Experiment 5 Rolling Motion

"Our business is with the causes of sensible effects"

I. Newton (1642-1727)

OBJECTIVES

To derive and test a model of rolling with slip.

THEORY

A round object placed on a tilted flat surface as shown in Fig. 5-1 can both roll and slip, depending on the frictional force available and the angle of tilt. The actual behavior should be derivable from Newton's laws and a model of the frictional force.

Referring to the quantities labeled in Fig. 5-1, the equations of motion are

$$Ma = Mg\sin\phi - f \qquad (5\text{-}1)$$

$$I\alpha = -Rf \qquad (5\text{-}2)$$

where M is the mass and I is the moment of inertia about the center of mass. The minus sign in Eq. 5-2 arises because the torque is in the $-\hat{k}$ direction for the coordinates shown. The frictional force can be approximated with the usual coefficient of friction

$$f = \mu N = \mu Mg\cos\phi \qquad \text{slipping} \qquad (5\text{-}3)$$

$$f \le \mu Mg\cos\phi \qquad \text{not slipping} \qquad (5\text{-}4)$$

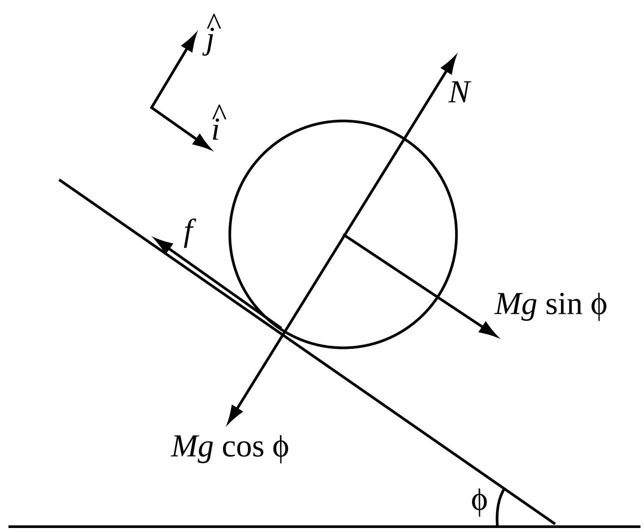

Fig. 5-1 A round object of mass M, radius R and moment of inertia I which can roll and slip on the inclined plane.

neglecting any difference between static and sliding coefficients. If the conditions are such that the object rolls without slipping, there is also a geometric relationship between the displacement of the center of mass and the angle turned or, equivalently, between the accelerations:

$$x = -R\theta \quad \Rightarrow \quad a = -\alpha R \tag{5-5}$$

where the minus sign again occurs because of our choice of coordinates.

The qualitative features of the motion can now be deduced from the equations. For small tilt angles the linear acceleration down the incline will be small. The frictional force can then be large enough to produce the needed angular acceleration to cause rolling without slip and satisfy Eq. 5–5. As the tilt increases, the angular acceleration required to avoid slip and the frictional force needed to produce that angular acceleration also increase. At a critical angle the maximum possible frictional force, given by Eq. 5-4, will be reached and the object will start to slip. The object will still move down the incline with calculable linear and angular accelerations, but they will no longer be related by Eq. 5-5.

Quantitative analysis of each regime is straightforward. For tilts below the critical angle, Eqs. 5-1, 5-2 and 5-5 can be solved for a, α and f in terms of the mass, gravitational acceleration g, and the various geometric factors. You will find, unsurprisingly, that the motion occurs with constant linear and angular accelerations, whose magnitudes increase with tilt. Above the critical angle, f is maximum at the value given by Eq. 5-3. Substituting the maximal f into Eqs. 5–1 and 5-2 yields different accelerations, which now depend on μ as well as the other parameters. Interestingly, the accelerations are still constant in this case. Finally, the critical angle can be found by equating the expression for f in the no-slip case to the maximum possible non-slip f given by Eq. 5-4.

EXPERIMENTAL PROCEDURE

The experimental goal is check the expressions you have derived. You can use the video system to measure *x(t)* and $\theta(t)$ to determine if the accelerations are constant and of the expected magnitude. You can also find out if the transition to slipping occurs at the expected critical angle.

