

CHAPTER 3

3.1

(a) $F(x, y, z) = \Sigma(0, 2, 4, 6) = z'$

x \ y ²	00	01	11	10
	0	1	0	1
0	1	0	0	1
1	1	0	0	1

(b) $F(x, y, z) = \Sigma(0, 2, 4, 5, 6, 7)$

x \ y ²	00	01	11	10
	0	1	0	1
0	1	0	0	1
1	1	1	1	1

$$F = x + z'$$

(c) $F(x, y, z) = \Sigma(0, 1, 2, 3, 4, 6)$

x \ y ²	00	01	11	10
	0	1	0	1
0	1	1	1	1
1	1	0	0	1

$$F = x' + z'$$

(d) $F(x, y, z) = \Sigma(1, 4, 5, 7)$

x \ y ²	00	01	11	10
	0	1	0	1
0	0	1	0	0
1	1	1	1	0

$$F = xy' + xz + y'z$$

3.2

(a) $F(x, y, z) = \Sigma(0, 1, 4, 7)$

x \ y ²	00	01	11	10
	0	1	0	1
0	1	1	0	0
1	1	0	1	0

$$F = x'y' + y'z' + xyz$$

(b) $F = y + x'z$

x \ yz	00	01	11	10
	m_0	m_1	m_3	m_2
0	0	1	1	1
1	m_4	m_5	m_7	m_6
			1	1

(c) $F(x, y, z) = \Sigma(2, 3, 5, 6)$

x \ y ²	00	01	11	10
	0	1	0	1
0	0	0	1	1
1	0	1	0	1

$$F = x'y + yz' + xy'z$$

(d) $F(x, y, z) = \Sigma(1, 2, 4, 7)$

x \ y ²	00	01	11	10
	0	1	0	1
0	0	1	0	1
1	1	0	1	0

$$F = x'y'z + x'yz' + xy'z' + xyz$$

$$= x'(y \oplus z) + x(y \oplus z)'$$

$$= x \oplus y \oplus z$$

(e) $F(x, y, z) = \Sigma(0, 2, 4, 6)$

		y			
		00	01	11	10
x	yz	m_0	m_1	m_3	m_2
0		1			1
1		1			1

$F = z'$

(f) $F(x, y, z) = \Sigma(3, 4, 5, 6, 7)$

		y			
		00	01	11	10
x	yz	m_0	m_1	m_3	m_2
0				1	
1		1	1	1	1

$F = x + yz$

3.3

(a) $F(x, y, z) = xyz + x'y + xyz'$
 $= xyz + x'yz + x'yz' + xyz'$
 $= \Sigma(2, 3, 6, 7)$

		y ²			
		00	01	11	10
x					
0		0	0	1	1
1		0	0	1	1

$F = y$

(b) $F(x, y, z) = x'yz + xyz' + xyz + x'yz' + xy'z'$
 $= \Sigma(2, 3, 4, 6, 7)$

		y ²			
		00	01	11	10
x					
0		0	0	1	1
1		1	0	1	1

$F = y + xz'$

(c) $F(x, y, z) = x'yz + xz$
 $= x'yz + xyz + xy'z$
 $= \Sigma(3, 5, 7)$

		y ²			
		00	01	11	10
x					
0		0	0	1	0
1		0	1	1	0

$F = xz + yz$

(d) $F(x, y, z) = xyz + x'y + xyz' + x'y'z'$
 $= xyz + x'yz + x'yz' + xyz' + x'y'z'$
 $= \Sigma(0, 2, 3, 6, 7)$

		y ²			
		00	01	11	10
x					
0		1	0	1	1
1		0	0	1	1

$F = y + x'z'$

3.4

(a) $F(x, y, z) = \Sigma(0, 1, 4, 5)$

$x \backslash yz$	00	01	11	10
	1	1	0	0
1	1	1	0	0

$F = y'$

(b) $F(A, B, C) = \Sigma(0, 2, 3, 7)$

$A \backslash BC$	00	01	11	10
	1	0	1	1
1	0	0	1	0

$F = BC + A'C'$

(c) $F(A, B, C, D) = \Sigma(1, 5, 9, 12, 13, 15)$

$AB \backslash CD$	00	01	11	10
	0	1	0	0
01	0	1	0	0
11	1	1	1	0
10	0	1	0	0

$F = C'D + ABC' + ABD$

(d) $F(w, x, y, z) = \Sigma(0, 2, 3, 8, 10, 11)$

$w \backslash xz$	00	01	11	10
	1	0	1	1
01	0	0	0	0
11	0	0	0	0
10	1	0	1	1

$F = x'y + x'z'$

$w \backslash xz$	y			
	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

$F = wx + wyz$

(e)

$w \backslash xz$	y			
	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

$F = wz' + xy'w$

(f)

3.5

(a) $F(w, x, y, z) = \Sigma(0, 4, 6, 8, 14, 15)$

wx \ yz	yz			
	00	01	11	10
00	1	0	0	0
01	1	0	0	1
11	0	0	1	1
10	1	0	0	0

$$F = w'xz' + x'y'z' + wxy$$

(b) $F = AC' + ABC' + ABD'$

AB \ CD	CD			
	00	01	11	10
00	m_0	m_1	m_3 1	m_2 1
01	m_4	m_5	m_7 1	m_6 1
11	m_{12} 1	m_{13} 1	m_{15}	m_{14} 1
10	m_8	m_9	m_{11}	m_{10}

D

B

wx \ yz	yz			
	00	01	11	10
00	m_0	m_1 1	m_3 1	m_2
01	m_4 1	m_5 1	m_7 1	m_6 1
11	m_{12}	m_{13} 1	m_{15} 1	m_{14}
10	m_8	m_9 1	m_{11} 1	m_{10}

z

x

(c) $F = z + xw'$

(d) $F(A, B, C, D) = \Sigma(0, 2, 4, 6, 8, 10, 12, 14)$

AB \ CD	CD			
	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$$F = D'$$

3.6

(a) $B'D'(A'C' + C) + AC'D' + BD(A'C + C')$
 $= A'B'C'D' + B'CD' + AC'D' + A'BCD + BC'D$
 $= \Sigma(0, 2, 5, 7, 8, 10, 12, 13)$

AB \ CD	CD			
	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	1	1	0	0
10	1	0	0	1

$$F = B'D' + ABC' + A'BD$$

(b) $F = xy' + x'z + wx'y$

wx \ yz		yz			
		00	01	11	10
00	x	m_0	m_1	m_3	m_2
			1	1	
01	x	m_4	m_5	m_7	m_6
		1	1		
11	x	m_{12}	m_{13}	m_{15}	m_{14}
		1	1		
10	x	m_8	m_9	m_{11}	m_{10}
			1	1	1

(c) $F = A'BCD + ABC + CD + B'D$
 $= \Sigma(1, 3, 7, 9, 11, 14, 15)$

AB \ CD	CD			
	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	0	0	1	1
10	0	1	1	0

$$F = B'D + CD + ABC$$

(d) $F = C'D + A'BD + A'B'C'$

AB \ CD	CD			
	00	01	11	10
00	m_0 1	m_1 1	m_3	m_2
01	m_4	m_5 1	m_7 1	m_6
11	m_{12}	m_{13} 1	m_{15}	m_{14}
10	m_8	m_9 1	m_{11}	m_{10}

$\underbrace{\hspace{100px}}_D$
 $\underbrace{\hspace{100px}}_C$

$\underbrace{\hspace{100px}}_B$

$\underbrace{\hspace{100px}}_A$

3.7

wx \ yz	yz			
	00	01	11	10
00	m_0	m_1 1	m_3 1	m_2 1
01	m_4	m_5 1	m_7 1	m_6
11	m_{12}	m_{13} 1	m_{15} 1	m_{14}
10	m_8	m_9 1	m_{11} 1	m_{10} 1

$\underbrace{\hspace{100px}}_z$
 $\underbrace{\hspace{100px}}_y$

$\underbrace{\hspace{100px}}_x$

$\underbrace{\hspace{100px}}_w$

(a) $F = z + x'y$

(b) $ACD' + B'C'D + BCD + BC'$
 $= \Sigma(1, 4, 5, 7, 9, 10, 12, 13, 14, 15)$

$AB \backslash CD$		00	01	11	10
		00	01	11	10
00		0	1	0	0
01		1	1	1	0
11		1	1	1	1
10		0	1	0	1

