Surface Representations

- Polygon meshes
  - Representations
    - Polygon list – each polygon is represented by a list of vertex coordinates

      \+ simplest representation

      – shared edges are drawn twice (wasted storage)

      – common processing operations are expensive and unreliable
    - Vertex list – define polygons using pointers to vertices.

      \+ compact and moving vertices is easy

      – difficult to find polygons that share an edge

      – shared edges are drawn twice (wasted storage)
    - Edge list – all edges, and vertices are stored and each polygon is defined in terms of edges and vertices

      \+ shared edges are not drawn twice

      \+ adjacent polygons are readily apparent from the edge records

      – takes up a lot of storage
  - Polygon normals

$$Ax + By + Cz + D = 0$$

$$A = \frac{1}{2}\sum_{i=1}^{n}(z_i + z_{i\oplus 1})(y_i - y_{i\oplus 1}) \qquad B = \frac{1}{2}\sum_{i=1}^{n}(x_i + x_{i\oplus 1})(z_i - z_{i\oplus 1}) \qquad C = \frac{1}{2}\sum_{i=1}^{n}(y_i + y_{i\oplus 1})(x_i - x_{i\oplus 1})$$

- **Parametric Curves**
  - Cubic curve representation

$$\begin{bmatrix} x(t) & y(t) & z(t) \end{bmatrix} = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{bmatrix}$$

$$\boldsymbol{Q}(t) \qquad = \qquad \boldsymbol{T} \qquad\qquad \boldsymbol{C}$$

$$\boldsymbol{Q}'(t) \qquad = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} \qquad \boldsymbol{C}$$

  - Continuity
    - geometric continuity, G^n: the direction of the nth derivative of the curve is continuous.
    - parametric continuity, C^n: the magnitude of the nth derivative of the curve is continuous.

  - Forms of parametric cubic curves

$$\boldsymbol{Q}(t) = \boldsymbol{TMG} = \boldsymbol{BG}$$

$\boldsymbol{M} \in \mathbb{R}^{4\times 4}$: basis matrix, $\quad \boldsymbol{G} \in \mathbb{R}^{4\times 3}$: geometry matrix (4 constraints)

$\boldsymbol{B}$: blending function (curve is a weighted sum of geometry matrix)

- **Hermite**
  - constraints: positions and tangent vectors of start and end points

$$\boldsymbol{C} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{P_1} \\ \boldsymbol{P_4} \\ \boldsymbol{R_1} \\ \boldsymbol{R_4} \end{bmatrix} = \boldsymbol{M}_h \boldsymbol{G}_h$$

- **Bezier**
  - constraints: specified start and end positions, tangent vector specified by endpoints

$$\boldsymbol{R}_1 = 3(\boldsymbol{P}_2 - \boldsymbol{P}_1), \qquad \boldsymbol{R}_4 = 3(\boldsymbol{P}_4 - \boldsymbol{P}_3)$$

$$\boldsymbol{C} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{P_1} \\ \boldsymbol{P_2} \\ \boldsymbol{P_3} \\ \boldsymbol{P_4} \end{bmatrix} = \boldsymbol{M}_b \boldsymbol{G}_b$$

  - Convex hull property: a point lies within the convex hull of a set of points if it can be written as a linear sum of the points, with the weights all lying in the range 0 and 1 and summing to 1
  - Can easily be subdivided to produce control points that are good poly-line approximations to the curve.
- **B-splines**
  - constraints: four control points shared between adjacent segments for approximation

$$\boldsymbol{C}_i = \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{P}_{i-1} \\ \boldsymbol{P}_i \\ \boldsymbol{P}_{i+1} \\ \boldsymbol{P}_{i+2} \end{bmatrix}$$

  - does not pass through the control points
  - can be transformed in an affine manner by applying the transformation to control points
- **Catmull-Rom**
  - constraints: four control points shared between adjacent segments for interpolation
    - specified start 2nd and end 3rd points, tangent vectors specified by 1st and 4th points.

$$\boldsymbol{C}_i = \frac{1}{2} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{P}_{i-1} \\ \boldsymbol{P}_i \\ \boldsymbol{P}_{i+1} \\ \boldsymbol{P}_{i+2} \end{bmatrix}$$

  - passes through the control points

	Hermite	**Bezier**	**B-spline**	**Catmull-Rom**
Convex hull property	N/A	Yes	Yes	No
Interpolates some control points	Yes	Yes	No	Yes
Interpolates all control points	Yes	No	No	Yes
Ease of subdivision	Good	Best	Avg	Avg
Inherent continuity	C^0 G^0	C^0 G^0	C^2 G^2	C^1 G^1
Easily achieved continuity	C^1 G^1	C^1 G^1	C^2 G^2	C^1 G^1