Cálculo diferencial e integral

Respostas dos exercícios

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1 Integral indefinida: parte I

1.

1) $4x + \frac{3x^2}{2} + \frac{2x^3}{3} + \frac{1}{4}x^4 + c$

2) $\frac{3x^{4/3}}{4} + \frac{2x^{3/2}}{3} + c$

3) $\sqrt{2}x - ex + \pi x + c$

4) $e^x + \frac{1}{2}(-4 + x)x + c$

5) $x\left[-2 + \log\left(x^2\right)\right] + c$

6) $-\operatorname{sen} x + c$

7) $-\ln\left[\cos(x)\right] + c$

8) $\frac{1}{4}\left(4x + x^2 + \frac{x^3}{3}\right) + c$

9) $\frac{4 + 12x^2 + 3x^7 + 2x^9}{12x^3} + c$

10) $-cos(x) + c$

11) $\frac{1}{3}\log(1 - 3x^2 + x^3) + c$

12) $\log(2 + x) + c$

13) $dx + \frac{cx^2}{2} + \frac{bx^3}{3} + \frac{ax^4}{4} + C$

14) $\frac{2}{25}\sqrt{x}\left(25 + x^2\right)$

15) $\frac{2w^{3/2}}{3} + \frac{w^4}{4}$

16) $-8\log(1 - x) + 8\log(1 + x)$

17) $\frac{25\left[2 + x\log\left(\frac{1-x}{1+x}\right)\right]}{2x}$

18) $e^{-x} + e^x$

2. $f(x) = -\frac{1}{x} + 15x - \frac{239}{4}$

3. $f(x) = e^x + \log(x) + 3 - e$

4. *a)* $f(x) = 2(1 + x)$*b)* $f(x) = \frac{1}{x} + 2x$*c)* $f(x) = e^x - 2x - \operatorname{sen}(x)$

5. $F(x) = cx + \frac{ax^3}{3} + \frac{\log(x)}{b}$.

6. (a) $\frac{x^4}{4a} + \frac{x^{1+c}}{1+c} + \frac{2}{3}x\sqrt{bx}$, tal que $\{x \in \mathbb{R} | a \neq 0, b > 0, c \in \mathbb{N}\}$

   (b) $\frac{x^4}{4a} + \frac{x^{1+c}}{1+c} + \frac{2}{3}x\sqrt{bx}$, tal que $\{x \in \mathbb{R} | a \neq 0, b > 0, c \in \mathbb{Z}_-\}$

   (c) $\frac{x^4}{4a} + \frac{x^{1+c}}{1+c} + \frac{2}{3}x\sqrt{bx}$, tal que $\{x \in \mathbb{R} | a \neq 0, b < 0, c \in \mathbb{N}\}$

7.

1) $x^2 + \frac{1}{6}(5+x)^6 + c$

2) $\frac{8}{27}\left(-4+x^3\right)^{9/8} + c$

3) $-\frac{5}{9}(6-3t^2)^{3/2} + c$

4) $\frac{(1+3x^2)\sqrt{x^2+3x^4}}{9x} + c$

5) $-\frac{4+x\ e^{1/x}}{x} + c$

6) $\frac{\operatorname{sen}^6(x)}{6} + c$

7) $\frac{2x^2}{\sqrt{x\,(x+2)}} + \frac{4x}{\sqrt{x\,(x+2)}} - \frac{4\sqrt{x\,(x+2)}\log\left[\sqrt{\frac{x}{2}+1}+\sqrt{\frac{x}{2}}\right]}{\sqrt{x\,(x+2)}} + c$

8) $\frac{1}{4}\operatorname{arctg}\frac{x}{4} + c$

9) $\frac{|1+x|^3}{3} + c$

10) $\frac{ae^{x^2}}{2} + bx + c$

11) $\frac{1}{4}\left(-\log(\cos(2e^x)) + \log(\operatorname{sen}(2e^x))\right)$

12) $\frac{x^4}{4}$

8.

