Cálculo diferencial e integral

Integral dupla

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1. (Larson & Edwards, 2014) Calcule as seguintes integrais:

$$(a)\ \int_0^x (x+2y)\ dy \qquad (b)\ \int_x^{x^2} \frac{y}{x}\ dy \qquad (c)\ \int_1^{2y} \frac{y}{x}\ dx,\ y>0$$

$$(d)\ \int_0^{\cos y} y\ dx \qquad (e)\ \int_0^{\sqrt{4-x^2}} x^2y\ dy \qquad (f)\ \int_{x^3}^{\sqrt{x}} \left(x^2+3y^2\right)\ dy$$

$$(g)\ \int_{e^y}^{y} \frac{y\ln x}{x}\ dx,\ y>0 \qquad (h)\ \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \left(x^2+y^2\right)\ dx \qquad (i)\ \int_0^{x^3} ye^{-\frac{y}{x}}\ dy$$

2. (Stwart, 2010) Calcule a integral dupla, identificando-a antes com o volume de um sólido.

$$(a)\ \int\int_R 3\ dx\ dy, \quad R=\{(x,y)|-2\le x\le 2, 1\le y\le 6\}$$

$$(b)\ \int\int_R (5-x)\, dx\ dy, \quad R=\{(x,y)|0\le x\le 5, 0\le y\le 3\}$$

$$(c)\ \int\int_R (4-2y)\, dx\ dy, \quad R=\{(x,y)|0\le x\le 1, 0\le y\le 1\}$$

$$(d)\ \int\int_R \left(6x^2y^3-5y^4\right)\ dx\ dy \quad R=\{(x,y)|0\le x\le 3, 0\le y\le 1\}$$

$$(e)\ \int\int_R \cos(x+2y)\ dx\ dy \quad R=\left\{(x,y)|0\le x\le \pi, 0\le y\le \frac{\pi}{2}\right\}$$

$$(f)\ \int\int_R \frac{xy^2}{x^2+1}\ dx\ dy \quad R=\{(x,y)|0\le x\le 1, -3\le y\le 3\}$$

$$(g)\ \int\int_R \frac{1+x^2}{1+y^2}\ dx\ dy \quad R=\{(x,y)|0\le x\le 1, 0\le y\le 1\}$$

$$(h)\ \int\int_R \frac{x}{1+xy}\ dx\ dy \quad R=\{(x,y)|0\le x\le 1, 0\le y\le 1\}$$

3. (Stwart, 2010)

   (a) Estime o volume do sólido que está abaixo da superfície $z=xy$ e acima do retângulo

   $$R=\{(x,y)|0\le x\le 6, 0\le y\le 4\}.$$

   Utilize a soma de Riemann com com $m=3$, $n=2$ e tome como ponto amostral o canto superior direito de cada sub-retângulo.

   (b) Use a Regra do Ponto Médio para estimar o volume do sólido da parte (a).

4. (Stwart, 2010) Calcule as seguintes integrais:

$(a)\ \displaystyle\int_1^3\int_0^1 (1+4xy)\ dx\ dy$ $(b)\ \displaystyle\int_2^4\int_{-1}^1 \left(x^2+y^2\right)\ dy\ dx$ $(c)\ \displaystyle\int_1^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \operatorname{sen} x\ \cos y\ dy\ dx$

$(d)\ \displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_{-1}^5 \cos y\ dx\ dy$ $(e)\ \displaystyle\int_0^2\int_0^1 (2x+y)^8\ dx\ dy$ $(f)\ \displaystyle\int_0^1\int_1^2 \frac{xe^x}{y}\ dy\ dx$

$(g)\ \displaystyle\int_1^4\int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right)\ dy\ dx$ $(h)\ \displaystyle\int_0^1\int_0^3 e^{x+3y}\ dx\ dy$ $(i)\ \displaystyle\int_0^1\int_0^1 (u-v)^5\ du\ dv$

$(j)\ \displaystyle\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\ dy\ dx$ $(l)\ \displaystyle\int_0^2\int_0^\pi r\operatorname{sen}^2\theta\ d\theta\ dr$ $(m)\ \displaystyle\int_0^1\int_0^1 \sqrt{s+t}\ ds\ dt$

5. (Stwart, 2010) Determine o volume do sólido que se encontra abaixo do plano $3x+2y+z=12$ e acima do retângulo $R=\{(x,y)|0\leq x\leq 1, -2\leq y\leq 3\}$.

