Optimizing Parking Space using OR Techniques:

Case Study at Thilakawardena Car Park (Kiribathgoda)

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Abstract

Due to the increasing number of private vehicle use among people, many mainstream shop owners are facing the difficulty of providing enough parking spaces for their customers. Availability of parking space has recently started to play a huge role in deciding what shops a customer visits since everyone seek for the most comfortable experience while shopping. In this case study, it is tried to optimize the available parking space at Thilakawardena Car Parking area according to pre-identified parking requirements for its customers. Thilakawardena group is serving a lot of customers in a daily schedule and by this case study, it is tried to see whether the available parking space successfully meets the demand from customers and provide information to decrease the delays in waiting queues for those customers who seek for parking space. Moreover, a linear programming approach is being used for this optimization problem and requirements are being gathered by observing the parking area. And the optimizations have been done upon two customer needs, namely; parking space and parking duration.

Keywords: LINEAR PROGRAMMING, PARKING SPACE OPTIMIZATION

1. Introduction

In deciding a shop to buy their needs, people tend to reach out for the venues that provide the most pleasant shopping experience for them. With the development that has taken place during the past decade in Sri Lanka, almost everyone owns a private vehicle of their own and it’s seen that individuals like to use these own methods of transportation other than public transportation. Due to this reason, when it comes to shopping, providing efficient parking space have turned out to be a crucial fact when deciding a venue to shop.

It’s observed that, everyone prefers to shop at a venue that provides enough and efficient parking space for its customers. So major shopping malls and similar venues are trying to provide the best possible pleasant shopping experience to their customers by enabling them to park their vehicles close to the shop in walking distance. But due to this high demand for parking space and since more and more are being attracted to such shopping malls, eventually, every shop face the difficulty of providing enough parking space for its growing number of customers. Here, the shop owners don’t like to lose their old customers as well as wanting to attract more new ones also. So they do not want any of their customers, no matter new or old, to face any kind of a trouble when finding a proper parking slot when they visit the shop.

Management's usual approach when finding solutions to this problem is to keep buying new land to accumulate enough space for the growing customer community. But when deciding when to buy new land and where to buy it from, the management does not have any scientific method of deciding so it is usually done upon using common sense. Due to this lack of scientific method, there have been many failure attempts which have eventually lead towards losses so the demand for parking slot optimization rises at this point.

When fulfilling customers' needs with parking space, there are two main things in concern. They are the accumulated space for the vehicle and the duration of occupying the parking space. Here, a linear approach can be taken towards solving and optimizing the parking space considering the above two facts as 3 cases; each at a time and both of them together. But it should be noted that, due to the fact that this is a real world problem, successfully constructing a model to satisfying all customers' needs and obtaining one specific optimal solution would be merely impossible as the true parking space would be decide upon the traffic flow at a specific time.

Usually, this type of a case study will result in determining the best allocation of a limited parking space among different varieties of vehicle types. Similarly, here it is tried to construct a linear mathematical model to represent the real world scenario which takes place at Thilakawardena Car Park, considering the vehicle types, time durations they keep parked and the space each vehicle type accumulate. Since the increasing customer count results with not enough space for all vehicles to be parked, using Operations Research techniques, we try to determine the optimal allocation of parking slots so that each vehicle type would get the most favourable parking area within the limited (total) parking space.

2. Literature Review

The term 'Parking' is recognized as stopping a vehicle at a specific place at an end of a traffic circulation. And considering this parking locations, it can be divided into two groups, namely: 'on street parking' and 'off street parking' (Widanengsih dan Elkhasnet, 2004). And the case study in concern falls under the 'off street parking' category since the parking area is located on a private land.

To represent the parking space that is occupied by each vehicle type, the term 'Parking Control Unit' (PCU) is used. PCU depends on each vehicle's dimension and the additional space that is required by the vehicle to move around freely when parking with a certain angle. Since this case study is conducted on a parking space owned by the Thilakawardena Group, here, specific types of vehicles are being recognized that are often parked inside the parking area and respective PCU values for these vehicles in concern are represented in below table.

