Pre-Lab 08 – Rotational Motion and Moment of Inertia Overview

Read the information in this document and answer the practice questions throughout. Some topics may be a review from your lecture course. When finished, check your answers to the practice questions and then complete the ***Pre-Lab 08 Quiz*** *on Canvas.*

Learning Objectives:

- Draw valid conclusions of rotational motion based on analogy with linear motion.
- Apply understanding of moment of inertia and torque to predict which objects require more force to change rotational motion.

I. Rotational Motion and the Moment of Inertia

Physics is the description of motion and how that motion might change given certain conditions. One-dimensional motion involves motion along a line and can be described using terms such as velocity v, acceleration a, and momentum p. For two dimensional motion, such as circular motion, new quantities can be defined based on polar rather than rectangular coordinates. For every linear quantity, there is an analogous rotational quantity, including angular velocity ω, angular acceleration α, and angular momentum L. These angular quantities are related to each other in the same manner that the linear quantities are: for instance $a = \Delta v / \Delta t$ and $\alpha = \Delta\omega / \Delta t$. In addition, angular and linear quantities are related, such as $v = r\omega$. Consult your textbook for more information.

Newton's first law of motion is sometimes referred to as the *law of inertia*. Inertia is the resistance of an object to a change in linear motion. More massive objects have greater inertia. For example, if you kick both a soccer ball and a bowling ball initially at rest, you feel a greater resistance to motion from the bowling ball due to its greater mass.

Likewise, an object moving in a circle resists a change in its motion. This is referred to as its **moment of inertia**. For a point mass m, moving in a circle of radius r as shown in Figure 1, its moment of inertia is given by $I = mr^2$. Here, its resistance to change in rotational motion varies with both the size of the circle and its mass.

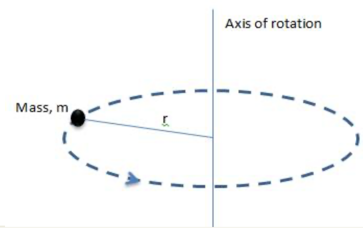
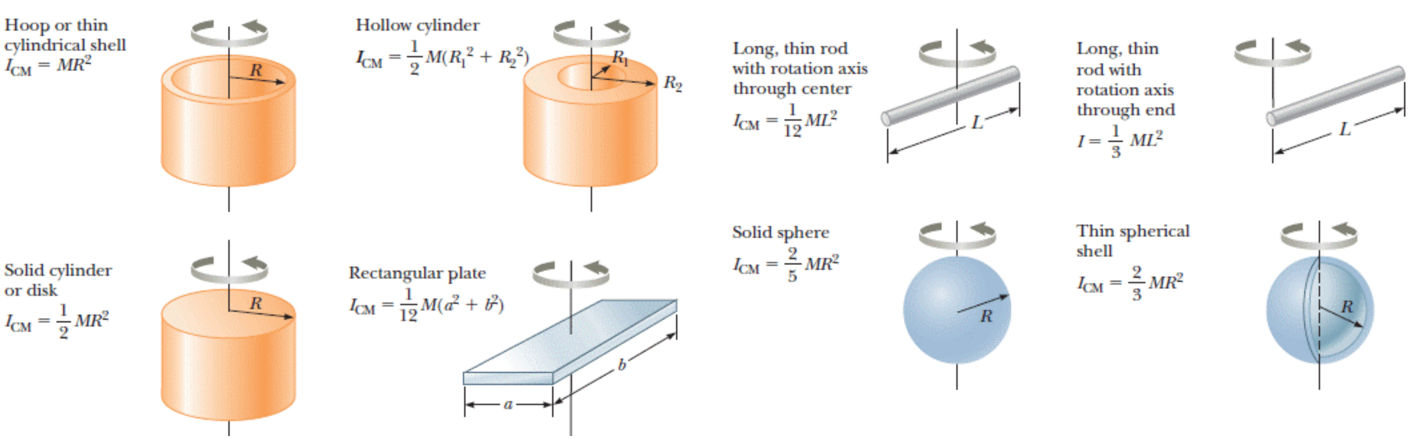


Figure 1 Point mass moving in a circle

Real world objects are essentially point masses distributed over a volume. The moment of inertia I for an extended object can be found by adding up all of the moments of inertia of the point masses (see your physics textbook for a more thorough description). Table 1 below provides the moments of inertia I for various regularly shaped objects.