BC' (points to row 10)
 $C'D$ (points to column 01)
 BD (points to column 11)
 ACD' (points to cell 10, 10)

$$F = BC' + BD + C'D + ACD'$$

(c) $AB'C + B'C' + A'BCD + ACD' + AB'C' + A'C'D$
 $= \Sigma(0, 1, 5, 7, 8, 9, 10, 11, 14)$

$AB \backslash CD$		00	01	11	10
		00	01	11	10
00		1	1	0	0
01		0	1	1	0
11		0	0	0	1
10		1	1	1	1

$A'BD$ (points to cell 00, 01)
 ACD' (points to cell 11, 10)
 $B'C'$ (points to cell 00, 00)
 AB' (points to cell 10, 00)

$$F = B'C' + AB' + A'BD + ACD'$$

(d) $wxy + xz + w'xz + y'z + wy'$
 $= \Sigma(1, 5, 7, 8, 9, 12, 13, 14, 15)$

$wx \backslash yz$		00	01	11	10
		00	01	11	10
00		0	1	0	0
01		0	1	1	0
11		1	1	1	1
10		1	1	0	0

xz (points to cell 01, 11)
 wx (points to cell 11, 11)
 wy' (points to cell 10, 00)
 $y'z$ (points to cell 01, 00)

$$F = wy' + y'z + wx + xz$$

3.8

(a) $wxy + yz + xy'z + wz'$ $wxy \rightarrow 111_ \rightarrow 1110(14), 1111(15)$ $yz \rightarrow _ _ 11 \rightarrow 0011(3), 0111(7), 1011(11), 1111(15)$ $xy'z \rightarrow _ 101 \rightarrow 0101(5), 1101(13)$ $wz' \rightarrow 1_ _ 0 \rightarrow 1000(8), 1010(10), 1100(12), 1110(14)$

$wx \backslash yz$	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	1	1	1	1
10	1	0	1	1

Annotations: yz (pointing to row 01), wxy (pointing to cell 1111), $xy'z$ (pointing to cell 0101), wz' (pointing to cell 1000).

$$F = \Sigma(3, 5, 7, 8, 10, 11, 12, 13, 14, 15)$$

(b) $AC'D + BC'D + ACD' + A'B'D + A'D'$

$AB \backslash CD$	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	0	1	0	1
10	0	1	1	1

Annotations: $A'D'$ (pointing to cell 0000), $BC'D$ (pointing to cell 0101), $AC'D$ (pointing to cell 1001), $AB'D$ (pointing to cell 1010).

$$F = \Sigma(0, 2, 4, 5, 6, 9, 10, 11, 13, 14)$$

(c) $wyz + w'x' + wx'z' + x'z'$

$$= \Sigma(0, 1, 2, 3, 8, 10, 11, 15)$$

$wx \backslash yz$	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	1	0
10	1	0	1	1

Annotations: $w'x'$ (pointing to cell 0000), $w'z'$ (pointing to cell 0000), $wx'z'$ (pointing to cell 1000), wyz (pointing to cell 0000).

(d) $F = \Sigma(3, 4, 5, 7, 11, 12)$

AB \ CD		C			
		00	01	11	10
A	00	m_0	m_1	m_3 1	m_2
	01	m_4 1	m_5 1	m_7 1	m_6
	11	m_{12} 1	m_{13}	m_{15}	m_{14}
	10	m_8	m_9	m_{11} 1	m_{10}

D

3.9

(a) $F(w, x, y, z) = \Sigma(0, 2, 4, 6, 8, 10, 12, 14)$

$wx \backslash yz$		00	01	11	10
		00	01	11	10
wx	00	1	0	0	1
	01	1	0	0	1
	11	1	0	0	1
	10	1	0	0	1

Essential Prime Implicant: z'

$$F = z'$$

(b) $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 6, 8, 9, 11, 12, 14, 15)$

$AB \backslash CD$		00	01	11	10
		00	01	11	10
00	1	0	1	1	
01	0	1	0	1	
11	1	0	1	1	
10	1	1	1	0	

Prime Implicants: $A'BC'D, A'B'D', B'C'D', A'B'C,$
 $A'CD', AB'C', AC'D', B'CD, AB'D,$
 ABD', ACD, ABC, BCD'

One set of Essential Prime Implicants: $A'BC'D, B'CD, AB'C', A'B'D'$
 $BCD', ABC, AB'C'$

(c)

		CD		C	
		00	01	11	10
AB	00	m_0 1	m_1 1	m_3	m_2 1
	01	m_4	m_5 1	m_7 1	m_6
	11	m_{12}	m_{13} 1	m_{15} 1	m_{14}
	10	m_8 1	m_9 1	m_{11}	m_{10} 1
A		B			
		D			

Essential: BC' , AC , $A'B'D$ **Non-Essential:** $A'B$

$$F = BC' + AC + A'B'D$$

(d) $F(w, x, y, z) = \Sigma(1, 3, 5, 7, 9, 11, 13, 15)$

		yz			
		00	01	11	10
wx	00	0	1	1	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	1	1	

Essential Prime Implicant = z

(e)

		CD		C	
		00	01	11	10
AB	00	m_0	m_1	m_3 1	m_2 1
	01	m_4 1	m_5 1	m_7 1	m_6 1
	11	m_{12} 1	m_{13} 1	m_{15}	m_{14}
	10	m_8	m_9 1	m_{11} 1	m_{10}
A		B			
		D			

Essential: BD , $B'C'$, $C'D$

$$F = BD + B'C' + C'D$$

(f) $F(w, x, y, z) = \Sigma(1, 3, 4, 6, 7, 9, 10, 12, 13, 15)$

$wx \backslash yz$	00	01	11	10
00	0	1	1	0
01	1	0	1	1
11	1	1	1	0
10	0	1	0	1

Prime Implicants: $wx'yz'$, $w'x'z$, $x'y'z$, $w'xy'$,
 $xy'z'$, $w'yz$, $w'xy$, $wy'z$,
 wxy' , xyz , wxz .

One set of Essential Prime Implicants: $wx'yz'$, $w'x'z$, $wy'z$,
 $x'y'z$, $w'yz$, $w'xz'$, xyz , wxy'

3.10 (a) $F(w, x, y, z) = \Sigma(0, 2, 5, 7, 8, 10, 13, 14, 15)$

Using K-map:

$wx \backslash yz$	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	1
10	1	0	0	1

Using Quine-McCluskey method:

<u>0 (0000)</u> ✓	<u>0,2 (00_0)</u> ✓	<u>0, 2, 8, 10 (0 0)</u>
<u>2 (0010)</u> ✓	<u>0,8 (_000)</u> ✓	
<u>8 (1000)</u> ✓	<u>2,10(_010)</u> ✓	
<u>5 (0101)</u> ✓	<u>8,10(10_0)</u> ✓	<u>5, 7, 13, 15 (_1_1)</u>
<u>10(1010)</u> ✓	<u>5, 7 (01_1)</u> ✓	
<u>7 (0111)</u> ✓	<u>5,13(_101)</u> ✓	
<u>13(1101)</u> ✓	<u>10,14(1_10)</u>	
<u>14(1110)</u> ✓	<u>7,15(_111)</u> ✓	
<u>15(1111)</u> ✓	<u>13,15(11_1)</u> ✓	
	<u>14,15(111_)</u>	

Prime Implicant Table:

	0	2	5	7	8	10	13	14	15
0, 2, 8, 10	x	x			x	x			
5, 7, 13, 15			x	x			x		x
10, 14					x			x	
14, 15							x	x	

$$F = (-0-0) + (-1-1) + (1-10) = x'z' + xz + wyz'$$

(b) $F = \Sigma(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

Essential: $AC, B'D', CD, A'BD$

$$F = AC + B'D' + CD + A'BD$$

		CD				C
		00	01	11	10	
AB	00	m_0 1	m_1	m_3 1	m_2 1	B
	01	m_4	m_5 1	m_7 1	m_6	
	11	m_{12}	m_{13}	m_{15} 1	m_{14} 1	
	10	m_8 1	m_9	m_{11} 1	m_{10} 1	
		D				A
		00	01	11	10	

(c) $F(w, x, y, z) = \Sigma(2, 3, 6, 7, 10, 11, 14, 15)$

$wx \backslash yz$		yz			
		00	01	11	10
wx	00	0	0	1	1
	01	0	0	1	1
	11	0	0	1	1
	10	0	0	1	1

