Probability on Trees and Networks

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p. 22: End the last sentence of Exercise 2.1(c) with "for irreducible chains".

p. 41: Replace "the (almost surely unique) closest path to L in the Hausdorff metric" by "recursively growing a path by successively adding a vertex closest to L". Change "Figure 2.2" to "Figure 2.3".

p. 61: In Exercise 2.35, change "$x, y, z \in V$" to "x, y, z be vertices in a finite network."

pp. 64–65: In each of Exercises 2.62 and 2.63, insert "simple" before "network".

p. 78: On line 12, change $x \neq a$ to $x \neq a, z$. On line 13, change "all vertices x" to "all vertices $x \neq z$".

p. 118: In Corollary 4.9, change the first occurrence of "graph" to "network".

p. 119, line −2: Insert "simple" before "network".

p. 189: Before (6.28), change $n \geq 1$ to $n \geq 0$.

p. 196, line −2: Change "contains" to "intersects" and add "and edges" after "vertices".

p. 201: In (6.38), change $\mathcal{P}_i A$ to $\mathcal{P}_i(A)$.

p. 298, line −17: Change Γ to γ.

p. 350: Change "In general, suppose that G is a simple plane network whose plane dual $G^\dagger$ is locally finite." to "In general, suppose that G is a proper, simple plane network whose plane dual $G^\dagger$ is locally finite."

p. 365: In Theorem 10.36, insert "proper," before "simple".

p. 366: In Proposition 10.37, insert "proper," before "simple".

p. 386: In Exercise 10.33, both subtractions of "2" should be replaced by subtractions of expected degrees in the wired spanning forest.

p. 414, line −3: Change $X\mu$ to $X\nu$.

p. 424, line 3: Change $|Y_n|$ to Y_n^2.

p. 430: In the conclusion of Lemma 13.5, change $\left|T_k\big(P/\|P\|_\pi\big)\right|_\pi$ to $\left\|T_k\big(P/\|P\|_\pi\big)\right\|_\pi$.

p. 435, line −1: Change "last" to "penultimate".

p. 443: V_i should be assumed finite.

p. 447, line −2: Change λ^{-j} to λ^j.

p. 457: Replace the sentence beginning "An extension to the much harder case" by "An extension to the much harder case of (biased) random walks on directed covers was achieved by Takacs (1997), with a further extension to certain random trees by Takacs (1998)."

p. 460: Just before Exercise 3.15, change "smaller" to "greater".

p. 463: In Exercise 13.23, add the hypothesis that G be simple.

p. 464: In Exercise 13.26, change -2 to -3 and $4/3$ to $(5 + \sqrt{7})/6 = 1.27^+$.

p. 464: In Exercise 13.31, change $\sqrt{d}$ to $\sqrt{d+2}$.

pp. 465–466: In each of Exercises 13.44–13.47, insert "simple" before "network".

p. 470, line -3: Change x to o.

p. 472: Remove the period in (14.3).

p. 476: Move the sentence preceding Example 14.14 to between that example and the next one. Change "Theorem 14.16" to "Lemma 14.16".

p. 479: In the first display, change $\mathbf{1}_{\{o\}}$ to $\mathbf{0}$.

p. 482: In Definition 14.26, precede (i) by "there is a metric on $\mathsf{V} \cup \Theta$ such that".

p. 484: In the proof that *(iii)* $\Rightarrow$ *(iv)*, add after $\mathbf{b}(\Omega_1)$, ": Here, we extend ν_o to the σ-field generated by $\mathscr{F}_\Theta$ and those $E \subset \Theta$ for which $\mathbf{b}^{-1}(E)$ lies in a subset of $\mathscr{B}_{\mathsf{V}^{\mathbb{N}}}$ that has $\mathbf{P}_o$-measure 0. With this extension, $\nu_o\big(\mathbf{b}(\Omega_1)\big) = 1$".

