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Rectified Optics and Edge Birefringence

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A well rectified polarizing microscope is constructed and applied to examine the edge birefringence.

An interpretation is proposed to clarify the origin of the edge birefringence by using the theory of birefringence introduced by Bragg.

§1. Introduction

Only low aperture lenses can be used with the polarizing microscope for detecting small retardation. If a high aperture objective is used, the field no longer remains dark due to the stray light and the detection of the small retardation becomes very difficult. Inoué and Hyde reported that the stray light arised by depolarization at the lens surfaces and could be made extinct with the polarization rectifier.[1] Furthermore, Inoué found with his rectified optics that there were the differential phase shifts to each side of a phase boundary of an isotropic material. The phenomenon is called the edge birefringence.[2] The purpose of this report is to present data of the rectified lenses and to propose an interpretation to clarify the nature of edge birefringence.

§2. Rectified Optics

Let the light fall on a plate of glass with the plane of vibration making an angle to the plane of incidence. The incident light may be resolved into p and s components. As the transmittance of the p vibration is larger than that of the s vibration, the vector sum of the refracted components i.e. the plane of vibration of the refracted light rotates toward the plane of incidence. The rotation of the plane of vibration occurs between crossed polars in the polarizing microscope and the rotation of the plane polarized light passing through different points of the exit pupil of the objective is schematically shown in Fig. 1. This rotation introduces the stray light which lowers the sensitivity to detect the weak birefringence.

Now it is verified that much of the rotation can be eliminated by the use of polarization rectifiers built into the objective and condenser lenses.[1] The rectifier consists of a $\lambda/2$ birefringent plate which reverses the above mentioned rotation and zero power lenses which introduce sufficient additional rotation to cancel out the reversed rotation. Figure 2 shows the function of the rectifier and the polarization conditions in different planes between the crossed polarizers. This system gives high extinction factor up to $\sim 10^5$ and reduces the detectable retardation to the order of 1 Å. Figures 3 and 4 are the muscles of chick taken with the non-rectified and rectified polarizing microscope, respectively.

§3. Edge Birefringence

By rectification, the sensitivity for detecting weak retardation is improved and hitherto unnoticed phenomenon becomes apparent. The observed phenomenon is as follows:[3] at the edges of any specimen including isotropic materials, light is diffracted as though each edge were covered with a double layer of extremely thin, birefringent material, the slow axis on the high index side lying parallel, and on the low index side lying perpendicular, to the edge. We have named this phenomenon edge birefringence. It is found at almost every boundary without depending on the conditions that materials of both sides of the boundary are

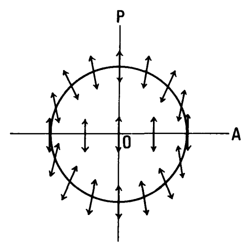

Fig. 1. Rotation of the plane of polarized light passing through different points of the exit pupil. P and A indicate the polarizer and analyzer directions, respectively.

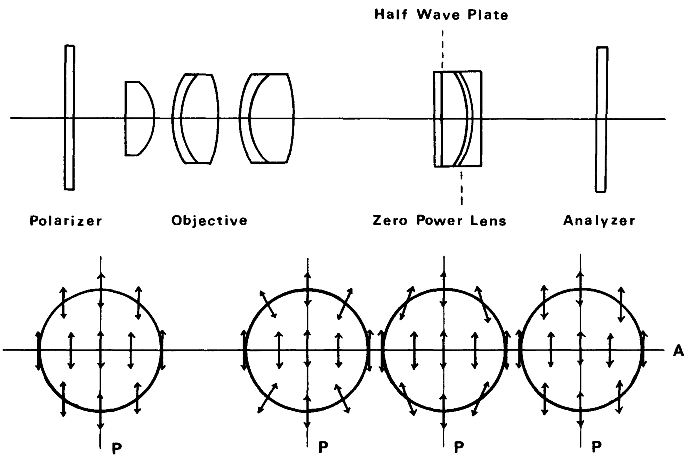

Fig. 2. Function of the rectifier and the polarization conditions in the objective.