<u>2</u> (0010) ✓	2, 3 (001_) ✓	2, 3, 6, 7(0_1_) ✓	(2, 3, 6, 7, 10, 11, 14, 15)
3 (0011) ✓	2, 6 (0_10) ✓	2, 3, 10, 11(_01_) ✓	
6 (0110) ✓	<u>2, 10</u> (_010) ✓	<u>2, 6, 10, 14</u> (__10) ✓	(_ _1_)
<u>10</u> (1010) ✓	3, 7(0_11) ✓	3, 7, 11, 15(_ _11) ✓	
7 (0111) ✓	3, 11(_011) ✓	6, 7, 14, 15(_11_) ✓	
11 (1011) ✓	6, 7(011_) ✓	10, 11, 14, 15(1_1_) ✓	
<u>14</u> (1110) ✓	6, 14(_110) ✓		
15 (1111) ✓	10, 11(101_) ✓		
	<u>10, 14</u> (1_10) ✓		
	7, 15 (_111) ✓		
	11, 15(1_11) ✓		
	14, 15(111_) ✓		

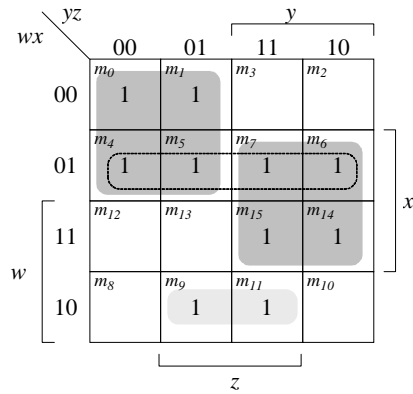
$$F = C$$

(d) $F = \Sigma(0, 1, 4, 5, 6, 7, 9, 11, 14, 15)$

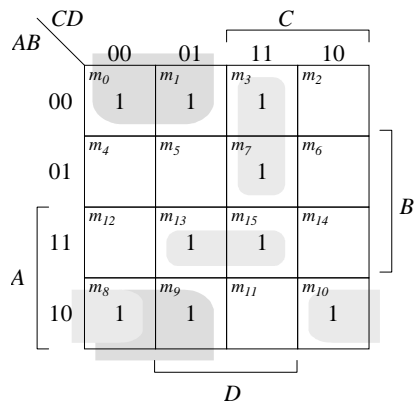
Essential: $w'y', xy, wx'z$

Non-essential: $wx, x'y'z, w'wz, w'x'z$

$$F = w'y' + xy + wx'z$$



(e) $F(A, B, C, D) = \Sigma(0, 1, 3, 7, 8, 9, 10, 13, 15)$

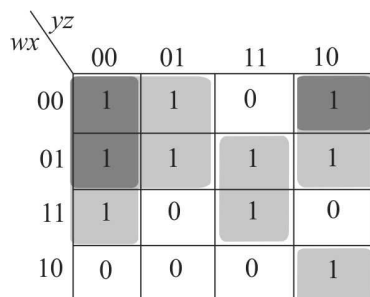


Essential: $B'C', AB'D'$

Non-essential: $ABD, A'CD, BCD$

$$F = B'C' + AB'D' + A'CD + ABD$$

(f) $F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 7, 10, 12, 15)$



$$0 \text{ (0000)} \checkmark \quad 0, 1 \text{ (000_)} \checkmark \quad 0, 1, 4, 5 \text{ (0_0_)} \checkmark$$

$$1 \text{ (0001)} \checkmark \quad 0, 2 \text{ (00_0)} \checkmark \quad 0, 2, 4, 6 \text{ (0_ _0)} \checkmark$$

2 (0010) ✓	0, 4(0_00) ✓	4, 5, 6, 7 (01_ _)
4 (0100) ✓	1, 5(0_01) ✓	
5 (0101) ✓	2, 6(0_10) ✓	
6 (0110) ✓	2, 10(010_0) ✓	
10 (1010) ✓	4, 5(010_0) ✓	
12 (1100) ✓	4, 6(01_0) ✓	
7 (0111) ✓	4, 12(_100) ✓	
15 (1111) ✓	5, 7(01_1) ✓	
	6, 7(011_0) ✓	
	7, 15(_111) ✓	

Essential Prime Implicants: (0, 1, 4, 5)(2, 10)(4, 12)(7, 15)(0, 2, 4, 6)

$$F = w'y' + x'yz' + xy'z' + xyz + w'z'$$

3.11 $F(w, x, y, z) = \Sigma(0, 1, 3, 5, 7, 9, 10, 13, 15)$

		yz			
		00	01	11	10
wx	00	1	1	1	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	1	0	1

Annotations: $(w + y' + z)$ points to the first row (wx=00); $(x' + z)$ points to the first column (yz=00); $(w' + y + z)$ points to the first two rows (wx=00, 01); $(w' + x + y' + z')$ points to the last two rows (wx=11, 10).

$$F = (x' + z)(w' + y + z)(w + y' + z)(w' + x + y' + z')$$

3.12 (a) $F(A, B, C, D) = \pi(0, 2, 4, 6, 8, 10, 12, 14)$

		CD			
		00	01	11	10
AB	00	0	1	1	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	1	1	0

$$F = D$$

(b) $F(A, B, C, D) = \pi(1, 3, 5, 7, 9, 11, 13, 15)$

		CD			
		00	01	11	10
AB	00	1	0	0	1
	01	1	0	0	1
	11	1	0	0	1
	10	1	0	0	1

Annotation: D' points to the last two columns (CD=11, 10).

$$F = D'$$

It shows a logical product of all even maxterms is equal to the complement of logical product of all odd maxterms. For n variable,

$$\pi(M_2i) = (\pi(M_2i + 1))' \quad \text{where, } i = 0, 1, 2, \dots, (2^n/2 - 1)$$

3.13 (a) $F = xz' + y'z' + yz' + xy' = \Sigma(0, 2, 4, 5, 6)$

$x \backslash yz$		00	01	11	10
		0	1	0	1
0	1	1	0	0	1
1	1	1	1	0	1

$$F = z' + xy' \quad (\text{Sum of Product})$$

$$F = (x + z')(y' + z') \quad (\text{Product of Sum})$$

(b) $F = AC'D' + C'D + AB' + AB'CD$
 $= \Sigma(1, 5, 8, 9, 10, 11, 12, 13)$
 $= \pi(0, 2, 3, 4, 6, 7, 14, 15)$

$AB \backslash CD$		00	01	11	10
		00	01	11	10
00	0	0	1	0	0
01	0	0	1	0	0
11	1	1	1	0	0
10	1	1	1	1	1

$$F = AC' + C'D + AB' \quad (\text{Sum of Product})$$

$$F = (A + D)(A + C')(B' + C') \quad (\text{Product of Sum})$$

(c) $F = (A' + B + D')(A' + B' + C')(A' + B' + C)(B' + C + D')$
 $F' = AB'D + ABC + ABC' + BC'D$

$AB \backslash CD$		00	01	11	10
		m_0	m_1	m_3	m_2
00			0	0	
01		m_4	m_5	m_7	m_6
11		m_{12}	m_{13}	m_{15}	m_{14}
10		m_8	m_9	m_{11}	m_{10}

$\underbrace{\hspace{10em}}_D$
 $\left. \begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right\} B$

$$F' = AB + BC'D$$

$$F = (A' + B')(B' + C + D')$$

$$F = A'D' + A'BC + AB'$$

AB \ CD		C			
		00	01	11	10
A	00	m_0 1	m_1 0	m_3 0	m_2 1
	01	m_4 1	m_5 0	m_7 1	m_6 1
	11	m_{12} 0	m_{13} 0	m_{15} 0	m_{14} 0
	10	m_8 1	m_9 1	m_{11} 1	m_{10} 1

(d) $F = BD' + AC'D + BC' + AB'CD$
 $= \Sigma(4, 5, 6, 9, 11, 12, 13, 14)$
 $= \pi(0, 1, 2, 3, 7, 8, 10, 15)$

AB \ CD		00	01	11	10
		00	01	11	10
A	00	0	0	0	0
	01	1	1	0	1
	11	1	1	0	1
	10	0	1	1	0

$$F = BC' + BD' + AB'D \rightarrow \text{SOP}$$

$$F = (A + B)(B + D)(B' + C' + D') \rightarrow \text{POS}$$

3.14 $F(A, B, C, D) = A'B'D' + AB'D' + BD + ABCD'$

AB \ CD		00	01	11	10
		00	01	11	10
A	00	1	0	0	1
	01	0	1	1	0
	11	0	1	1	1
	10	1	0	0	1

Annotations: $B'D'$ (points to cell 00,00), BD (points to cell 11,11), ABC (points to cell 01,11), ACD' (points to cell 11,10).

