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A practice-ready relocation model for free-floating carsharing systems with electric vehicles – Mesoscopic approach and field trial results

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ABSTRACT

This paper introduces a relocation model for free-floating Carsharing (FFCS) systems with conventional and electric vehicles (EVs). In case of imbalances caused by one-way trips, the approach recommends profit maximizing vehicle relocations. Unlike existing approaches, two types of relocations are distinguished: inter zone relocations moving vehicles between defined macroscopic zones of the operating area and intra zone relocations moving vehicles within such zones. Relocations are combined with the unplugging and recharging of EVs and the refueling of conventional vehicles. In addition, remaining pure service trips are suggested. A historical data analysis and zone categorization module enables the calculation of target vehicle distributions. Unlike existing approaches, macroscopic optimization steps are supplemented by microscopic rule-based steps. This enables relocation recommendations on the individual vehicle level with the exact GPS coordinates of the relocation end positions. The approach is practice-ready with low computational times even for large-scale scenarios.

To assess the impact of relocations on the system's operation, the model is applied to a FFCS system in Munich, Germany within three real world field tests. Test three shows the highest degree of automation and represents the final version of the model. Its evaluation shows very promising results. Most importantly, the profit is increased by 5.8% and the sales per vehicle by up to 10%. The mean idle time per trip end is decreased by 4%.

1. Introduction

Carsharing (CS) systems contribute to solving problems in transportation, land use, environment and society (Shaheen et al., 2012). They offer an advantageous alternative to private vehicle ownership. During the last 30 years, CS systems have become innovative mobility concepts all over the world. CS membership rates are steeply rising. In Germany, more than 750.000 registered users had access to almost 14.000 CS vehicles in 2013 (Bundesverband CarSharing e. V., 2014). Originally, CS systems were station-based. A fleet of vehicles is distributed throughout a network of CS stations with different capacities. The CS operator owns or leases the vehicles. The CS members reserve the automobiles before using them at time-dependent and often distance-dependent fees (Shaheen et al., 2012). At the end of the trip, the user generally has to return the vehicle to its home station. This prevents stations from running out of vehicles or flowing over. However, some operators allow for one-way trips to other stations. Imbalances in vehicle stocks at stations are caused by the gravitational

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effect due to vehicle stock tendencies of stations and the tide phenomena due to demand oscillations (Waserhole and Jost, 2013). For instance, Hondas Diracc system in Singapore already had to stop operation due to those phenomena (The Straits Times, 2008).

During the last decade, the station-based systems were supplemented by the new so-called "free-floating" or "flexible" CS systems (FFCS). Those systems allow trips to almost any parking spot within a defined operating area. The customer does not have to reserve in advance and is not bound to stations. The fee is time-dependent. Like for station-based one-way systems, one-way trips are likely to cause imbalances between vehicle supply and demand. The prediction of the system behavior is more complex as the customers access the vehicles spontaneously without reservation and do not specify their destinations in advance. Weikl and Bogenberger (2014) have proven that vehicle imbalances occur for FFCS systems. Data of a FFCS system in Munich, Germany was used exemplarily. Vehicles were missing in central zones of the operating area on Monday mornings as they conglomerated in southern zones during the weekends. The reason were different usage patterns on weekends and workdays. Another study showed for the same FFCS system, that the main trip purposes on Sundays are "driving home", "leisure activities" and "picking someone up". In contrast, on workdays shopping trips and trips to work dominate (Lenz and Bogenberger, 2014).

Firnkorn and Mueller (2011) highlighted that the goal of global megacities that citizens predominantly abstain from private vehicles requires a solution to the challenging logistic problems of FFCS systems. If serious vehicle imbalances cannot be avoided, they have to be eliminated by dynamically relocating vehicles from oversupplied to undersupplied regions.

1.1. Vehicle imbalance problem – state of the art and research gaps

1.1.1. Station-based vehicle sharing systems

In the past, the focus was on vehicle imbalances of station-based one-way vehicle sharing (VS) systems. Table 1 summarizes the most important studies. For a more comprehensive overview of existing literature on CS systems with the focus on demand modeling and the vehicle imbalance problem of one-way CS systems see Jorge and Correia (2013). Morency et al. (2011) stated that the degree, to which relocations are necessary in Bikesharing (BS) systems, is related to the stations' locations. The same assumptions apply for CS systems. Consequently, CS vehicle stock imbalances can be partly avoided or reduced by deploying the optimal fleet size, the optimal number of stations and their locations as well as the optimal station capacities. Some studies treating this topic are listed in the first part of Table 1. The focus is on how to avoid deviations between supply and demand by optimal system design rather than on how to eliminate existing imbalances during operation.