Fig. 3. Muscle of chick—no rectifier.

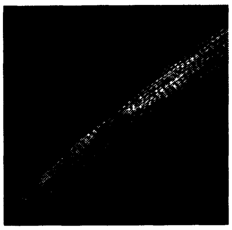

Fig. 4. Muscle of chick—with rectifiers.

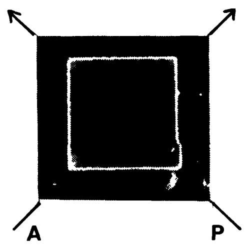

Fig. 5. Edge birefringence of sodium chloride—crossed polarizers.

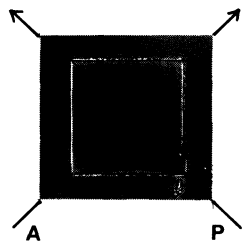

Fig. 6. Edge birefringence of sodium chloride—compensator turned from the case of Fig. 5.

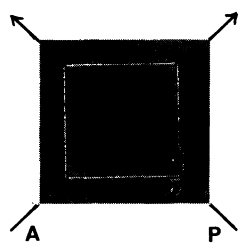

Fig. 7. Edge birefringence of sodium chloride—compensator turned in the opposite direction to Fig. 6.

solids, liquid or gas, so long as the boundary is sharp and there exists a refractive index difference between both sides of the boundary. As far as we know, it is not based on the presence of a membrane at the optical interface, but is a basic diffraction phenomenon taking place at every edge. Edge birefringence disappears when the refractive indices on both sides of the boundary are matched. It reverses in sign when the relative magnitude of refractive indices on both sides of the boundary are reversed.

Figures 5–7 are the edge birefringence of sodium chloride. In the figures, *P* and *A* indicate the polarizer and analyzer directions, respectively. In Figs. 6 and 7, the rotation direction of the compensator is opposite.

Incidentally, it is discussed that in order to clarify the edge birefringence, many assumptions are possible and those are as follows:[4)]

1) the phenomenon is due to the rotation of the plane of polarized light at the edge,

2) the phenomenon stems from diffraction, so the rigorous diffraction theory explains it,

3) the phenomenon is the interference phenomenon like the Lloyd's mirror experiment,

but the nature of edge birefringence is still incompletely understood.

§4. Theory of Birefringence

Firstly, let us consider how to calculate the local electric field.[5)] The local field $\boldsymbol{E}_{\mathrm{loc}}$ at any atom may be written as a sum

$$\boldsymbol{E}_{\mathrm{loc}}=\boldsymbol{E}_0+\boldsymbol{E}_{\mathrm{P}}, \tag{1}$$

where $\boldsymbol{E}_0$ is the field from external sources and $\boldsymbol{E}_{\mathrm{P}}$ is the total effect of all the other atoms in the specimen. $\boldsymbol{E}_{\mathrm{P}}$ is given by

$$\boldsymbol{E}_{\mathrm{P}}=\sum_i \frac{3(\boldsymbol{p}_i\cdot\boldsymbol{r}_i)\boldsymbol{r}_i-r_i^2\boldsymbol{p}_i}{r_i^5}, \tag{2}$$

where $\boldsymbol{p}_i$ is the dipole moment of atom i, and $\boldsymbol{r}_i$ is the position vector from the reference point to the atom. Furthermore, $\boldsymbol{E}_{\mathrm{P}}$ may be expressed as a sum

$$\boldsymbol{E}_{\mathrm{P}}=\boldsymbol{E}_1+\boldsymbol{E}_2+\boldsymbol{E}_3. \tag{3}$$

$\boldsymbol{E}_1$ is the depolarization field, $\boldsymbol{E}_2$ is the Lorentz field and $\boldsymbol{E}_3$ is the field of the atoms within the Lorentz sphere.