Two SOP form of F is,

No. of literals

$$F(A, B, C, D) = B'D' + BD + ACD' \quad 2 + 2 + 3 = 7$$

and

$$F(A, B, C, D) = B'D' + BD + ABC \quad 2 + 2 + 3 = 7$$

		CD			
		00	01	11	10
AB	00	1	0	0	1
	01	0	1	1	0
	11	0	1	1	1
	10	1	0	0	1

$(B'+C+D)$ $(B+D')$

→ $(A+B'+D)$

POS of form of F is,

$$F(A, B, C, D) = (B + D') (B' + C + D) (A + B' + D)$$

$$\text{No. of literals} = 2 + 3 + 3 = 7$$

- 3.15** (a) $F(x, y, z) = \Sigma(0, 1, 3, 5, 7)$
 $d = \Sigma(2, 4, 6)$

		yz			
		00	01	11	10
x	0	1	1	1	-
	1	-	1	1	-

$$F = 1$$

- (b) $F = \Sigma(0, 4, 8, 10, 14)$
 $d = \Sigma(2, 6, 12)$

		CD			
		00	01	11	10
AB	00	1	0	0	-
	01	1	0	0	-
	11	-	0	0	1
	10	1	0	0	1

$$F = D'$$

(c) $F = \Sigma(5, 6, 7, 11, 14, 15)$
 $d = \Sigma(3, 9, 13)$

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00	0	0	-	0
	01	0	1	1	1
	11	0	-	1	1
	10	0	-	1	0

$$F = BD + BC + AD$$

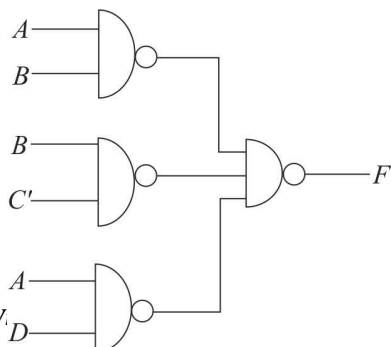
(d)

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00	-	0	-	1
	01	1	0	1	-
	11	1	-	0	1
	10	-	0	0	1

$$F = D' + A'C$$

3.16 (a) $F(A, B, C, D) = AD + BC'D' + ABC + A'BC'D$

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00	0	0	0	0
	01	1	1	0	0
	11	1	1	1	1
	10	0	1	1	0



$$F = AB + BC' + AD$$

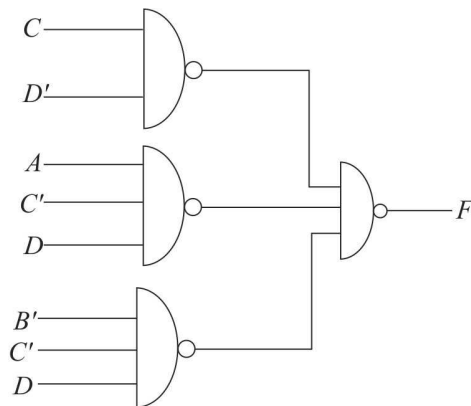
$$= [(AB)'(BC')'(AD)']'$$

(b) $F(A, B, C, D) = A'B'C'D + CD' + AC'D$

		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00	0	1	0	1
	01	0	0	0	1
	11	0	1	0	1
	10	0	1	0	1

$$F(A, B, C, D) = CD' + AC'D + B'C'D$$

$$= [(CD)'(AC'D)'(B'C'D)']'$$



(c) $F = (A' + C' + D')(A' + C')(C' + D')$

$$F' = (A' + C' + D')' + (A' + C')' + (C' + D')'$$

$$F' = ACD + AC + CD$$

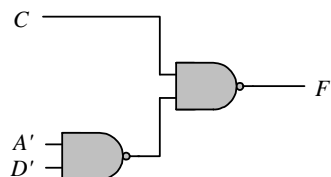
		<i>CD</i>			
		00	01	11	10
<i>AB</i>	00	m_0 1	m_1 1	m_3 0	m_2 1
	01	m_4 1	m_5 1	m_7 0	m_6 1
	11	m_{12} 1	m_{13} 1	m_{15} 0	m_{14} 0
	10	m_8 1	m_9 1	m_{11} 0	m_{10} 0

A *B* *D*

$$F = C' + A'D'$$

$$F = (C(A + D))'$$

$$F = (C(A'D'))'$$

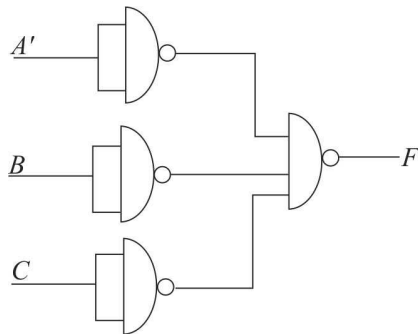


(d) $F(A, B, C, D) = A' + AB + B'C + ACD$

		CD			
		00	01	11	10
AB	00	1	1	1	1
	01	1	1	1	1
	11	1	1	1	1
	10			1	1

$$F = A' + B + C$$

$$= [(A'A)'(BB)'(CC)']'$$



3.17 $F(A, B, C, D) = \Sigma(4, 5, 6, 7, 9, 13, 15)$

		CD			
		00	01	11	10
AB	00	0	0	0	0
	01	1	1	1	1
	11	0	1	1	0
	10	0	1	0	0

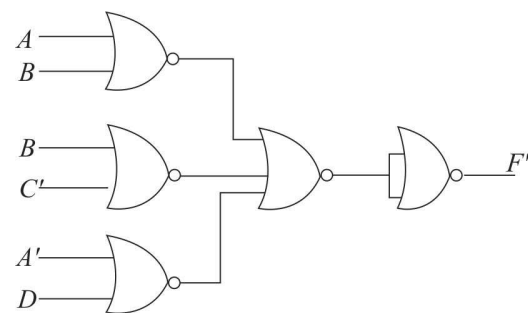
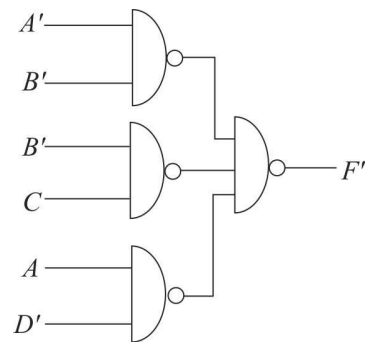
$$F' = B'A' + B'C + AD'$$

By NAND:

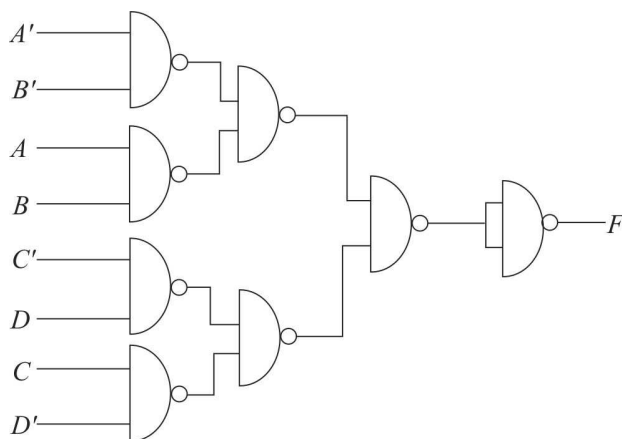
$$= [(B'A)'(B'C)'(AD')']'$$

By NOR:

$$= [(A + B)' + (B + C)' + (A' + D)']'$$



3.18 $F(A, B, C, D) = (A \oplus B)'(C \oplus D)$
 $= (A'B + AB')'(C'D + CD')$
 $= (A + B')(A' + B)(C'D + CD')$
 $= (A'B' + AB)(C'D + CD')$
 $= [(A'B')'(AB)]' + [(C'D)'(CD)']'$

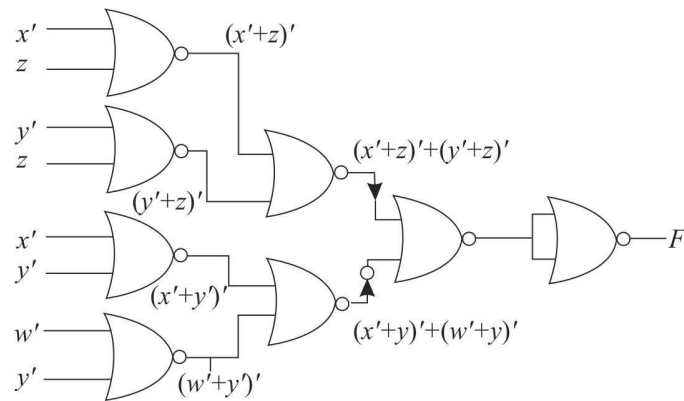


3.19 (a) $F(w, x, y, z) = wx'y + xy'z' + w'yz' + xy$

$wx \backslash yz$	00	01	11	10
00	0	0	0	1
01	1	0	1	1
11	1	0	1	1
10	0	0	1	1

Simplified Boolean equation:

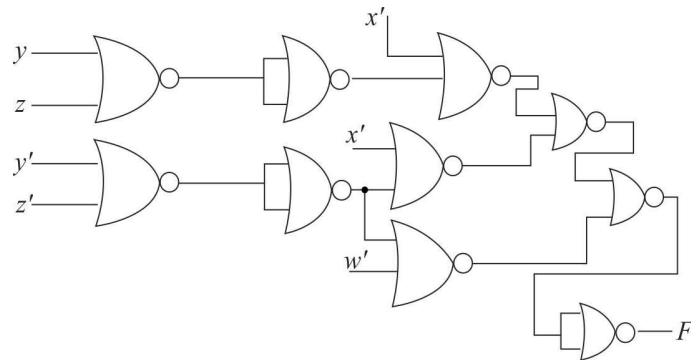
$$\begin{aligned}
 F(w, x, y, z) &= xz' + yz' + xy + wy \\
 &= (x' + z)' + (y' + z)' + (x' + y)' + (w' + y)'
 \end{aligned}$$



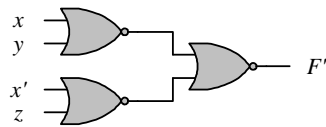
(b) $F(w, x, y, z) = \Sigma(4, 7, 11, 12, 15)$

$wx \backslash yz$	00	01	11	10
00	0	0	0	0
01	1	0	1	0
11	1	0	1	0
10	0	0	1	0

$$\begin{aligned}
 F &= xy'z' + xyz + wyz \\
 &= [(x' + (y + z))' + (x' + (y' + z'))' + (w' + (y' + z'))']
 \end{aligned}$$

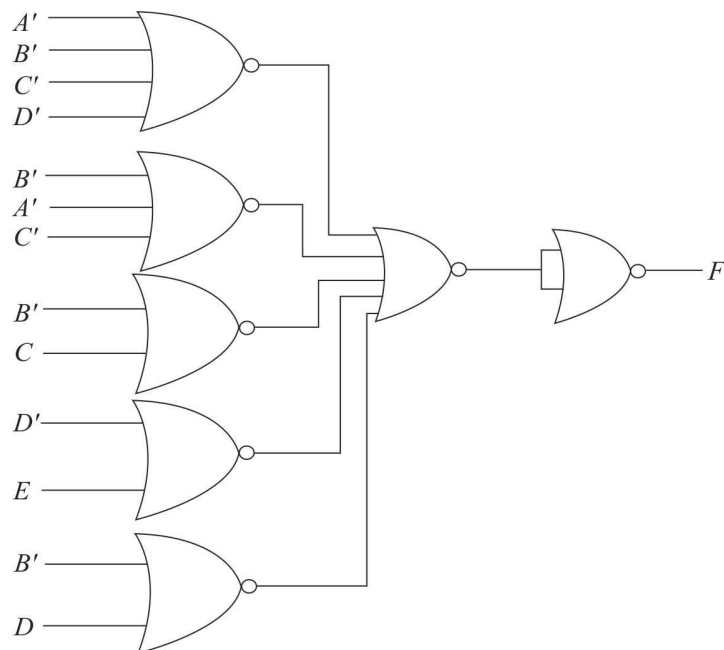


(c) $F = [(x + y)(x' + z)]' = (x + y)' + (x' + z)'$
 $F' = [(x + y)' + (x' + z)']'$



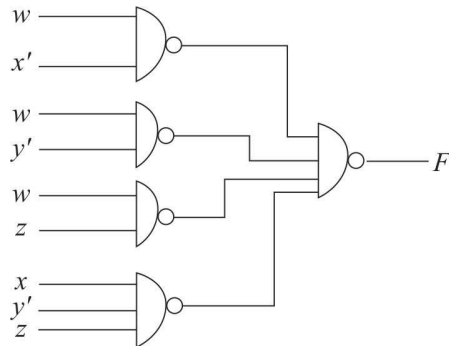
3.20

$$\begin{aligned}
 F &= BC(D + C)A + (BC' + DE') + BD' \\
 &= ABCD + ABC + BC' + DE' + BD' \\
 &= [[(A' + B' + C' + D')' + (A' + B' + C')' + (B' + C)' + (D' + E)' + (B' + D)']']'
 \end{aligned}$$



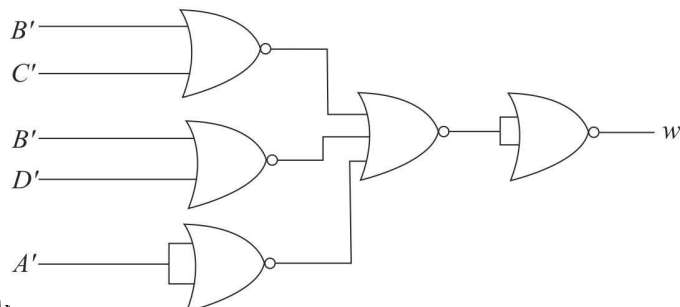
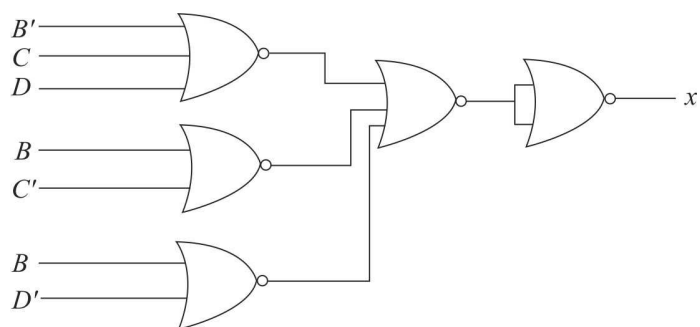
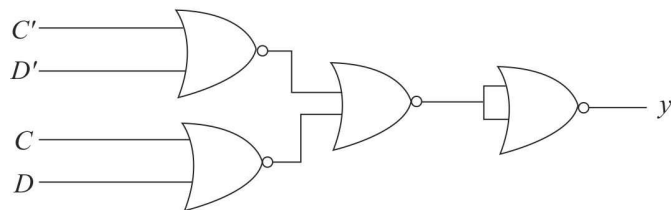
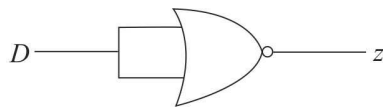
3.21

$$\begin{aligned}
 F &= w(x' + y' + z) + xy'z \\
 &= wx' + wy' + wz + xy'z \\
 &= [(wx')'(wy')'(wz)'(xy'z)']'
 \end{aligned}$$



3.22

$$\begin{aligned}
 z &= D' \\
 y &= CD + (C + D)' = [(C' + D')' + (C + D)']' \\
 x &= B(C + D)' + B'(C + D) \\
 &= BC'D' + B'C + B'D \\
 &= [(B' + C + D)' + (B + C)' + (B + D)']'
 \end{aligned}$$

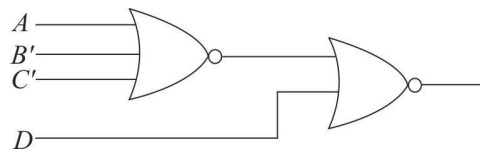


$$\begin{aligned}
 w &= B(C + D) + A \\
 &= BC + BD + A \\
 &= [(B' + C')' + (B' + D')' + (A'A')']'
 \end{aligned}$$

3.23 $F(A, B, C, D) = \Sigma(0, 2, 4, 12, 14) + d(1, 5, 8, 10)$

		CD			
		00	01	11	10
AB	00	1	-	0	1
	01	1	-	0	0
	11	1	0	0	1
	10	-	0	0	-

$$\begin{aligned}
 F &= C'D' + AD' + B'D' \\
 \text{OR} \\
 F' &= D + A'BC \\
 F &= [D + A'BC]' \\
 &= [D + (A + B' + C')]
 \end{aligned}$$