p. 485: In Exercise 14.10, change $\mathsf{V}^{\mathbb{N}}$ to Ω. Change $\mathbf{b} \circ \pi_n$ to $\pi_n \circ \mathbf{b}$.

p. 492: In the proof of Theorem 14.39, replace $\widehat{X}_n$ by (Ψ_n, X_n).

p. 494: Add to Exercise 14.14, "Show also that if $f \ge f \circ T$ a.s., then $f = f \circ T$ a.s."

p. 498: Change $\beta = \beta \circ T$ to $\beta \ge \beta \circ T$.

p. 499: Change "Exercise 13.2(iii)" to "Exercise 13.2(c)".

p. 500: Change "Exercise 13.2(ii)" to "Exercise 13.2(b)".

p. 505: In Exercise 14.20, change "constant $c > 0$" to "constants $c, d > 0$".

p. 510: In Exercise 14.24, change γX_j to $X_j \gamma$ and change $C(K) := \sum_{\gamma \in K} \mathbf{P}_\gamma[\forall n \ge 1 \ X_n \notin K]$ to $C(K) := \sum_{\gamma \in K} \mathbf{P}_{\gamma^{-1}}[\forall n \ge 1 \ X_n \notin K^{-1}]$. In Exercise 14.32, replace "satisfies Definition 14.26, except that it is not necessarily metrizable" with "allows a Poisson representation in the sense that there is a family $\{\nu_x \,;\ x \in \mathsf{V}\}$ of probability measures on Ξ such that the Gelfand isomorphism $\varphi \colon \mathbf{BH}(\mathsf{V}, P) \to C(\Xi)$ satisfies $u(x) = \int_\Xi \varphi(u)\, d\nu_x$ for all $u \in \mathbf{BH}(\mathsf{V}, P)$ and $x \in \mathsf{V}$".

p. 511: In Exercise 14.36(a), replace the final ν by "ν_0 for some ν_0". In Exercise 14.39, remove the negative sign in the display.

p. 523: In Example 15.14, replace $M^{2\alpha}$ by M^α.

p. 526, line -7: Change $\mathbf{T}$ to T^υ.

p. 538: In line -3, change $z^{\dim E}$ to $z^{-\dim E}$.

p. 540: In the next-to-last line of the proof of Proposition 6.5, add a factor of 3 after the equality sign. In Proposition 6.8, change b-ary to b-adic, 2^{-d} to $2^{-d}b^{-d\ell}$, $3^d b^{d(1+\ell)}$ to $3^d b^d$, and $\ell := (1/2)\log_b d$ to $\ell := \lceil (1/2)\log_b d \rceil$.

p. 550: In the line beginning "Now assume (iii)," change $\xi_k \cap T$ to $\xi_k \cap T^*$. Two lines below that, change $u \not\geq x_k$ to $u \not\leq x_k$.

p. 554: In Exercise 16.9, change the first occurrence of $\mu^i(E)$ to $\mu^i(E)^2$. In Exercise 16.13, A should be assumed closed. In Exercise 16.14, Λ should be assumed compact.

p. 555: In Exercise 16.18, change "supremum" to "suprema". In Exercise 16.19, change $\mathrm{cap}_1(\Lambda)$ to $\mathrm{cap}_2(\Lambda)$. In Exercise 16.20, replace "Remove" by "Can one remove" and end with a question mark.

p. 556: Change "*ergodic theorem* theorem" to "*ergodic theorem*".

p. 557: At the end of Exercise 17.1, clarify that (iii) then means that $(\mathbf{p} \times \mu)\big(f(X_0) \neq f(X_1)\big) = 0$.

p. 562: In the proof of Lemma 17.5, μ should be replaced by $\mathsf{DSRW} \times \mu$ and μ' by $\mathsf{SRW} \times \mu'$.