Table 2.1: PCU for concerned vehicle types

Vehicle Type	Width (meter)	Parking Width (meter)	Length (meter)	Parking Length (meter)
Three Wheeler	1	1.5	2.2	2.7
Motor Bike	0.8	1.3	1.9	2.4
Car	1.5	2.5	4.1	5.1
Van	1.8	2.8	5.0	6.0

When it comes to parking, there are certain characteristics that have to be provided to determine the demanded parking space by the customers (Hobbs, 1995). These parking characteristics include;

1. Number of vehicles entering the parking area. (*Parking Volume*)
2. Number of vehicles parked in the area at a certain time (*Parking Accumulation*)
3. Amount of time a certain type of vehicle is been parked in the parking area (*Parking Duration*)

3. Case Study Procedure

The concerned car park is situated at the heart of Kiribathgoda town and is owned by the Thilakawardena business group. And firstly, to observe the general behaviour at the parking area, a survey was conducted from 9 a.m to 6 p.m and the required information for the accumulated vehicle types and other relevant information have been gathered through direct observation. Dimensions of the vehicle types were obtained and the PCU values were calculated using the surveyed data (presented in table 2.1). Behaviour of the parked vehicles also obtained through this field survey by observing the parking site and counting the number of vehicles of each vehicle type parked in the area and also the respective parking durations. Total parking space was also measured so that this limited parking space could be included into a mathematical model to determine the optimal allocation for each vehicle type within this concerned parking space. Final data processing procedures were conducted using Microsoft Excel software.

Furthermore, using the data obtained from the survey, mathematical model was constructed by including the user parking space demand proportionally to average parking accumulation and average parking duration for each vehicle type. Here, the proportionality of average parking accumulation was computed for every 15 minutes for the 4 types of vehicles. I.e. these proportions for Motor Bikes, Cars, Three Wheelers and Vans were calculated using the following formulation:

$$\frac{P_1}{P_1+P_2+P_3+P_4}(x_1+x_2+x_3+x_4), \qquad \frac{P_2}{P_1+P_2+P_3+P_4}(x_1+x_2+x_3+x_4),$$

$$\frac{P_3}{P_1+P_2+P_3+P_4}(x_1+x_2+x_3+x_4), \qquad \frac{P_4}{P_1+P_2+P_3+P_4}(x_1+x_2+x_3+x_4),$$

Where;

P_1 : Average Parking accumulation of Motor Bike (Number of Motor Bikes per 15 mins)
P_2 : Average Parking accumulation of Car (Number of Cars per 15 mins)
P_3 : Average Parking accumulation of Three Wheeler (Number of Three Wheelers/ 15 mins)
P_4 : Average Parking accumulation of Van (Number of Vans/ 15 mins)

$x_1 + x_2 + x_3 + x_4$: Parking space capacity.

Moreover, proportionality to average parking duration in every 15 minutes for Motor Bike, Car, Three Wheeler and Van are formulated as follows:

$$\frac{t_1}{t_1 + t_2 + t_3 + t_4}(x_1 + x_2 + x_3 + x_4), \qquad \frac{t_2}{t_1 + t_2 + t_3 + t_4}(x_1 + x_2 + x_3 + x_4),$$
$$\frac{t_3}{t_1 + t_2 + t_3 + t_4}(x_1 + x_2 + x_3 + x_4), \qquad \frac{t_4}{t_1 + t_2 + t_3 + t_4}(x_1 + x_2 + x_3 + x_4),$$

Where;
t_1: Average Parking duration for Motor Bike (minutes)
t_2: Average Parking duration for Car (minutes)
t_3: Average Parking duration for Three Wheeler (minutes)
t_4: Average Parking duration for Van (minutes)

Now the problem that needs to be solved is to maximize the parking space capacity at Thilakawardena Car Park. And this maximization has to be done under the constraints of not exceeding the total available parking area and also to meet the demand of parking for each type of vehicle within a certain amount of time (here, for each 15 minutes cycles)

So the main purpose is to allocate parking space for all 4 types of vehicle. And based on the PCU values, parking space needed for each type of vehicle concerned in this study can be clarified as follows:

1. Parking space for a Motor Bike : 3.12 m^2
2. Parking space for a Car : 12.75 m^2
3. Parking space for a Three Wheeler : 4.05 m^2
4. Parking space for a Van : 16.8 m^2

Note that the proportion of vehicle average parking accumulation (for every 15 minutes cycle) for each type of vehicle is computed using the vehicle's average parking accumulation divided by the total average parking accumulation for each type of vehicle and multiply it the total parking space capacity (as formulated before)

So their mathematical representations can be presented as follows:

a. Proportion of average parking accumulation for Motor Bike:

$$\frac{P_1}{P_1 + P_2 + P_3 + P_4}(x_1 + x_2 + x_3 + x_4)$$

b. Proportion of average parking accumulation for Car:

$$\frac{P_2}{P_1 + P_2 + P_3 + P_4}\ (x_1 + x_2 + x_3 + x_4)$$

c. Proportion of average parking accumulation for Three Wheeler:

$$\frac{P_3}{P_1 + P_2 + P_3 + P_4}\ (x_1 + x_2 + x_3 + x_4)$$

d. Proportion of average parking accumulation for Van:

$$\frac{P_4}{P_1 + P_2 + P_3 + P_4}\ (x_1 + x_2 + x_3 + x_4)$$

Where;

