

CÁLCULO DIFERENCIAL E INTEGRAL

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## Respostas dos exercícios

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## 1 Integral indefinida: parte I

1.

- 1)  $4x + \frac{3x^2}{2} + \frac{2x^3}{3} + \frac{1}{4}x^4 + c$     2)  $\frac{3x^{4/3}}{4} + \frac{2x^{3/2}}{3} + c$     3)  $\sqrt{2}x - ex + \pi x + c$
- 4)  $e^x + \frac{1}{2}(-4 + x)x + c$     5)  $x[-2 + \log(x^2)] + c$     6)  $-\operatorname{sen} x + c$
- 7)  $-\ln[\cos(x)] + c$     8)  $\frac{1}{4}\left(4x + x^2 + \frac{x^3}{3}\right) + c$     9)  $\frac{4 + 12x^2 + 3x^7 + 2x^9}{12x^3} + c$
- 10)  $-\cos(x) + c$     11)  $\frac{1}{3}\log(1 - 3x^2 + x^3) + c$     12)  $\log(2 + x) + c$
- 13)  $dx + \frac{cx^2}{2} + \frac{bx^3}{3} + \frac{ax^4}{4} + C$     14)  $\frac{2}{25}\sqrt{x}(25 + x^2)$     15)  $\frac{2w^{3/2}}{3} + \frac{w^4}{4}$
- 16)  $-8\log(1 - x) + 8\log(1 + x)$     17)  $\frac{25\left[2 + x\log\left(\frac{1-x}{1+x}\right)\right]}{2x}$     18)  $e^{-x} + e^x$

2.  $f(x) = -\frac{1}{x} + 15x - \frac{239}{4}$
3.  $f(x) = e^x + \log(x) + 3 - e$
4. a)  $f(x) = 2(1 + x)b$  b)  $f(x) = \frac{1}{x} + 2xc$  c)  $f(x) = e^x - 2x - \operatorname{sen}(x)$
5.  $F(x) = cx + \frac{ax^3}{3} + \frac{\log(x)}{b}$ .
6. (a)  $\frac{x^4}{4a} + \frac{x^{1+c}}{1+c} + \frac{2}{3}x\sqrt{bx}$ , tal que  $\{x \in \mathbb{R} | a \neq 0, b > 0, c \in \mathbb{N}\}$
- (b)  $\frac{x^4}{4a} + \frac{x^{1+c}}{1+c} + \frac{2}{3}x\sqrt{bx}$ , tal que  $\{x \in \mathbb{R} | a \neq 0, b > 0, c \in \mathbb{Z}_-\}$
- (c)  $\frac{x^4}{4a} + \frac{x^{1+c}}{1+c} + \frac{2}{3}x\sqrt{bx}$ , tal que  $\{x \in \mathbb{R} | a \neq 0, b < 0, c \in \mathbb{N}\}$

7.

$$1) x^2 + \frac{1}{6}(5+x)^6 + c \quad 2) \frac{8}{27}(-4+x^3)^{9/8} + c \quad 3) -\frac{5}{9}(6-3t^2)^{3/2} + c$$

$$4) \frac{(1+3x^2)\sqrt{x^2+3x^4}}{9x} + c \quad 5) -\frac{4+x e^{1/x}}{x} + c \quad 6) \frac{\text{sen}^6(x)}{6} + c$$

$$7) 2\sqrt{x(x+2)} - 2\ln\left|\sqrt{x(x+2)} + x + 1\right| + c$$

$$8) \frac{1}{4} \text{arctg} \frac{x}{4} + c \quad 9) \frac{|1+x|^3}{3} + c$$

$$10) \frac{ae^{x^2}}{2} + bx + c \quad 11) \frac{1}{4}(-\log(\cos(2e^x)) + \log(\text{sen}(2e^x)))$$

$$12) \frac{x^4}{4}$$

8.

1)  $2 \operatorname{sen}(\sqrt{x}) + c$

2)  $\ln(\sec(x+3) + \operatorname{tg}(x+3)) + c$

3)  $-\frac{1}{2} \cos(x^2 + 2x) + c$

4)  $\frac{1}{12} [\cos(9-3x) - 9 \cos(3-x)] + c$

5)  $\frac{1}{2} \left[ \ln \left( \operatorname{sen} \left( \frac{x^2}{2} \right) \right) - \ln \left( \cos \left( \frac{x^2}{2} \right) \right) \right] + c$