Table 1 Moments of inertia for various homogeneous objects of mass M.



II. Moment of Inertia Exercises: *(Check answers at end of document.)*

Question 1

Consider two identical rods rotating around two different axes as shown in Figure 2. If each rod were spinning at the same rev/min, which would be easier to stop? ____________

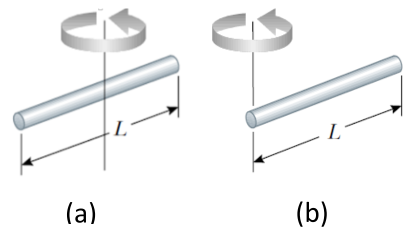


Fig. 2. Same rod, different spin axes.

Question 2

Consider a hollow and solid sphere of the same mass and radius. If both have an axis of rotation through their centers, which would take more force to start rotating from rest? Refer to Table 1.

Question 3

If the radius of a thin cylindrical shell were doubled, by how much would the mass need to change to maintain its original moment of inertia (I)? Assume axis passes through its center of mass.

Question 4

Is the following statement true or false?

Every object has a single mass, so every object has a single ***moment of inertia****.*

Question 5

Identify the correctness of statements (a) and (b). Fix a false statement.

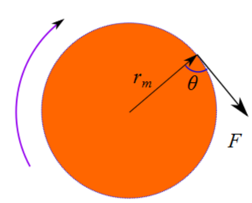
a. *The minimum value for the moment of inertia of a rotating object is when the axis of rotation passes through the center of mass of the object.*

b. *There is a maximum value for the moment of inertia of an object rotating in a specific plane where the axis of rotation actually passes through the object; this is where the axis passes through the center of mass of the object.*

III. Torque

The rotational analog to force is **torque** τ*.* It is what is necessary to change an object's rotational motion. When a force F is applied to an object at a point a distance r from the object's axis of rotation (called the moment arm, r_m), the torque is given by:

$$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow |\tau| = rF \sin\theta$$



Spin increases at rate α

Note that a force applied along a line that passes through the axis of rotation ($\theta = 0^o$) will **NOT** cause the object to rotate, so τ is simply 0. As noted previously, a force F applied to a an object with mass m causes a linear acceleration a, such that $F = ma$. Similarly, a torque τ applied to an object with **moment of inertia** I causes an angular acceleration α, with $\tau = I\alpha$ and where $a_t = r\alpha$.

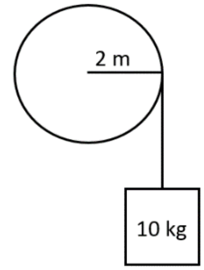
IV. Torque Exercises:

Question 6

A 10 kg disk is rotated by pulling a string wrapped around it, similar to the disk above. The radius of the disk is 0.5 m and the force applied by the string is 105 N. What is the net torque?

Question 7

A student wants to determine the mass of a pulley. He attaches a 10 kg box to it and lets it fall causing the pulley to rotate. The pulley is a solid disk of radius 2 m. He measures the angular acceleration of the disk to be 2.45 rad/s^2. The questions will guide you to the answer. *(Understanding this process will be useful for lab this week.)*



a. Determine the tangential acceleration of the pulley.
b. Determine the net force on the pulley. This is essentially the tension in the string.
c. Determine the net torque on the pulley.
d. Determine the experimental moment of inertia I.
e. Determine the mass of the pulley (consider it a solid disk).

V. Answers to Exercises

Question 1

Consider two identical rods rotating around two different axes as shown in Figure 2. If each rod were spinning at the same rev/min, which would be easier to stop? ____________

Both rods have the same mass, but rod (a) has a smaller moment of inertia than rod (b) because on average, more of the mass has a smaller circle to travel. This highlights the fact that the moment of inertia I of an object depends on both its mass and the distribution of that mass around an axis of rotation.