P_1 : Motor Bike average parking accumulation (Number of Motor Bikes in 15 mins)
P_2 : Car average parking accumulation (Number of Cars in 15 mins)
P_3 : Three Wheeler average parking accumulation (Number of Three Wheelers in 15 mins)
P_4 : Van average parking accumulation (Number of Vans in 15 mins)
$(x_1 + x_2 + x_3 + x_4)$: Parking slot capacity that is to be allocated

Also note that the proportion of vehicle average parking duration (for every 15 minutes cycle) for each type of vehicle can be computed using the vehicle's average parking duration divided by the total average parking duration for each type of vehicle and multiply it the total parking space capacity (as formulated before)

Their mathematical representation is as follows:

a. Proportion of average parking duration for Motor Bike:

$$\frac{t_1}{t_1 + t_2 + t_3 + t_4}\ (x_1 + x_2 + x_3 + x_4)$$

b. Proportion of average parking duration for Car:

$$\frac{t_2}{t_1 + t_2 + t_3 + t_4}\ (x_1 + x_2 + x_3 + x_4)$$

c. Proportion of average parking duration for Three Wheeler:

$$\frac{t_3}{t_1 + t_2 + t_3 + t_4}\ (x_1 + x_2 + x_3 + x_4)$$

d. Proportion of average parking duration for Van:

$$\frac{t_4}{t_1 + t_2 + t_3 + t_4}\ (x_1 + x_2 + x_3 + x_4)$$

Where;

t_1: Average parking duration for Motor Bike (minutes)
t_2: Average parking duration for Car (minutes)
t_3: Average parking duration for Three Wheeler (minutes)
t_4: Average parking duration for Van (minutes)

3.1. Constructing the Linear Programming Model

In order to obtain the optimum parking slot allocations, this maximization problem can be structured as the following linear programming model:

Objective Function: Maximize $Z = x_1 + x_2 + x_3 + x_4$

Subject to:

$$3.12x_1 + 12.75x_2 + 4.05x_3 + 16.8x_4 \leq \text{ Total Parking Space available}$$

$x_1 \geq \frac{P_1}{P_1+P_2+P_3+P_4}\,(x_1 + x_2 + x_3 + x_4) \times \alpha$ [Motor Bike parking accumulation proportional to all]

$x_2 \geq \frac{P_2}{P_1+P_2+P_3+P_4}\,(x_1 + x_2 + x_3 + x_4) \times \alpha$ [Car parking accumulation proportional to all]

$x_3 \geq \frac{P_3}{P_1+P_2+P_3+P_4}\,(x_1 + x_2 + x_3 + x_4) \times \alpha$ [Three Wheeler parking accumulation proportional to all]

$x_4 \geq \frac{P_4}{P_1+P_2+P_3+P_4}\,(x_1 + x_2 + x_3 + x_4) \times \alpha$ [Van parking accumulation proportional to all]

$x_1 \geq \frac{t_1}{t_1+t_2+t_3+t_4}\,(x_1 + x_2 + x_3 + x_4) \times \alpha$ [Motor Bike parking duration proportional to all]

$x_2 \geq \frac{t_2}{t_1+t_2+t_3+t_4}\,(x_1 + x_2 + x_3 + x_4) \times \alpha$ [Car parking duration proportional to all]

$x_3 \geq \frac{t_3}{t_1+t_2+t_3+t_4}\,(x_1 + x_2 + x_3 + x_4) \times \alpha$ [Three Wheeler parking duration proportional to all]

$x_4 \geq \frac{t_4}{t_1+t_2+t_3+t_4}\,(x_1 + x_2 + x_3 + x_4) \times \alpha$ [Van parking duration proportional to all]

The non negativity constraints:

$$x_1, x_2, x_3, x_4, P_1, P_2, P_3, P_4, t_1, t_2, t_3, t_4 \geq 0 \quad \text{and} \quad 0 \leq \alpha \leq 1$$

Here, α is the level of satisfaction for each constraint and x_1, x_2, x_3, x_4 denote the allocated parking slots for Motor Bike, Car, Three Wheeler and Van respectively.

To find the coefficient $(P_1, P_2, P_3, P_4, t_1, t_2, t_3, t_4)$ values, consider the data obtained through the field survey.