6. (Stwart, 2010) Determine o volume do sólido que se encontra abaixo do paraboloide hiperbólico $z=4+x^2-y^2$ e acima do quadrado $R=\{(x,y)|-1\leq x\leq 1, 0\leq y\leq 2\}$.

7. (Stwart, 2010) Determine o volume do sólido que se encontra abaixo do paraboloide elíptico $z=-\dfrac{x^2}{4}-\dfrac{y^2}{9}+1$ e acima do retângulo $R=\{(x,y)|-1\leq x\leq 1, -2\leq y\leq 2\}$.

8. (Larson & Edwards, 2014) Calcule as seguintes integrais:

$(a)\ \displaystyle\int_{-1}^5\int_0^{3y} \left(3+x^2+\frac{1}{4}y^2\right)\ dx\ dy$ $(b)\ \displaystyle\int_{-4}^4\int_0^{x^2} \sqrt{64-x^3}\ dy\ dx$ $(c)\ \displaystyle\int_1^4\int_1^{\sqrt{x}} 2ye^{-x}\ dy\ dx$

$(d)\ \displaystyle\int_0^2\int_{3y^2-6y}^{2y-y^2} 3y\ dx\ dy$ $(e)\ \displaystyle\int_0^2\int_0^{\sqrt{1-y^2}} (x+y)\ dx\ dy$ $(f)\ \displaystyle\int_0^2\int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y^2}}\ dx\ dy$

$(g)\ \displaystyle\int_1^3\int_0^y \frac{4}{x^2+y^2}\ dx\ dy$ $(h)\ \displaystyle\int_0^{\frac{\pi}{2}}\int_0^{2\cos\theta} r\ dr\ d\theta$ $(i)\ \displaystyle\int_0^{\frac{\pi}{4}}\int_{\sqrt{3}}^{\sqrt{3}\cos\theta} r\ dr\ d\theta$

9. (Larson & Edwards, 2014) Avalie as seguintes integrais impróprias:

$(a)\ \displaystyle\int_1^\infty\int_0^{\frac{1}{x}} y\ dy\ dx$ $(b)\ \displaystyle\int_0^3\int_0^\infty \frac{x^2}{1+y^2}\ dy\ dx$

$(c)\ \displaystyle\int_1^\infty\int_1^\infty \frac{1}{xy}\ dx\ dy$ $(d)\ \displaystyle\int_0^\infty\int_0^\infty xye^{-\left(x^2+y^2\right)}\ dx\ dy$

Referências

Larson, R.; Edwards, B. **Calculus**. Cengage Learning: Boston, Ed. 10, 2014.

Stwart, J. **Cálculo: volume 2**. Cengage Learning: São Paulo, Ed. 6, 2010.

Respostas de alguns exercícios

1. (a) $2x^2$; (c) $y\ln(2y)$; (e) $\frac{4x^2-x^4}{2}$; (g) $\frac{y}{2}\left[(\ln y)^2 - y^2\right]$; (i) $x^2\left(1-e^{-x^2}-x^2e^{-x^2}\right)$

2. (a) 60; (c) 3; (d) $\frac{21}{2}$; (f) $9\ln 2$

3. (a) 288; (b) 144

4. (a) 10; (c) 1; (e) $\frac{261.632}{45}$; (g) $\frac{21}{2}\ln 2$; (i) 0; (l) π

5. $47,5$

7. $\frac{166}{27}$

8. (a) 1629; (b) $\frac{2048\sqrt{2}}{9}$; (c) $\frac{e^3-4}{e^4}$; (d) 16 ; (e) 213; (f) 4; (g) $\pi\ln 3$; (h) $\pi/2$;
   (i) $-\frac{3}{16}(\pi-2)$

9. (a) $1/2$; (c) Divergente.