Cálculo diferencial e integral

Integral definida: parte I

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1. Calcular a integral definida

1) $\int_{-3}^{5} \left(3x^3 + 2x^2 + 3x + 8\right)\, dx$ 2) $\int_{0}^{10} \left(2\sqrt{x} + 3\sqrt[3]{x}\right)\, dx$ 3) $\int_{-1}^{1} \left(3\pi + \sqrt{5}\right)\, dx$

4) $\int_{2}^{5} \left(1 + x + e^x - e^{\log 3}\right)\, dx$ 5) $\int_{1}^{8} \ln(x+1)\, dx$ 6) $\int_{\pi}^{2\pi} \frac{\operatorname{sen}^2 x - 1}{\cos x}\, dx$

7) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4} \operatorname{tg} x\, dx$ 8) $\int_{c}^{d} \frac{x^2 + 2x + 4}{x^3}\, dx$ 9) $\int_{3}^{a} \left(x^6 - x^{-4} + x^3\right)\, dx$

10) $\int_{0}^{\frac{\pi}{4}} \frac{1}{2} \operatorname{tg} x \cos x\, dx$ 11) $\int_{k}^{3} \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dx$ 12) $\int_{2k+\pi}^{r} \frac{x-2}{x^2 - 4}\, dx$

13) $\int_{0}^{1} \frac{ax^3 + bx^2 + cx + d}{k}\, dx$ 14) $\int_{0}^{1} \left(\frac{1}{\sqrt{x}} + \frac{2x\sqrt{x}}{10}\right)\, dx$ 15) $\int_{-3}^{2} 4x^3 + \sqrt{x+1}\, dx$

16) $\int_{0}^{0.9} \frac{16}{1 - x^2}\, dx$ 17) $\int_{-1}^{3} \frac{5}{2x^4 - x^2}\, dx$ 18) $\int_{-5}^{10} \left(2e^x - \frac{1}{2}e^{-x}\right)\, dx$

2. Avalie a integral e a interprete em termos de cálculo de área

a) $\int_{0}^{3} \left(\frac{1}{2}x - 1\right)\, dx$ *b*) $\int_{2}^{-2} \sqrt{4 - x^2}\, dx$ *c*) $\int_{-3}^{0} \left(1 + \sqrt{9 - x^2}\right)\, dx$

d) $\int_{-1}^{2} |x|\, dx$ *e*) $\int_{-1}^{3} (3 - 2x)\, dx$ *f*) $\int_{0}^{10} |x - 5|\, dx$

3. Avalie a integral $\int_{\frac{\pi}{2}}^{\pi} \operatorname{sen}^2 x \cos^4 x\, dx$

4. Dada que a integral $\int_{0}^{1} 3x\sqrt{x^2 + 4}\, dx = 5\sqrt{5} - 8$, o que é a integral $\int_{1}^{0} 3u\sqrt{u^2 + 4}\, dx$? Justifique.

5. Escreva a integral a seguir na forma de uma integral simples do tipo $\int_{a}^{b} f(x)\, dx$:

$$\int_{-5}^{10} f(x)\, dx + \int_{10}^{12} f(x)\, dx - \int_{-7}^{-5} f(x)\, dx$$

6. Se $\int_1^5 f(x)\ dx = 12$ e $\int_4^5 f(x)\ dx = 3,6$ determine $\int_1^4 f(x)\ dx$.

7. Determine a integral $\int_0^5 f(x)\ dx$ tal que

$$f(x) = \begin{cases} 3, & \text{para} \quad x < 3 \\ x, & \text{para} \quad x \geq 3 \end{cases}$$

8. Avalie as integrais

1) $\int_{-2}^{3} \left(x^2 - 3\right)\ dx$ 2) $\int_1^2 x^{-2}\ dx$ 3) $\int_0^2 \left(x^4 - \frac{3}{4}x^2 + \frac{2}{3}x - 1\right)\ dx$

4) $\int_0^1 \left(1 + \frac{1}{2}u^4 - \frac{2}{5}u^9\right)\ du$ 5) $\int_0^1 x^{\frac{4}{5}}\ dx$ 6) $\int_1^8 \sqrt[3]{x}\ dx$

7) $\int_{-1}^{0} \left(2x - e^x\right)\ dx$ 8) $\int_0^1 x\left(\sqrt[3]{x} + \sqrt[4]{x}\right)\ dx$ 9) $\int_0^2 (y-1)(2y+1)\ dy$

10) $\int_{-1}^{1} t\,(1-t)^2\ dt$ 11) $\int_0^{\frac{\pi}{4}} \sec\theta \operatorname{tg}\theta\ d\theta$ 12) $\int_0^{\frac{\pi}{4}} \sec^2 x\ dx$

13) $\int_1^9 \frac{1}{2x}\ dx$ 14) $\int_1^2 \frac{x^3 + 3x^6}{x^4}\ dx$ 15) $\int_0^{\frac{\pi}{3}} \frac{\operatorname{sen} x + \operatorname{sen} x \operatorname{tg}^2 x}{\sec^2 x}\ dx$

16) $\int_0^{\frac{\pi}{4}} \frac{1 + \cos^2 x}{\cos^2 x}\ dx$ 17) $\int_a^b \frac{4t}{t^2 + 1}\ dx$ 18) $\int_1^2 \frac{(x-1)^3}{x^2}\ dx$

9. Determine a área da região formada pelas curvas

1) $f(x) = x^2 + 4x + 2,\ x = 1,\ y = 0$ 2) $f(x) = \ln(x),\ x = 4,\ y = 0$

3) $f(x) = e^x,\ x = -1,\ x = 0,\ y = 0$ 4) $f(x) = \frac{1}{x},\ x = 5,\ x = 1,\ y = 0$

Alguns exercícios foram retirados do livro *Single variable calculus: concepts & contexts* (Stewart, 2010).

Referências

Stewart, J. *Single variable calculus: concepts and contexts*. Brooks/Cole, 4 ed., 630 p., 2010.