3.24 $F(A, B, C, D) = \Sigma(1, 5, 8, 9, 10, 11, 12, 13, 15)$

		CD			
		00	01	11	10
AB	00	0	1	0	0
	01	0	1	0	0
	11	1	1	1	0
	10	1	1	1	1

Annotations: $C'D$ points to the 1 in row 00, column 01; AD points to the 1 in row 01, column 11; AB' points to the 1 in row 11, column 10; AC' points to the 1 in row 10, column 00.

$$\begin{aligned}
 F(A, B, C, D) &= C'D + AB' + AC' + AD \\
 &= ((C'D)'(AB')'(AC')'(AD))' \\
 &= ((C+D')(A'+B)(A'+C)(A'+D))' \\
 &= ((C + D')' + (A' + B)' + (A' + C)' + (A' + D)')' \\
 &= ((C'D)'(AB')'(AC')'(AD))'
 \end{aligned}$$

→ (a) AND-OR
→ (b) OR-NAND
→ (c) NOR-OR
→ (d) NAND-NAND

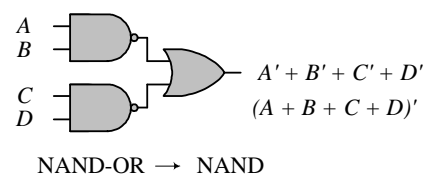
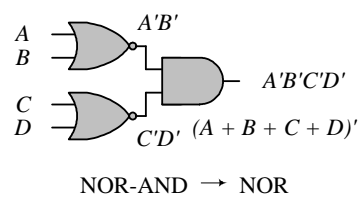
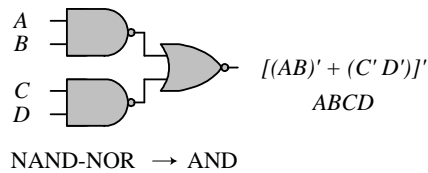
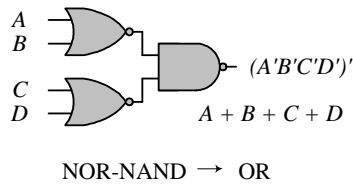
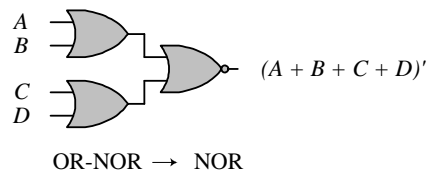
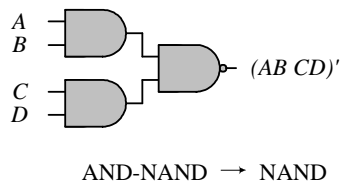
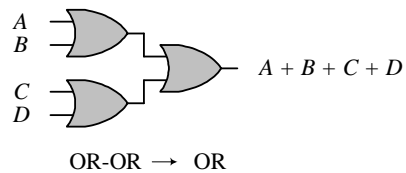
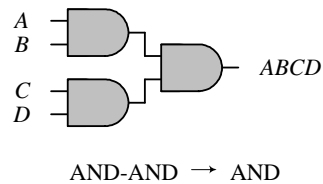
		CD			
		00	01	11	10
AB	00	0	1	0	0
	01	0	1	0	0
	11	1	1	1	0
	10	1	1	1	1

$(A+C')$ points to the top row (AB=00, 01).
 $(A+D)$ points to the first column (CD=00, 01).
 $(B'+C'+D)$ points to the bottom-right cell (AB=11, CD=10).

$$\begin{aligned}
 F(A, B, C, D) &= (A + D)(A + C')(B' + C' + D) \\
 &= ((A + D)' + (A + C')' + (B' + C' + D)')' \\
 &= (A'D)'(A'C)'(BCD)'
 \end{aligned}$$

\rightarrow (e) OR-AND
 \rightarrow (f) NOR-NOR
 \rightarrow (g) NAND-AND

3.25



The degenerate forms use 2-input gates to implement the functionality of 4-input gates.

3.26

$$f = abc' + b'd' + a'd' + b'cd'$$

$ab \backslash cd$	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	1	0	
10	1	0	0	1

$$g = (a + b + c' + d')(a' + b' + d)(a' + d')$$

$ab \backslash cd$	00	01	11	10
00	1	1	0	1
01	1	1	1	1
11	0	0	0	0
10	1	0	0	1

Group the overlapping is

$$F = fg = \Sigma(0, 2, 4, 5, 8, 10)$$

$ab \backslash cd$	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	1	0	0	1

$$F = a'd' + b'd'$$

3.27

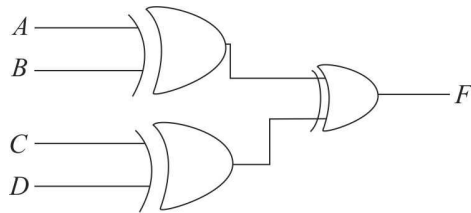
$$(x \text{ XNOR } y) = x'y' + xy$$

$$\begin{aligned}
 \text{Dual of } (x \text{ XNOR } y) &= (x + y)(x' + y') \\
 &= xy' + x'y \\
 &= ((x' + y)(x + y'))' \\
 &= (x'y' + xy)' \\
 &= (x \text{ XNOR } y)'
 \end{aligned}$$

3.28

4 bit even parity generator

$$F = \Sigma(1, 2, 4, 7, 8, 11, 13, 14)$$

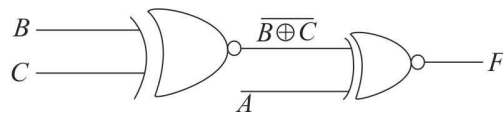


$$= A \oplus B \oplus C \oplus D$$

3-bit parity checker using even parity

$$F(A, B, C) = \Sigma(0, 3, 6, 5)$$

$$= A \oplus B \oplus C$$



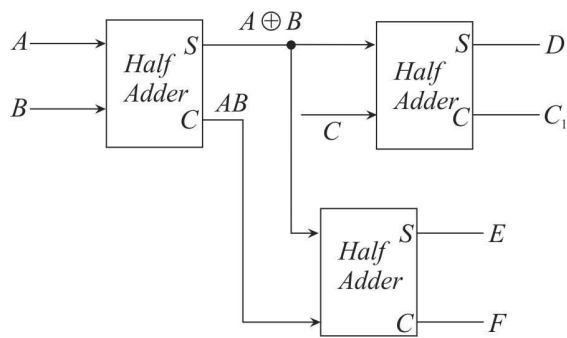
3.29

$$D = (\overline{A \oplus B}) C + (A \oplus B) C'$$

$$E = (A \oplus B) \oplus AB$$

$$F = (A'B + AB')AB$$

$$C_1 = (A \oplus B) C$$

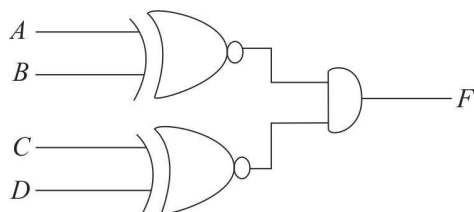


3.30

$$F = A'B'C'D' + ABC'D' + A'B'CD + ABCD$$

$$= C'D'(\overline{A \oplus B}) + CD(A \oplus B)$$

$$= (\overline{A \oplus B})(\overline{C \oplus D})$$



3.31

Note: It is assumed that a complemented input is generated by another circuit that is not part of the circuit that is to be described.

- (a) **module** Fig_3_20a_gates (F, A, B, C, C_bar, D);
 output F;
 input A, B, C, C_bar, D;
 wire w1, w2, w3, w4;
 and (w1, C, D);
 or (w2, w1, B);
 and (w3, w2, A);
 and (w4, B, C_bar);
 or (F, w3, w4);
endmodule
- (b) **module** Fig_3_20b_gates (F, A, B, B_bar, C, C_bar, D);
 output F;
 input A, B, B_bar, C, C_bar, D;
 wire w1, w2, w3, w4;
 not (w1_bar, w1);
 not (w3_bar, w3);
 not (w4_bar, w4);
 nand (w1, C, D);
 or (w2, w1_bar, B);
 nand (w3, w2, A);
 nand (w4, B, C_bar);
 or (F, w3_bar, w4_bar);
endmodule
- (c) **module** Fig_3_21a_gates (F, A, A_bar, B, B_bar, C, D_bar);
 output F;
 input A, A_bar, B, B_bar, C, D_bar;
 wire w1, w2, w3, w4;
 and (w1, A, B_bar);
 and (w2, A_bar, B);
 or (w3, w1, w2);
 or (w4, C, D_bar);
 and (F, w3, w4);
endmodule
- (d) **module** Fig_3_21b_gates (F, A, A_bar, B, B_bar, C_bar, D);
 output F;
 input A, A_bar, B, B_bar, C_bar, D;
 wire w1, w2, w3, w4, F_bar;
 nand (w1, A, B_bar);
 nand (w2, A_bar, B);
 not (w1_bar, w1);
 not (w2_bar, w2);
 or (w3, w1_bar, w2_bar);
 or (w4, w5, w6);
 not (w5, C_bar);
 not (w6, D);
 nand (F_bar, w3, w4);
 not (F, F_bar);
endmodule
- (e) **module** Fig_3_24_gates (F, A, A_bar, B, B_bar, C, D_bar);
 output F;
 input A, A_bar, B, B_bar, C, D_bar

```

wire    w1, w2, w3, w4, w5, w6, w7, w8, w7_bar, w8_bar;
not     (w1_bar, w1);
not     (w2_bar, w2);
not     (w3, E_bar);
nor     (w1, A, B);
nor     (W2, C, D);
and     (F, w1_bar, w2_bar, w3);
endmodule

