= D + 3 + ((2^{d_k} - 1)(D + 3) - 2^{d_k} d_k) + (k - 2^{d_k}) \cdot N_{d_k} \\ &= D + 3 + (2^{d_k} (D + 3) - (D + 3) - 2^{d_k} d_k) + k \cdot N_{d_k} - 2^{d_k} \cdot N_{d_k} \\ &= 2^{d_k} (D + 3) - 2^{d_k} d_k - 2^{d_k} (D - d_k + 1) + k \cdot N_{d_k} \\ &= 2^{d_k} (D + 3 - d_k - D + d_k - 1) + k \cdot N_{d_k} \\ &= 2^{d_k} (2) + k \cdot N_{d_k} \\ &= 2^{d_k+1} + k \cdot N_{d_k} \end{aligned}$$