Parking Characteristic	**Motor Bike**	**Car**	**Three Wheeler**	**Van**
Parking Volume	93	54	23	11
Parking Capacity	63	14	44	10
Parking accumulation at a Peak Time (5 p.m – 5.15 p.m)	16	7	3	1
Average parking duration (minutes)	38	71	43	62

After calculating P_1, P_2, P_3, P_4 and t_1, t_2, t_3, t_4 values with the use of above observed survey data, the mathematical model for this maximization problem can be written:

$$\text{Maximize Z } = x_1 + x_2 + x_3 + x_4$$

Subject to:

$$3.12x_1 + 12.75x_2 + 4.05x_3 + 16.8x_4 \leq 216$$

$$x_1 \geq 0.59(x_1 + x_2 + x_3 + x_4) \times \alpha$$

$$x_2 \geq 0.26(x_1 + x_2 + x_3 + x_4) \times \alpha$$

$$x_3 \geq 0.11(x_1 + x_2 + x_3 + x_4) \times \alpha$$

$$x_4 \geq 0.04(x_1 + x_2 + x_3 + x_4) \times \alpha$$

$$x_1 \geq 0.18(x_1 + x_2 + x_3 + x_4) \times \alpha$$

$$x_2 \geq 0.33(x_1 + x_2 + x_3 + x_4) \times \alpha$$

$$x_3 \geq 0.20(x_1 + x_2 + x_3 + x_4) \times \alpha$$

$$x_4 \geq 0.29(x_1 + x_2 + x_3 + x_4) \times \alpha$$

Where;

$$x_1 + x_2 + x_3 + x_4 \geq 0 \quad \text{and} \quad 0 \leq \alpha \leq 1$$

4. Results and Discussion

Mathematical model was tested for values of α between 0.75 to 1 using Lingo 11.0 in seek of the optimal solution. But in any case, no feasible solution was achieved. As an example, when testing the model for $= 0.80$, it means that, when trying to satisfy both the average parking accumulation and parking duration constraints simultaneously for every vehicle type (4) at more than 80% satisfaction, constraints were unable to provide any realistic point within the convex set. Taking the above obtained results into account, approach towards trying to find an optimal solution was driven into more realistic scenarios as follows:

1. Only consider the average parking accumulation constraints with values of α between 0.75 to 1 and try to optimize the model.
2. Only consider the average parking duration constraints with values of α between 0.75 to 1 and try to optimize the model.
3. Consider both average parking accumulation and parking duration constraints with values of α between 0.75 to 1 and try to optimize the model.

4.1. Optimizing the model only considering Parking Accumulation Constraints.

Here, the parking duration constraints are not considered and the mathematical model is optimized for values of α from 0.75 to 1 using LINGO 11.0. Obtained results are shown in Table 4.1.

Table 4.1. Solutions for Model Only Considering Average Parking Accumulations

Variable	$\alpha = 0.75$	$\alpha = 0.80$	$\alpha = 0.85$	$\alpha = 0.90$	$\alpha = 0.95$	$\alpha = 1$
x_1	27.27086	25.72413	24.26148	22.87625	21.56244	20.31467
x_2	7.679160	7.962230	8.229912	8.483426	8.723869	8.952226
x_3	3.248875	3.368636	3.481886	3.589142	3.690868	3.787480
x_4	1.181409	1.224959	1.266140	1.305143	1.342134	1.377266
Z	39.38031	38.27995	37.23942	36.25396	35.31931	34.43164

Considering the solutions obtained in Table 4.1, its clear that when the level of α increase (i.e. satisfaction level for the constraints) number of parking slots decrease for all vehicle types as well as the total. It's also seen that in every case, Motor Bike is having the highest parking slot accumulation among all 4 types of vehicles.

4.2. Optimizing the model only considering Parking Duration Constraints.

Here, the parking accumulation constraints are not considered and the mathematical model is optimized for values of α from 0.75 to 1 using LINGO 11.0. Obtained results are shown in Table 4.2.

Table 4.2. Solutions for Model Only Considering Average Parking Durations

Variable	$\alpha = 0.75$	$\alpha = 0.80$	$\alpha = 0.85$	$\alpha = 0.90$	$\alpha = 0.95$	$\alpha = 1$
x_1	9.649207	8.269893	6.998712	5.823426	4.733580	3.720183
x_2	6.203061	6.346662	6.479006	6.601365	6.714829	6.820335
x_3	3.759431	3.846462	3.926670	4.000827	4.069594	4.133536
x_4	5.451175	5.577370	5.693672	5.801200	5.900911	5.993627
Z	25.06287	24.04039	23.09806	22.22682	21.41891	20.66768

