

$$(1) A \vee (B \vee C) \vdash A \vee (B \vee (C \vee D))$$

$$\begin{array}{l} \overline{\Gamma_{\text{ано}}} A \vee (B \vee C) \vdash_{\Pi_1} B \vee (A \vee (B \vee C)) \vdash_{\Pi_2} (B \vee A) \vee (B \vee C) \vdash_{\Pi_3} \\ \vdash_{\Pi_3} (B \vee C) \vee (B \vee A) \vdash_{\Pi_4} ((B \vee C) \vee B) \vee A \vdash_{\Pi_4} (B \vee (C \vee D)) \vee A \vdash_{\Pi_3} A \vee (B \vee (C \vee D)) \end{array}$$

$$(2) A \vee (B \vee (A \vee C)) \vdash B \vee (A \vee C)$$

$$\begin{array}{l} \overline{\Gamma_{\text{ано}}} A \vee (B \vee (A \vee C)) \vdash_{\Pi_1} B \vee (A \vee (A \vee C)) \vdash_{\Pi_2} (B \vee (A \vee C)) \vee A \vdash_{\Pi_3} \\ \vdash_{\Pi_3} (B \vee A) \vee (A \vee C) \vdash_{\Pi_4} C \vee ((B \vee (A \vee C)) \vee A) \vdash_{\Pi_4} ((B \vee (A \vee C)) \vee A) \vee C \vdash_{\Pi_4} \\ \vdash_{\Pi_4} (B \vee (A \vee C)) \vee (A \vee C) \vdash_{\Pi_3} (A \vee C) \vee (B \vee (A \vee C)) \vdash_{\Pi_2} B \vee ((A \vee C) \vee \\ \vee (B \vee (A \vee C))) \vdash_{\Pi_1} (B \vee (A \vee C)) \vee (B \vee (A \vee C)) \vdash_{\Pi_2} B \vee (A \vee C) \vdash_{\Pi_2} \# \end{array}$$

$$(3) A \vee (B \vee C) \vdash A \vee (C \vee B)$$

$$\begin{array}{l} \overline{\Gamma_{\text{ано}}} A \vee (B \vee C) \vdash_{\Pi_1} (B \vee C) \vee A \vdash_{\Pi_2} B \vee (C \vee A) \vdash_{\Pi_3} (C \vee A) \vee B \vdash_{\Pi_4} \\ \vdash_{\Pi_4} (C \vee B) \vee A \vdash_{\Pi_3} A \vee (C \vee B) \end{array}$$

$$(4) \vdash (A \vee A) \rightarrow A \quad \vdash \neg(A \vee A) \vee A$$

$$\begin{array}{l} \overline{\Gamma_{\text{Ax}}} \neg(A \vee A) \vee (A \vee A) \vdash_{\Pi_1} (\neg(A \vee A) \vee A) \vee A \vdash_{\Pi_2} \\ \vdash_{\Pi_2} A \vee (\neg(A \vee A) \vee A) \vdash_{\Pi_1} A \vee (A \vee (\neg(A \vee A) \vee A)) \vdash_{\Pi_1} \\ \vdash_{\Pi_1} (A \vee A) \vee (\neg(A \vee A) \vee A) \vdash_{\Pi_2} \neg(A \vee A) \vee A \end{array}$$

$$(5) (A \rightarrow B) \rightarrow C \vdash A \vee C \quad \neg(\neg A \vee B) \vee C \vdash A \vee C$$

$$\overline{\Gamma_{\text{ано}}} \neg(\neg A \vee B) \vee C \vdash$$

$$\vdash_{\text{Ax}} \neg A \vee A \vdash_{\Pi_1} A \vee \neg A \vdash_{\Pi_1} B \vee (A \vee \neg A) \vdash_{\Pi_2} (A \vee \neg A) \vee B \vdash_{\Pi_1}$$

$$\vdash_{\Pi_1} A \vee (\neg A \vee B) \vdash_{\Pi_1} A \vee C$$