Now birefringence means that $\boldsymbol{E}_{\mathrm{loc}}$ depends on the direction. Many birefringence phenomena can be explained by considering the anisotropies of $\boldsymbol{E}_1$, $\boldsymbol{E}_2$ and $\boldsymbol{E}_3$ terms.[6-8)]

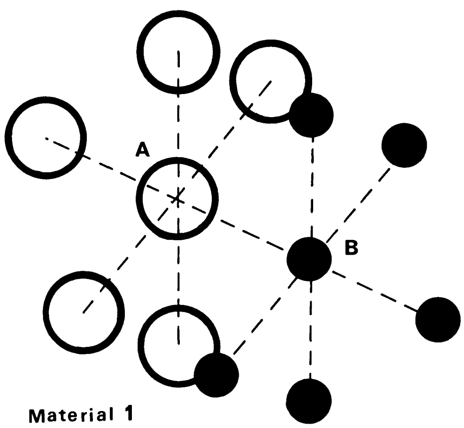

Fig. 8. Model of two isotropic materials.

§5. Origin of Edge Birefringence

We apply the above theory to the edge of two isotropic materials and show that the edge is anisotropic. As a model of the edge of two materials, let us suppose that the two materials consist of different monoatomic crystals, whose lattice constants are the same, as shown in Fig. 8. Let the polarizability and polarization of the material $i(i=1, 2)$ be α_i and $\boldsymbol{P}_i$, respectively, and we postulate $\alpha_1>\alpha_2$.

Since we can assume that the dimensions of materials are much larger than the wavelength of light, we do not need to take into consideration $\boldsymbol{E}_1$ term.[9)]

And at the point on the edge, $\boldsymbol{E}_2$ is given by

$$\boldsymbol{E}_2=\frac{2\pi}{3}(\boldsymbol{P}_1+\boldsymbol{P}_2) \tag{4}$$

and this term does not depend on the direction of vibration of the light. Therefore, birefringence does not occur from $\boldsymbol{E}_2$ term. But since the refractive index near the boundary is given by

$$\frac{n^2-1}{n^2+2}=\frac{2\pi}{3}N(\alpha_1+\alpha_2), \tag{5}$$

where N is the number per unit volume of atoms, the refractive index at the edge n is between n_1 and n_2,[10)] where n_i is the refractive index of the material i.

Lastly, we consider $\boldsymbol{E}_3$ term. Near the edge, $\boldsymbol{E}_3$ depends on the direction of vibration of the incident light. Calculating $\boldsymbol{E}_{\mathrm{loc}}$ at the atom A of Fig. 8, $\boldsymbol{E}_{\mathrm{loc}}$ may be expressed by

$$\begin{aligned}\boldsymbol{E}_{\mathrm{loc}\parallel}&=\frac{1}{1-\delta_\parallel}\left\{\boldsymbol{E}_0+\frac{2\pi}{3}(\boldsymbol{P}_1+\boldsymbol{P}_2)\right\},\\ \boldsymbol{E}_{\mathrm{loc}\perp}&=\frac{1}{1+\delta_\perp}\left\{\boldsymbol{E}_0+\frac{2\pi}{3}(\boldsymbol{P}_1+\boldsymbol{P}_2)\right\},\end{aligned} \tag{6}$$

where $\parallel$ and $\perp$ mean that the directions of vibration of the light are parallel and perpendicular to the edge, respectively, and $\delta_\parallel$ and $\delta_\perp$ are the positive values which may be expressed in terms of the polarizabilities and the lattice constant. Then the Lorenz-Lorentz equations are given by

$$\begin{aligned}\frac{n_\parallel^2-1}{n_\parallel^2+2}&=\frac{1}{1-\delta_\parallel}\,\frac{2\pi}{3}N(\alpha_1+\alpha_2),\\ \frac{n_\perp^2-1}{n_\perp^2+2}&=\frac{1}{1+\delta_\perp}\,\frac{2\pi}{3}N(\alpha_1+\alpha_2).\end{aligned} \tag{7}$$

Let us consider an implication of eq. (7). The polarization of the atom A depends on the direction of vibration of the light, i.e. the polarization is anisotropic and $n_{\parallel}$ is greater than $n_{\perp}$. If we consider the atom B of Fig. 8, we see that the polarization of the atom is anisotropic but in this case $n_{\parallel}$ is smaller than $n_{\perp}$. Considering this way, we see that the edge is not isotropic, and the distributions of refractive indices are like shown in Fig. 9(a). Thus, the distribution of the coefficient of birefringence becomes as shown in Fig. 9(b) and this coincides with the observed signs.