1. Physical arrangement

The round object is a hoop with reference marks at the center and on the rim. It can roll down a smooth ramp whose tilt can be increased by raising a support bar under one end. The actual tilt can be determined from the height difference of the two calibration marks. Be sure to position the ramp so that the motion is perpendicular to the camera axis.

The camera should already be set up on a support across the room from the drop area. Be sure the power is on (plugged in) and the round switch on the back is set to "P Scan" (progressive scan mode). You can check the orientation and field of view when you open the preview screen of the capture program.

The camera has been set for a fast shutter speed (1/500th s) in order to produce a sharp image of rapid motion. Auxiliary lamps, mounted on a stand, provide the additional light needed for a good exposure. Be sure the lights are turned on when you are taking data, and turn them off when you stop taking pictures.

2. Data acquisition

Start the program VideoPoint Capture as you did before. Check the preview screen and adjust the camera zoom or position slightly if necessary to get all of the ramp into the picture. Practice releasing the cylinder from near the top of the ramp until you can get a good recording of it rolling at least most of the way down without falling off. Save the frames that show the motion from just after the start to just before it hits at the bottom, and then go to VideoPoint for analysis.

Ask for two points in the Number of Points window, and then mark the center of the hoop as S1 and the rim as S2 in each frame. Pick Movie > Scale and follow directions to calibrate the distances. You can use the marks on the edge of the ramp for the distance reference. It is also convenient to align the coordinate axes with the ramp. To do this, drag Origin 1 from the lower left corner of the window and center it on one of the calibration marks. A circle with an ‘ear’ will appear around the origin. Drag the ‘ear’ to align the x-axis with the ramp, using the other calibration mark for reference. You can now shrink the window to uncover the data table and coordinate list.

To look at the motion of the center, go to View > New Graph, select time for Horizontal Axis, and Point S1, x, Position for Vertical Axis. Expand the graph and open the curve fitting dialog with Graph > Add/Edit Fit or by clicking the F tab. Choose Type of Fit: Polynomial, Order of Fit: 2 and then Apply or OK. If the quadratic fits the data well, as expected for constant linear acceleration, you can record the fitted value of a for this tilt.

The angular acceleration is found similarly, using a clever feature of VideoPoint which transforms the origin to point S1 in each frame of the movie. To do this, go to the Coord. Sys. window and select Point S1 in the group labeled Origin: Origin 1. Now select Options > Make Point Origin to create a new group headed Origin: Point S1 in the coordinate window. Drag the Point S2 line from Origin: Origin 1 to Origin: Point S1. This causes the position of point S2 to be computed relative to the position of point S1 in each frame of the movie. You can now create a new graph with time for Horizontal Axis, and Point S2, Angle, Angular Position for Vertical Axis. This plot of θ vs time should also be described by a quadratic, from which you can read the constant angular acceleration α.

The rolling condition can be checked directly by plotting x vs ϕ. According to Eq. 5-5 this should be a straight line with slope R if the hoop is rolling without slipping. It is also an interesting plot when slipping occurs.

You will want to repeat these measurements for several angles to include conditions in which the hoop does and does not slip. The maximum tilt will probably be limited by the erratic motion that occurs when the normal force becomes very small and the hoop can be deflected by minor irregularities in the ramp surface.

3. Analysis

There are several ways that you can compare your data with the theoretical model. Are the linear and angular accelerations constant for each angle? Below the critical angle, the accelerations are independent of the unknown coefficient of friction and a plot of a vs $\sin\theta$ should be a straight line. Is that observed? With the expected slope? Above the critical angle, you can show that a plot of α vs $\cos\theta$ should be straight, with a slope that depends on μ. Is that observed, and is the value of μ plausible? Finally, you can clearly display the transition to slipping by plotting $a/\alpha R$ vs $\tan\theta$, for reasons that you should explain in your report.

REPORT

Your report should include derivations of the accelerations above and below the critical angle, example plots demonstrating that the accelerations are constant at a given tilt, and the various analysis plots with explanations based on the derived acceleration equations.