p. 563: In the proof of Theorem 17.7, change $X_1 B$ to $X_1^{-1}B$, $X_1 A$ to $X_1^{-1}A$, sB to $s^{-1}B$, $s^{-1}A$ to sA, $s^{-1}A$ to sA, and in line -5, the first s^{-1} to s.

p. 575: In Exercise 17.9, change "a UNIF_T-path is $\widehat{\mathbf{GW}}_*$" to "(T, ξ) is $\widehat{\mathbf{GW}}_*$ when $T \sim W \cdot \mathbf{GW}$ and ξ is a UNIF_T-path in T."

p. 577, lines 5–6: Change $\int$ to $\iint$.

p. 578: In line 7, $\mathbf{AGW}_{\mathsf{Exit}}$ should be the pushforward of $(\mathsf{SRW} \times \mathbf{AGW})_{\mathsf{Exit}}$ under the map $(\vec{x}, T) \mapsto T$.

p. 579, line 6: Change $y = 1$ to $|y| = 1$.

p. 581: In the line preceding (17.17), change "limit" to "liminf": this is easy to see and sufficient, although the limit does exist a.s.

p. 590: Add "*In all the exercises about* $\mathbf{GW}$ *or* $\mathbf{AGW}$, *assume that* $p_0 = 0$ *unless stated otherwise*." In the definition of $D_k(\vec{x}, T)$ in Exercise 17.19, change $j \in \mathbb{N}$ to $j \in \mathbb{Z}$.

p. 592: In Exercise 17.29, remove $\displaystyle\sum_{n=0}^{N-1}$ and change n to N. In Exercise 17.30, change $\theta(x)$ to $\theta_0(x)$.

p. 593: In Exercise 17.34, change $m - 1$ to $m^2 - m$ and insert "when $\mathbf{E}[L^2] < \infty$" after "$\mathbf{E}\big[\mathscr{E}_c(\theta)\big] < \infty$". In Exercise 17.36, insert "augmented" before "Galton-Watson" and "a uniformly chosen neighbor of" after "(not loop-erased) simple random walk from".

p. 633: In the solution to Exercise 11.16, it should not be asserted that the bound is at least $(1 - \log 2)(1/\rho - 1)$. Such a bound holds, but requires a modified argument.

p. 636: For Exercise 13.23, if G is not simple, then one can make a similar Markov chain using the directed line graph of G.

p. 637: For Exercise 13.31, change $\sqrt{d + \epsilon}$ to $\sqrt{d + 2 + \epsilon}$.

p. 639: For Exercise 14.6, change $[k, n - 1]$ to $(k, n - 1]$. For Exercise 14.12, change "folows" to "follows".

p. 640: Add a comment on Exercise 14.14 to say, "For the second part, consider $\mathbf{E}[g \circ f - g \circ f \circ T]$ for a suitable g." For Exercise 14.23, change "$H(X_1) = 0$" to "$H(X_1) < \infty$". For Exercise 14.24, change "$\sum_{k=0}^{n-1} \sum_{\gamma \in K} \mathbf{P}_o[\forall j \in [k, n - 1] \ \gamma X_j \notin K X_k]$" to "$\sum_{i=0}^{n-1} \sum_{\gamma \in K} \mathbf{P}_o[\forall j \in (i, n - 1] \ X_i \gamma \notin X_j K]$".

p. 641: Replace the comment for Exercise 14.32 with "As noted by Fursternberg (1971b), the space Ξ can be too large to provide a compactification boundary."

p. 645: For Exercise 17.4, after "3", add "in the reduced tree".

p. 646: For Exercise 17.19(c), change "this is the average number" to "this is a weighted average number"; change Γ_k to $2\Gamma_k - 1$; and change $j \in \mathbb{N}$ to $j \in \mathbb{Z}$ in the definition of $D'_k(\overset{\leftrightarrow}{x}, T)$.

p. 655: The journal for Cayley should be *Quart. J. Pure Appl. Math.*