Here, we refer to the range where the edge is anisotropic. The theory which is stated in §4 is based on the fact that the range of interactions is less than the wavelength of light. Therefore, we may say that the edge birefringence stems from the region which has much less width than the wavelength of light.

§6. Numerical Example

We take up sodium chloride as a numerical example. We suppose the ideal crystal structure and neglect the influence of the outside material.

Data which we use are as follows:[5]

molecular mass	58.46
density	2.16 g/cm^3
lattice constant	2.81 Å
electronic polarizabilities	$Na^+ = 0.179$ Å^3
	$Cl^- = 3.66$ Å^3.

Now, as the polarizability of Na^+ ion is much less than that of Cl^- ion, we may neglect the interactions among the Na^+ ions in calculating $\delta_{\parallel}$ and $\delta_{\perp}$. Using the above data and eq. (2) and taking into account only 8 nearest Cl^- ions, we get

$$J = n_{\parallel} - n_{\perp} = 0.12 \tag{8}$$

as the coefficient of birefringence.

§7. Experiment

It is very difficult to measure the order of edge birefringence of sodium chloride, because if we want to make an ideal crystal, it becomes a small one. But as the edge birefringence does not appear when the refractive indices of the two specimens are the same, we may consider that the retardation depends on the difference of refractive indices of the two specimens finally, though we need to know the crystal structures

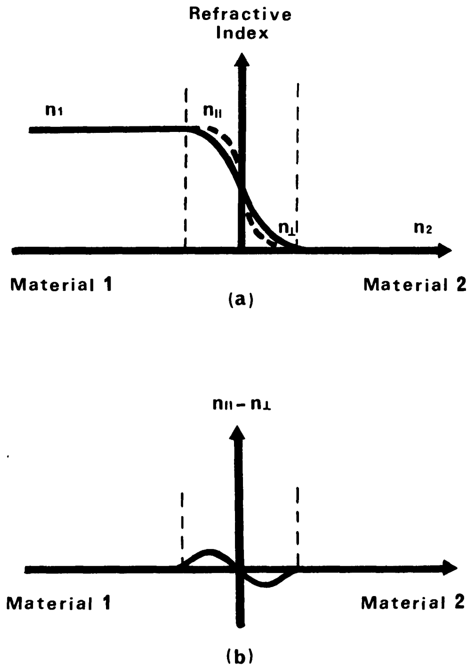

Fig. 9. (a) Distributions of the refractive indices. (b) Distribution of the coefficient of birefringence.

in order to calculate the retardation. So we choose the edge of epoxy resin as an experimental sample. We do not show the edge birefringence of epoxy resin, because it resembles that of sodium chloride which are shown in Figs. 5–7. Epoxy resin has very clear-cut edge and its refractive index 1.52 is nearly equal to that of sodium chloride (1.50). The measured coefficient of birefringence is given by

$$J = n_{\parallel} - n_{\perp} = 0.009. \tag{9}$$

Now let us consider the range where the edge is anisotropic by using the results given by eqs. (8) and (9) and considering that the edge birefringence looks like a diffraction phenomenon. If we take 2000 Å as the extention of diffraction, the range t is given by

$$t = \frac{2000 \times 0.009}{0.12} = 150 \text{ (Å)}.$$

§8. Conclusion

At first, the performance of the rectified optics is reported and then using the system, the nature of edge birefringence is examined and a theory to clarify it is proposed. The conclusion is that the origin of edge birefringence may be explained by considering the edge is anisotropic. However, as it is already stated, besides our theory many assumptions are proposed and the phenomenon is not still said to be solved definitely, so it would be necessary to expect further discussions and studies to be done.

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