6)  $\frac{e^{x^3}}{3} + c$

7)  $-\frac{\ln(x^2) + 2}{x \ln 10} + c$

8)  $-\ln \left[ \operatorname{sen} \left( \frac{1}{x} \right) \right] + c$

9)  $\frac{1}{2} \sec^2(\ln(x)) + \ln(\cos(\ln(x))) + c$

10)  $\frac{\operatorname{sen}^3 x}{3} + c$

11)  $\frac{e^{x^2}}{2} + c$

12)  $\frac{\operatorname{tg}^3 x}{3} + c$

13)  $-\operatorname{sen} \left( \frac{1}{x} \right) + c$

14)  $-e^{\frac{1}{x}} + c$

15)  $\frac{2}{3} [\ln(x) + 1]^{\frac{3}{2}} + c$

16)  $-\frac{1}{2} e^{6-2x^5} + c$

17)  $-\frac{1}{8} \cos(8x) + c$

18)  $-2 \ln [\cos(\sqrt{x})] + c$

19)  $-\frac{5}{4} \cos \left( \frac{4x-8}{5} \right) + c$

20)  $-\frac{8}{3} \sqrt{4-3x} + c$

21)  $\frac{5}{32(5-4x^2)^4} + c$

22)  $\frac{1}{2} \cos \left( \frac{1}{x^2} \right) + c$

23)  $\operatorname{arctg} \left( \frac{x}{3} \right) + c$

24)  $\frac{2}{9} (x(x^2+3))^{\frac{3}{2}} + c$

9.  $\frac{(ax+b)^{c+1}}{ac+a} + C$

10.  $\left( \frac{b}{a} + x \right) \ln(ax+b) + \frac{e^{cx}}{c} - x + C$

11.  $\ln(1+4x+x^2) - \ln(6)$

12.  $-\frac{x^2}{2} + \frac{1}{2} e^{x^2-2x+4} + x + \frac{1}{2} (35 - e^{19})$

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13.

- 1)  $\frac{1}{2} \operatorname{arctg} \left( \frac{x-2}{2} \right) + c$
- 2)  $\frac{1}{4} \ln ((1-x) - \ln x) + c$
- 3)  $\ln (x^2 - 2x + 2) - 4 \arctan (1-x) + c$
- 4)  $\sqrt{x^2 - 4x + 2} + 4 \ln \left( -\sqrt{x^2 - 4x + 2} - x + 2 \right) + c$
- 5)  $5\sqrt{x^2 - 2x + 2} + c$
- 6)  $5 \frac{\ln (-\sqrt{x^2 - 4x + 1} - x + 2)}{\sqrt{2}} + c$
- 7)  $-\operatorname{arctg} (3-x) + c$
- 8)  $\frac{(x-1)[x+2\ln(x-1)-1]}{\sqrt{(x-1)^2}} + c$
- 9)  $\frac{1}{2} \ln (x^2 + 12x - 10) + c$

14.

- 1)  $e^x (x-1) + c$
- 2)  $\sqrt{1-x^2} + x \operatorname{arcsen} (x) + c$
- 3)  $\frac{1}{2} [x + \operatorname{sen} (x) \cos (x)] + c$
- 4)  $x \operatorname{sen} (x) + \cos (x) + c$
- 5)  $x \operatorname{arccos} (x) - \sqrt{1-x^2} + c$
- 6)  $-\frac{1}{27} e^{-3x} (9x^2 + 6x + 2) + c$
- 7)  $2\sqrt{x} (\ln x - 2) + c$
- 8)  $\frac{5 \ln x + 1}{25x^5} + c$
- 9)  $\frac{2^x (x \ln 2 - 1)}{\ln^2 2} + c$
- 10)  $\frac{1}{x^2 + 2} + \frac{1}{2} \ln (x^2 + 2) + c$
- 11)  $\frac{1}{27} x^3 (9 \ln^2 (x) - 6 \ln (x) + 2) + c$
- 12)  $-\frac{1}{15} (2-x^2)^{\frac{3}{2}} (3x^2 + 4) + c$
- 13)  $\frac{1}{4} [\cos (1-4x) - 4x \operatorname{sen} (1-4x)] + c$
- 14)  $10e^{\frac{x}{5}} (3x - 14) + c$
- 15)  $(w+2) \operatorname{sen}(w) \cos(w) - \frac{1}{4} (2w^2 + 8w - 1) \cos(2w) + c$
- 16)  $\frac{1}{81} (6x^3 \operatorname{sen} (3x^3) + (2-9x^6) \cos (3x^3)) + c$
- 17)  $-e^{-x} (x^4 - 4x^3 + 4x^2 + 8x + 8) + c$
- 18)  $\frac{1}{6} \ln^2 (x^3)$

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15.