```

(f) **module** Fig_3_25_gates (F, A, A_bar, B, B_bar, C, D_bar);
output F;
input A, A_bar, B, B_bar, C, D_bar;
wire w1, w1_bar, w2, w2_bar;
wire w3, w4, w5, w6, w7, w8;
not (w1, A_bar);
not (w2, B);
not (w3, A);
not (w4, B_bar);
and (w5, w1_bar, w2_bar);
and (w6, w3, w4);
nor (w7, w5, w6);
nor (w8, c, d_bar);
and (F, w7, w8);
endmodule

3.32

Note: It is assumed that a complemented input is generated by another circuit that is not part of the circuit that is to be described.

Note: Because the signals here are all scalar-valued, the logical operators (!, &&, and ||) are equivalent to the bitwise operators (~, &, |). If the operands are vectors the bitwise operators produce a vector result; the logical operators would produce a scalar result (true or false).

(a) **module** Fig_3_20a_CA (F, A, B, C, C_bar, D);
output F;
input A, B, C, C_bar, D;
wire w1, w2, w3, w4;
assign w1 = C && D;
assign w2 = w1 || B;
assign w3 = !(w2 && A);
assign w4 = !w3;
assign w5 = !(B && C_bar);
assign w5_bar = !w5;
assign F = w4 || w5_bar;
endmodule

(b) **module** Fig_3_20b_CA (F, A, B, C, C_bar, D);
output F;
input A, B, B_bar, C, C_bar, D;
wire w2 = !w1;
wire w3 = !B_bar;
wire w4, w5, w5_bar, w6, w6_bar;
assign w1 = !(C && D);
assign w4 = w2 || w3;
assign w5 = !(w4 && A);
assign w5_bar = !w5;
assign w6 = !(B && C_bar);
assign w6_bar = !w6;

```

    assign F = w5_bar || w6_bar;
endmodule
(c) module Fig_3_21a_CA (F, A, A_bar, B, B_bar, C, D_bar);
    output F;
    input    A, A_bar, B, B_bar, C, D_bar;
    wire     w1, w2, w3, w4;
    assign w1 = A && B_bar;
    assign w2 = A_bar && B;
    assign w3 = w1 || w2;
    assign w4 = C || D_bar;
    assign F = w3 || w4;
endmodule

(d) module Fig_3_21b_CA (F, A, A_bar, B, B_bar, C_bar, D);
    output F;
    input    A, A_bar, B, B_bar, C_bar, D;
    wire     w1, w2, w1_bar, w2_bar, w3, w4, w5, w6, F_bar;
    assign w1 = !(A && B_bar);
    assign w2 = !(A_bar && B);
    assign w1_bar = !w1;
    assign w2_bar = !w2;
    assign w3 = w1_bar || w2_bar;
    assign w4 = !C_bar;
    assign w5 = !D;
    assign w6 = w4 || w5;
    assign F_bar = !(w3 && w6);
    assign F = !F_bar;
endmodule

(e) module Fig_3_24_CA (F, A, B, C, D, E_bar);
    output F;
    input    A, B, C, D, E_bar;
    wire     w1, w2, w1_bar, w2_bar, w3_bar;
    assign w1 = !(A || B);
    assign w1_bar = !w1;
    assign w2 = !(C || D);
    assign w2_bar = !w2;
    assign w3 = !E_bar;
    assign F = w1_bar && w2_bar && w3;
endmodule

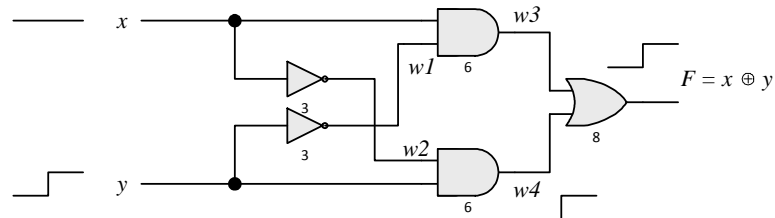
(f) module Fig_3_25_CA (F, A, A_bar, B, B_bar, C, D_bar);
    output F;
    input    A, A_bar, B, B_bar, C, D_bar;
    wire     w1, w2, w3, w4, w5, w6, w7, w8, w9, w10;
    assign w1 = !A_bar;
    assign w2 = !B;
    assign w3 = w1 && w2;
    assign w4 = !A;
    assign w5 = !B_bar;
    assign w6 = w4 && w5;
    assign w7 = !(C || D_bar);
    assign w8 = !(w3 || w6);
    assign w9 = !w8;
    assign w10 = !w7;
    assign F = w9 && w10;
endmodule

```

3.33

(a)

Initially, with $xy = 00$, $w1 = w2 = 1$, $w3 = w4 = 0$ and $F = 0$. $w1$ should change to 0 3ns after xy changes to 01. $w4$ should change to 1 6ns after xy changes to 01. F should change from 0 to 1 8ns after $w4$ changes from 0 to 1, i.e., 14 ns after xy changes from 00 to 01.



(b)

```
`timescale 1ns/1ps
```

```
module Prob_3_33 (F, x, y);
  wire w1, w2, w3, w4;
```

```
  and #6 (w3, x, w1);
  not #3 (w1, x);
  and #6 (w4, y, w1);
  not #3 (w2, y);
  or #8 (F, w3, w4);
```

```
endmodule
```

```
module t_Prob_3_33 ();
```

```
  reg x, y;
  wire F;
```

```
  Prob_3_33 M0 (F, x, y);
```

```
  initial #200 $finish;
```

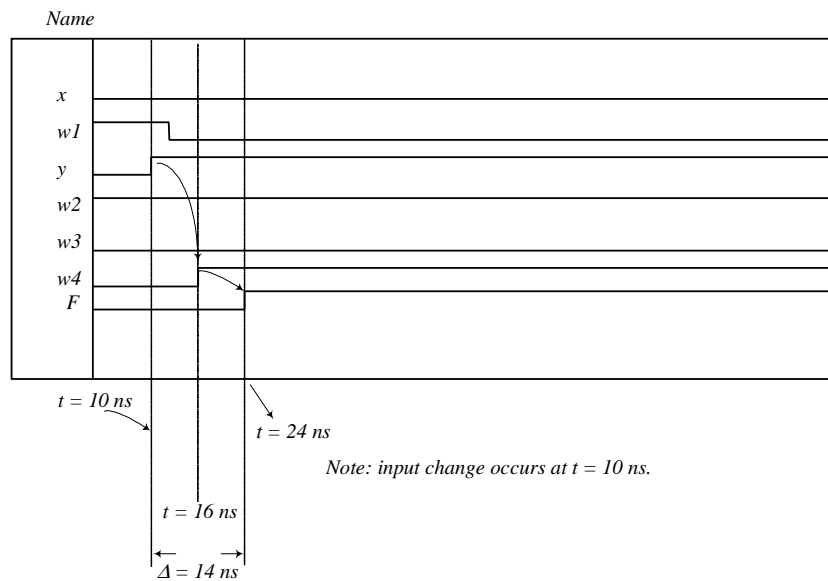
```
  initial fork
```

```
    x = 0;
    y = 0;
    #20 y = 1;
```

```
  join
```

```
endmodule
```

(c) To simulate the circuit, it is assumed that the inputs $xy = 00$ have been applied sufficiently long for the circuit to be stable before $xy = 01$ is applied. The testbench sets $xy = 00$ at $t = 0$ ns, and $xy = 1$ at $t = 10$ ns. The simulator assumes that $xy = 00$ has been applied long enough for the circuit to be in a stable state at $t = 0$ ns, and shows $F = 0$ as the value of the output at $t = 0$. For illustration, the waveforms show the response to $xy = 01$ applied at $t = 10$ ns.