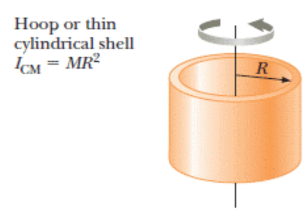
Question 2

Consider a hollow and solid sphere of the same mass and radius. If both have an axis of rotation through their centers, which would take more force to start rotating from rest?

The thin spherical shell has a larger moment of inertia as given in Table 1 so it will require more force (torque) to start it rotating from rest. This should make sense as both objects have the same mass which means the shell has more mass further away from the axis of rotation making it harder to rotate. This also means that once rotating, the shell will be harder to stop rotating!

Question 3

If the radius of a thin cylindrical shell were doubled, by how much would the mass of the shell need to be changed to maintain its original moment of inertia (I) if the axis of rotation passes through it center of mass as shown in Table 1?



The moment of inertia (I_{CM}) for the thin cylindrical shell depends on both M and R^2. In order to keep I_{CM} the same, if R is doubled then M must be reduced by a quarter ($\frac{1}{4}\,M$) due to the square of R.

Question 4

Is the statement below true or false:

Every object has a single mass, so every object has a single moment of inertia.

This is false. The moment of inertia depends on more than just the mass of the object. It also depends on the position of the axis of rotation for the object, so it has as many moments of inertia as there are possible axes of rotation.

Question 5

Which statement below is correct? Change any incorrect statements to read correctly.

a. *The minimum value for the moment of inertia of a rotating object is when the axis of rotation passes through the center of mass of the object.*

This statement is correct.

b. *There is a maximum value for the moment of inertia of an object rotating in a specific plane where the axis of rotation actually passes through the object; this is where the axis passes through the center of mass of the object.*

Incorrect. This statement would be correct if written as:
"There is a maximum value for the moment of inertia of an object rotating in a specific plane where the axis of rotation actually passes through the object; this is where the axis passes through the outermost edge of an object, such that the mass is distributed the farthest from the axis of rotation."

Question 6

A 10 kg disk is rotated by pulling a string wrapped around it, similar to the disk above. The radius of the disk is 0.5 m and the force applied by the string is 105 N. What is the net torque?

$$\vec{\tau} = rF\sin\theta$$
$$\tau = (0.5\text{m})(105\text{ N})\sin 90^{o} = 52.5\text{ N}\cdot\text{m}$$

Question 7

A student wants to determine the mass of a pulley. He attaches a 10 kg box to it and lets it fall causing the pulley to rotate. The pulley is a solid disk of radius 2 m. He measures the angular acceleration of the disk to be 2.45 rad/s^2. The questions will guide you to the answer.

a. $\boldsymbol{a_t = r\alpha = (2\text{ m})\left(2.45\frac{\text{rad}}{\text{s}^2}\right) = 4.90\text{ m/s}^2}$

b. $\boldsymbol{\sum F = ma_t = (10\text{ kg})\left(4.90\frac{\text{m}}{\text{s}^2}\right) = 49\text{ N}}$

Or

$$\boldsymbol{\sum F = T - mg = -ma \text{ where } T = m(g-a) = (10\text{ kg})(9.80 - 4.90) = 49\text{ N}}$$

c. $\boldsymbol{\vec{\tau} = rF\sin\theta = (2\text{ m})(49\text{ N})\sin 90^{o} = 98}\ \text{N}\cdot\text{m}$

d. $\boldsymbol{I_{experimental} = \frac{\tau}{\alpha} = \frac{98\ \text{N}\cdot\text{m}}{2.45\frac{\text{rad}}{\text{s}^2}} = 40}\ \text{kg}\cdot\text{m}^2$

e. $\boldsymbol{I_{theoretical,disk} = \frac{1}{2}M_{disk}r^2 = \frac{1}{2}(20\ \text{kg})(2\ m)^2 = 40}\ \text{kg}\cdot\text{m}^2$

such that

$$\boldsymbol{M_{disk} = M_{pulley} = \frac{2\,I}{r^2} = \frac{2\times 40\ \text{kg}\cdot\text{m}^2}{(2\text{m})^2} = \boxed{20\ \text{kg}}}$$