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|---|--|
| 1) $\sin(x) - x \cos(x) + c$  | 2) $-\frac{\ln x + 1}{x} + c$  |
| 3) $\frac{5}{4} [3(x^4 - 1) \operatorname{arctg}(x) - x(x^2 - 3)] + c$  | 4) $\frac{1}{9} x^3 (3 \ln x - 1) + c$   |
| 5) $\frac{5}{4} e^{2x} (2x - 1) + c$  | 6) $2x (\ln^2 x - 2 \ln x + 2) + c$  |
| 7) $e^x (x^2 - 2x + 2) + c$   | 8) $-\frac{3^x [x \ln 3 (x \ln 9 - 4 - 3 \ln 3 + 4 + \ln 27)]}{\ln^3 3} + c$       |
| 9) $e^x (5x - 2) + c$   | 10) $\frac{1}{16} x^4 (4 \ln x - 1) + c$   |
| 11) $(x^2 - 2) \sin x + 2x \cos x + c$  | 12) $\frac{1}{2} e^x (\sin x - \cos x) + c$  |
| 13) $x^2 \left( \frac{1}{2} \ln(x - 4) - \frac{1}{4} \right) - 2x - 8 \ln(4 - x) + c$   |  |
| 14) $\frac{x^6}{6} + c$   | 15) $\frac{5}{16} \sin(4x) + \cos\left(1 - \frac{5x}{4}\right) + c$                |
| 16) $\frac{1}{16} x^2 [-x^2 + 4(x^2 + 4) \ln x - 8] + c$  | 17) $4(3t + 5) \sin\left(\frac{t}{4}\right) + 48 \cos\left(\frac{t}{4}\right) + c$ |
| 18) $\frac{2}{15} (x + 1)^{\frac{3}{2}} (3x - 2) + c$   | 19) $\frac{2}{45} (x^3 + 1)^{\frac{3}{2}} (x^3 - 2) + c$                           |
| 20) $2e^{\frac{x}{2}} (x^4 - 8x^3 + 48x^2 - 192x + 384) + c$  | 21) $-\frac{2x^2 + 1}{4(x^2 + 1)^2} + c$   |
| 22) $x + \ln(1 - x) - \ln(x + 1) + c$   | 23) $\frac{\ln x}{2} - \frac{1}{4} [\ln(x^2) - 2 \ln x] \ln(\ln(x^2)) + c$         |
| 24) $\frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} - x + c$   | 25) $\sqrt{16 - x^2} + x \arcsin\left(\frac{x}{4}\right) + c$                      |
| 26) $\frac{x^{a+1} [(a+1) \ln x - 1]}{(a+1)^2} + c$   | 27) $(x - 2) \operatorname{tg} x + \ln(\cos x) + c$                                |
| 28) $\frac{1}{2} x [\sin(\ln x) + \cos(\ln x)] + c$   | 29) $\frac{2e^{1/x} (x - 1)}{x} + c$   |
| 30) $\frac{1}{2} \left[ -\ln\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right) + \ln\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) + \frac{\sin x}{\cos^2 x} \right] + c$ |  |

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$$\begin{aligned}
 &31) \ x (\ln (x^2 + 1) - 2) + 2 \operatorname{arctg} x + c \quad 32) \ \frac{1}{8} (2x (x + \operatorname{sen} (2x)) + \cos (2x)) + c \\
 &33) \ x (\ln (4x) (\ln (4x) (\ln (4x) - 3) + 6) - 6) + c
 \end{aligned}$$

## 2 Integral indefinida: Frações parciais

1.

$$\begin{aligned}
 a) \ f(x) &= \frac{3}{4(x+3)} - \frac{1}{4(3x+1)} & b) \ f(x) &= -\frac{1}{x+1} - \frac{1}{(x+1)^2} + \frac{1}{x} \\
 c) \ f(x) &= \frac{2}{3(x+2)} + \frac{1}{3(x-1)} & d) \ f(x) &= 1 - 2\frac{2x+1}{x^2+x+2} \\
 e) \ f(x) &= x + \frac{2x+1}{x^2+x-2} & f) \ \frac{33}{4(x-6)} - \frac{13}{4(x-2)}
 \end{aligned}$$

2.

$$\begin{aligned}
 a) \ \frac{3}{4} \ln |x+3| - \frac{1}{12} \ln |3x+1| + c & \quad b) \ \frac{1}{x+1} + \ln |x| - \ln |x+1| + c \\
 c) \ \frac{1}{3} [\ln |1-x| + 2 \ln |x+2|] + c & \quad d) \ x - 2 \ln |x^2+x+2| + c \\
 e) \ \frac{x^2}{2} + \ln |x^2+x+2| + c & \quad f) \ f(x) = \frac{33}{4} \ln |6-x| - \frac{13}{4} \ln |2-x| + c
 \end{aligned}$$