3.34 **module** Prob_3_34 (Out_1, Out_2, Out_3, A, B, C, D);
 output Out_1, Out_2, Out_3;
 input A, B, C, D;
 wire A_bar, B_bar, C_bar, D_bar;
 assign A_bar = !A;
 assign B_bar = !B;
 assign C_bar = !C;
 assign D_bar = !D;
 assign Out_1 = (A + B_bar) && C_bar && (C || D);
 assign Out_2 = ((C_bar && D) || (B && C && D) || (C && D_bar)) && (A_bar || B);
 assign Out_3 = (((A && B) || C) && D) || (B_bar && C);
 endmodule

3.35

```

module Exmpl-3(A, B, C, D, F)      // Line 1
inputs A, B, C, Output D, F,      // Line 2
output B                          // Line 3
and g1(A, B, B);                  // Line 4
not (D, B, A),                    // Line 5
OR (F, B, C);                     // Line 6
endofmodule;                     // Line 7

```

Line 1: Dash not allowed character in identifier; use underscore: Exmpl_3. Terminate line with semicolon (;).

Line 2: **inputs** should be **input** (no s at the end). Change last comma (,) to semicolon (;). *Output* is declared but does not appear in the port list, and should be followed by a comma if it is intended to be in the list of inputs. If *Output* is a misspelling of **output** and is to declare output ports, *C* should be followed by a semicolon (;) and *F* should be followed by a semicolon (;).

Line 3: B cannot be declared both as an input (Line 2) and output (Line 3). Terminate the line with a semicolon (;).

Line 4: A cannot be an output of the primitive if it is an input to the module

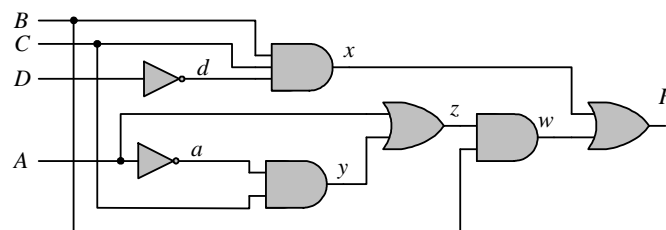
Line 5: Too many entries for the not gate (may have only a single input, and a single output). Terminate the line with a semicolon, not a comma.

Line 6: OR must be in lowercase: change to “or”. Replace semicolon by a comma (B,) in the list of ports.

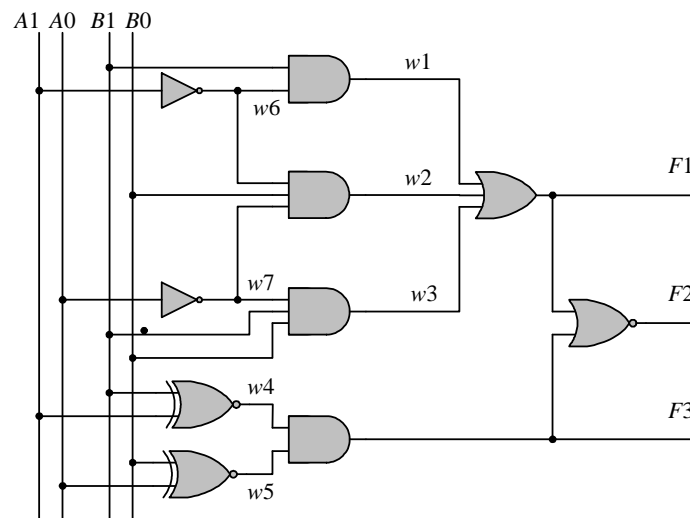
Line 7: Remove semicolon (no semicolon after endmodule).

3.36

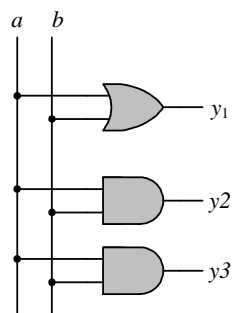
(a)



(b)



(c)



3.37

```

UDP_Majority_4 (y, a, b, c, d);
output y;
input a, b, c, d;
table
// a b c d : y
  0 0 0 0 : 0;
  0 0 0 1 : 0;
  0 0 1 0 : 0;
  0 0 1 1 : 0;
  0 1 0 0 : 0;
  0 1 0 1 : 0;
  0 1 1 0 : 0;
  0 1 1 1 : 1;

  1 0 0 0 : 0;
  1 0 0 1 : 0;
  1 0 1 0 : 0;
  1 0 1 1 : 0;
  1 1 0 0 : 0;
  1 1 0 1 : 0;
  1 1 1 0 : 1;
  1 1 1 1 : 1;
endtable
endprimitive

```

3.38

```

module t_Circuit_with_UDP_02467;
  wire E, F;
  reg A, B, C, D;
  Circuit_with_UDP_02467 m0 (E, F, A, B, C, D);

  initial #100 $finish;
  initial fork
    A = 0; B = 0; C = 0; D = 0;
    #40 A = 1;
    #20 B = 1;
    #40 B = 0;
    #60 B = 1;
    #10 C = 1; #20 C = 0; #30 C = 1; #40 C = 0; #50 C = 1; #60 C = 0; #70 C = 1;
    #20 D = 1;
  join
endmodule
// Verilog model: User-defined Primitive
primitive UDP_02467 (D, A, B, C);
  output D;
  input A, B, C;
  // Truth table for D = f (A, B, C) = . (0, 2, 4, 6, 7);
  table
  // A B C : D // Column header comment
    0 0 0 : 1;
    0 0 1 : 0;
    0 1 0 : 1;
    0 1 1 : 0;
    1 0 0 : 1;
    1 0 1 : 0;
    1 1 0 : 1;
    1 1 1 : 1;
  endtable

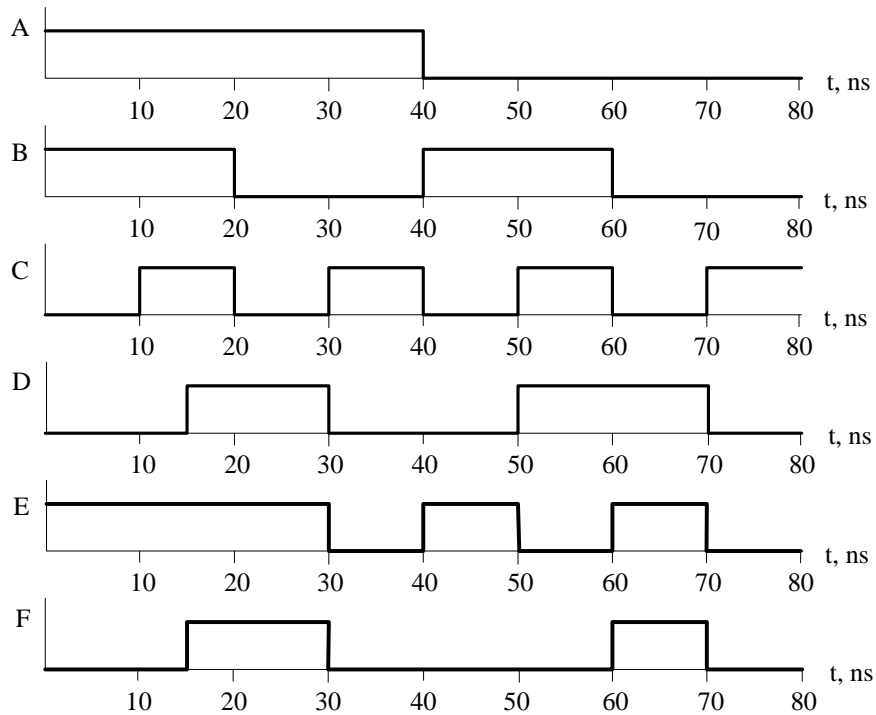
```

```

endtable
endprimitive
// Verilog model: Circuit instantiation of Circuit_UDP_02467
module Circuit_with_UDP_02467 (e, f, a, b, c, d);
  output e, f;
  input a, b, c, d;

  UDP_02467 M0 (e, a, b, c);
  and (f, e, d); //Option gate instance name omitted
endmodule

```



3.39

a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$s = a'b + ab' = a \oplus b$
 $c = ab = a \&\& b$

```
module Prob_3_39 (s, c, a, b);  
  input a, b;  
  output s, c;  
  
  xor (s, a, b);  
  and (c, a, b);  
endmodule
```