1) $2\,\mathrm{sen}\left(\sqrt{x}\right)+c$

2) $\ln\left(\sec\left(x+3\right)+\mathrm{tg}\left(x+3\right)\right)+c$

3) $-\frac{1}{2}\cos\left(x^2+2x\right)+c$

4) $\frac{1}{12}\left[\cos\left(9-3x\right)-9\cos\left(3-x\right)\right]+c$

5) $\frac{1}{2}\left[\ln\left(\mathrm{sen}\left(\frac{x^2}{2}\right)\right)-\ln\left(\cos\left(\frac{x^2}{2}\right)\right)\right]+c$

6) $\frac{e^{x^3}}{3}+c$

7) $-\frac{\ln\left(x^2\right)+2}{x\ln 10}+c$

8) $-\ln\left[\mathrm{sen}\left(\frac{1}{x}\right)\right]+c$

9) $\frac{1}{2}\sec^2\left(\ln\left(x\right)\right)+\ln\left(\cos\left(\ln\left(x\right)\right)\right)+c$

10) $\frac{\mathrm{sen}^3\,x}{3}+c$

11) $\frac{e^{x^2}}{2}+c$

12) $\frac{\mathrm{tg}^3\,x}{3}+c$

13) $-\mathrm{sen}\left(\frac{1}{x}\right)+c$

14) $-e^{\frac{1}{x}}+c$

15) $\frac{2}{3}\left[\ln\left(x\right)+1\right]^{\frac{3}{2}}+c$

16) $-\frac{1}{2}e^{6-2x^5}+c$

17) $-\frac{1}{8}\cos\left(8x\right)+c$

18) $-2\ln\left[\cos\left(\sqrt{x}\right)\right]+c$

19) $-\frac{5}{4}\cos\left(\frac{4x-8}{5}\right)+c$

20) $-\frac{8}{3}\sqrt{4-3x}+c$

21) $\frac{5}{32\left(5-4x^2\right)^4}+c$

22) $\frac{1}{2}\cos\left(\frac{1}{x^2}\right)+c$

23) $\mathrm{arctg}\left(\frac{x}{3}\right)+c$

24) $\frac{2}{9}\left(x\left(x^2+3\right)\right)^{\frac{3}{2}}+c$

9. $\frac{(ax+b)^{c+1}}{ac+a}+C$

10. $\left(\frac{b}{a}+x\right)\ln\left(ax+b\right)+\frac{e^{cx}}{c}-x+C$

11. $\ln\left(1+4x+x^2\right)-\ln(6)$

12. $-\frac{x^2}{2}+\frac{1}{2}e^{x^2-2x+4}+x+\frac{1}{2}\left(35-e^{19}\right)$

13.

1) $\frac{1}{2}\operatorname{arctg}\left(\frac{x-2}{2}\right)+c$

2) $\frac{1}{4}\ln\left((1-x)-\ln x\right)+c$

3) $\ln\left(x^2-2x+2\right)-4\arctan\left(1-x\right)+c$

4) $\sqrt{x^2-4x+2}+4\ln\left(-\sqrt{x^2-4x+2}-x+2\right)+c$

5) $5\sqrt{x^2-2x+2}+c$

6) $5\frac{\ln\left(-\sqrt{x^2-4x+1}-x+2\right)}{\sqrt{2}}+c$

7) $-\operatorname{arctg}\left(3-x\right)+c$

8) $\frac{(x-1)\left[x+2\ln\left(x-1\right)-1\right]}{\sqrt{(x-1)^2}}+c$

9) $\frac{1}{2}\ln\left(x^2+12x-10\right)+c$

14.

1) $e^x\left(x-1\right)+c$

2) $\sqrt{1-x^2}+x\operatorname{arcsen}\left(x\right)+c$

3) $\frac{1}{2}\left[x+\operatorname{sen}\left(x\right)\cos\left(x\right)\right]+c$

4) $x\operatorname{sen}\left(x\right)+\cos\left(x\right)+c$

5) $x\arccos\left(x\right)-\sqrt{1-x^2}+c$

6) $-\frac{1}{27}e^{-3x}\left(9x^2+6x+2\right)+c$

7) $2\sqrt{x}\left(\ln x-2\right)+c$

8) $\frac{5\ln x+1}{25x^5}+c$

9) $\frac{2^x\left(x\ln 2-1\right)}{\ln^2 2}+c$

10) $\frac{1}{x^2+2}+\frac{1}{2}\ln\left(x^2+2\right)+c$

11) $\frac{1}{27}x^3\left(9\ln^2\left(x\right)-6\ln\left(x\right)+2\right)+c$

12) $-\frac{1}{15}\left(2-x^2\right)^{\frac{3}{2}}\left(3x^2+4\right)+c$

13) $\frac{1}{4}\left[\cos\left(1-4x\right)-4x\operatorname{sen}\left(1-4x\right)\right]+c$

14) $10e^{\frac{x}{5}}\left(3x-14\right)+c$

15) $\left(w+2\right)\operatorname{sen}(w)\cos(w)-\frac{1}{4}\left(2w^2+8w-1\right)\cos\left(2w\right)+c$

16) $\frac{1}{81}\left(6x^3\operatorname{sen}\left(3x^3\right)+\left(2-9x^6\right)\cos\left(3x^3\right)\right)+c$

17) $-e^{-x}\left(x^4-4x^3+4x^2+8x+8\right)+c$

18) $\frac{1}{6}\ln^2\left(x^3\right)$

15.