Considering the solutions obtained in Table 4.2, as it was also seen in Table 4.1, when the level of satisfaction for Average Parking Durations (α) increases, number of optimum parking slots tend to get smaller. Also, among all vehicle types, Motor Bike gets the highest Parking Slot allocation at every level of α. As it was clearly visible in the obtained field survey results, average parking durations for all four types of vehicles did not differ in large numbers. Because of that reason, here the resulted optimal parking slot allocations does not vary in extreme numbers, i.e. the allocations are somewhat balanced between all four types of vehicles (unlike in Table 4.1. where there were significant changes in allocated parking slots for different vehicle types)

4.3. Optimizing the model considering both Parking Accumulation and parking Duration Constraints.

Here, both parking accumulation and parking duration constraints are considered and the mathematical model is optimized for values of α from 0.5 to 0.7 using LINGO 11.0. (For α values between 0.7 and 1, this problem becomes infeasible) Obtained results are shown in Table 4.3.

Table 4.3. Solutions for Model Considering Both Parking Accumulations and Parking Durations

Variable	$\alpha = 0.5$	$\alpha = 0.55$	$\alpha = 0.60$	$\alpha = 0.65$	$\alpha = 0.70$
x_1	18.78109	16.58029	14.59409	12.79255	11.15106
x_2	5.252338	5.481463	5.688248	5.875807	6.046703
x_3	3.183235	3.322099	3.447423	3.561095	3.664668
x_4	4.615691	4.817043	4.998763	5.163588	5.313769
Z	31.83235	30.20090	28.72852	27.39304	26.17620

Solutions in Table 4.3 show that, as it was also seen in both Table 4.1 and Table 4.2, higher satisfaction level for the constraints (higher α values) results with a smaller parking slot allocation. At each level, the highest average parking slot accumulation and parking duration is present in Motor Bike compared to the rest. But in contrary, as α increases, parking slots for Motor Bike decrease (even though it records the highest among rest) whereas, parking slots for Car, Three Wheeler and Van increases with α (satisfaction level)

Lastly, considering above three formulations for difference scenarios, when trying to find the optimal number of parking slots for the considered parking space of Thilakawardena shopping group, the best option would be the first one, i.e. the formulation which only considered parking accumulations in order to optimize the model. It’s because only that formulation provides the highest total number of optimal parking slots as the result of the maximization problem. Formulation which only consider the average parking duration as well as the formulation which took both scenarios into account, i.e. parking accumulation and parking duration, both of these models resulted with a less number of parking slots than the first one. Moreover, in the real world, the customers would not like to keep constraints on the parking duration as well because that would put pressure on their shopping experience. What would only matter is the availability of parking space at a specific time, which is directly related with the first model – average parking accumulations. So the first model not only results with the highest number of optimal parking slots, it’s also related with the customers’ satisfaction as well.

5. Conclusion

Solving this optimization problem to find the optimal allocation of parking slots demonstrated that, different scenarios have to be considered when formulating the mathematical model. That happens because there exist different types of requirements and demands which have to be satisfied in order to achieve the optimality. Attempting to fulfill all these parking requirements from owners' side and demands from the customers/users side may lead the optimization problem towards an infeasible solution. So to obtain a reasonable optimal solution that would successfully satisfy most of the conditions and constraints which has to be met, it is necessary to pinpoint and isolate the most important and realistic conditions/requirements/demands when formulating such real world optimization problem into a mathematical form. It becomes the only way to achieve a sensible optimal solution that can be used in the real world for the optimal parking allocation problem.

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Appendix

Complete LINGO Model for the Optimal Allocation Problem

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Model:

Max = x1 + x2 + x3 + x4;

! Total Available Parking Space Constaint;
3.12*x1 + 12.75*x2 + 4.05*x3 + 16.8*x4 <= 216;

! Satisfaction Level for Constraints;
A = 0.70;

! Average Parking Accumulation Constraints;
A* (0.59 * ( x1 + x2 + x3 + x4 ) ) <= x1;
A* (0.26 * ( x1 + x2 + x3 + x4 ) ) <= x2;
A* (0.11 * ( x1 + x2 + x3 + x4 ) ) <= x3;
A* (0.04 * ( x1 + x2 + x3 + x4 ) ) <= x4;

! Average Parking Duration Constraings;
A* (0.18 * ( x1 + x2 + x3 + x4 ) ) <= x1;
A* (0.33 * ( x1 + x2 + x3 + x4 ) ) <= x2;
A* (0.20 * ( x1 + x2 + x3 + x4 ) ) <= x3;
A* (0.29 * ( x1 + x2 + x3 + x4 ) ) <= x4;

end
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