1) $\operatorname{sen}(x) - x\cos(x) + c$

2) $-\dfrac{\ln x + 1}{x} + c$

3) $\dfrac{5}{4}\left[3\left(x^4 - 1\right)\operatorname{arctg}(x) - x\left(x^2 - 3\right)\right] + c$

4) $\dfrac{1}{9}x^3\left(3\ln x - 1\right) + c$

5) $\dfrac{5}{4}e^{2x}\left(2x - 1\right) + c$

6) $2x\left(\ln^2 x - 2\ln x + 2\right) + c$

7) $e^x\left(x^2 - 2x + 2\right) + c$

8) $-\dfrac{3^x\left[x\ln 3\left(x\ln 9 - 4 - 3\ln 3 + 4 + \ln 27\right)\right]}{\ln^3 3} + c$

9) $e^x\left(5x - 2\right) + c$

10) $\dfrac{1}{16}x^4\left(4\ln x - 1\right) + c$

11) $\left(x^2 - 2\right)\operatorname{sen} x + 2x\cos x + c$

12) $\dfrac{1}{2}e^x\left(\operatorname{sen} x - \cos x\right) + c$

13) $x^2\left(\dfrac{1}{2}\ln\left(x - 4\right) - \dfrac{1}{4}\right) - 2x - 8\ln\left(4 - x\right) + c$

14) $\dfrac{x^6}{6} + c$

15) $\dfrac{5}{16}\operatorname{sen}\left(4x\right) + \cos\left(1 - \dfrac{5x}{4}\right) + c$

16) $\dfrac{1}{16}x^2\left[-x^2 + 4\left(x^2 + 4\right)\ln x - 8\right] + c$

17) $4\left(3t + 5\right)\operatorname{sen}\left(\dfrac{t}{4}\right) + 48\cos\left(\dfrac{t}{4}\right) + c$

18) $\dfrac{2}{15}\left(x + 1\right)^{\frac{3}{2}}\left(3x - 2\right) + c$

19) $\dfrac{2}{45}\left(x^3 + 1\right)^{\frac{3}{2}}\left(x^3 - 2\right) + c$

20) $2e^{\frac{x}{2}}\left(x^4 - 8x^3 + 48x^2 - 192x + 384\right) + c$

21) $-\dfrac{2x^2 + 1}{4\left(x^2 + 1\right)^2} + c$

22) $x + \ln\left(1 - x\right) - \ln\left(x + 1\right) + c$

23) $\dfrac{\ln x}{2} - \dfrac{1}{4}\left[\ln\left(x^2\right) - 2\ln x\right]\ln\left(\ln\left(x^2\right)\right) + c$

24) $\dfrac{x^4}{4} + \dfrac{x^3}{3} - \dfrac{x^2}{2} - x + c$

25) $\sqrt{16 - x^2} + x\operatorname{arcsen}\left(\dfrac{x}{4}\right) + c$

26) $\dfrac{x^{a+1}\left[\left(a + 1\right)\ln x - 1\right]}{\left(a + 1\right)^2} + c$

27) $\left(x - 2\right)\operatorname{tg} x + \ln\left(\cos x\right) + c$

28) $\dfrac{1}{2}x\left[\operatorname{sen}\left(\ln x\right) + \cos\left(\ln x\right)\right] + c$

29) $\dfrac{2e^{1/x}\left(x - 1\right)}{x} + c$

30) $\dfrac{1}{2}\left[-\ln\left(\cos\dfrac{x}{2} - \operatorname{sen}\dfrac{x}{2}\right) + \ln\left(\cos\dfrac{x}{2} + \operatorname{sen}\dfrac{x}{2}\right) + \dfrac{\operatorname{sen} x}{\cos^2 x}\right] + c$

31) $x\left(\ln\left(x^2+1\right)-2\right)+2\operatorname{arctg}x+c$ 32) $\frac{1}{8}\left(2x\left(x+\operatorname{sen}\left(2x\right)\right)+\cos\left(2x\right)\right)+c$

33) $x\left(\ln\left(4x\right)\left(\ln\left(4x\right)\left(\ln\left(4x\right)-3\right)+6\right)-6\right)+c$