Other studies concentrated more intensely on relocation strategies for station-based one-way CS systems. Two different approaches were distinguished: operator-based relocation strategies with CS vehicle movements between stations conducted by the operator and user-based relocation strategies shifting this task to the CS users. The second part of Table 1 lists relevant papers. Operator-based approaches recommend vehicle relocations between stations to the operator. User-based strategies offer incentives to the customers to change their travel behaviors. For operator-based strategies, one question remains: how are the vehicle movements most efficiently conducted by a specific number of relocation workers? This is a complex many to many pickup and delivery problem. In the past, this aspect of the relocation problem of one-way CS systems was highlighted by some authors (see third part of Table 1).

The existing relocation algorithms for station-based systems are complex with high computational times. Mostly, aspects of the overall problem are treated without combining demand prediction, relocation algorithm and staff operation planning. Target vehicle distributions are mostly taken as given. Relocations are mostly conducted periodically at the end of the day. Some models assume a priori information on the users' destinations and reservations well in advance, which is not always given in reality.

1.1.2. Free-floating carsharing systems

For solving the relocation problem, FFCS systems could be transformed to station-based systems by defining artificial CS stations, e.g. by theoretically dividing the operating area into station-like zones. Transferring the existing relocation models for station-based systems to FFCS systems is however restricted, as the new systems have other dynamics resulting from spontaneous usage without reservation, without stations and without a priori information on the users' destinations.

First, the existing models for station-based systems mostly consider relocations at the end of the day. However, trips in FFCS systems are usually shorter and the number of trips per vehicle is higher. Imbalances might occur more often and relocations might have to be conducted dynamically throughout the whole day. This implies that future demand has to be known for smaller time steps than 24 h.

Second, the algorithms for the efficient execution of the proposed relocation movements are only partly transferable to FFCS systems. They act on a station basis. Mostly, a specific number of relocation workers get a list of stations to visit successively as well as the number of vehicles to move between stations. As the number of CS stations is limited, the problem size is manageable. In FFCS systems, each vehicle that has to be relocated possibly has a different origin and destination position. A single team of relocation workers thus has to visit much more different positions increasing the problem size. Moreover, the order execution algorithms for station-based BS systems are not suitable for FFCS systems as several bicycles can be moved altogether on a bicycle carrier. In FFCS systems, each vehicle has to be brought to its destination separately unless vehicles are connected with a tow-bar or build platoons.

Table 1
Previous studies on solving the vehicle imbalance problem of station-based vehicle sharing systems.

Study	Focus	Approach	Objective	Relocation	System
Barth and Todd (1999)	Measures of effectiveness; optimal fleet size	Simulation	Minimizing waiting times/ number of relocations	Operator-based	VS
Cepolina and Farina (2012)	Optimal fleet size; optimal vehicle distribution among stations	Optimization	Minimizing vehicle costs and waiting times	User-based	CS
Correia et al. (2014)	Optimal number, locations, size of stations	Optimization; simulation case study	Maximizing operator profit	Operator-based	CS
García-Palomares et al. (2012)	Optimal station locations	Geographic information system	Maximizing coverage	–	BS
Martinez et al. (2012)	Optimal station locations, optimal fleet size	Optimization; simulation case study	Maximizing operator profit	Operator-based	BS
Barth et al. (2004)	Relocation	Simulation; real world university setting	Minimizing the number of relocations	User-based (trip splitting/joining)	CS
Clemente et al. (2013)	Relocation	Unified modeling language; timed petri net; simulation case study	Best balancing vehicle supply and demand	User-based	CS with EVs
Di Febbraro et al. (2012)	Relocation	Discrete event systems; optimization; simulation case study	Maximizing operator profit (by minimal number of staff and vehicles)	User-based (fare discount)	CS
Fan and Xu (2013)	Relocation	Optimization	Maximizing operator profit	Operator-based	CS
Jorge et al. (2013)	Relocation, optimal station locations	Comparison of optimization with simulation	Maximizing operator profit	Operator-based	CS
Miller-Hooks and Nair (2010)	Relocation	Optimization; simulation case study	Minimizing relocation costs	Operator-based	VS
Nourinejad and Roorda (2014)	Relocation, optimal fleet size	Optimization; discrete event simulation; simulation case study	Maximizing operator profit	Operator-based	CS
Sayarshad et al. (2012)	Minimal fleet size, relocation	Optimization	Minimizing unmet demand, number of bicycles, number of relocations	Operator-based	BS
Bruglieri et al. (2014)	Staff operation planning: routes and schedules	Optimization; realistic test scenarios	Maximizing the number of served requests	Operator-based	CS with EVs
Chemla et al. (2013)	Staff operation planning: routes	Optimization	Minimizing relocation costs	Operator-based	BS
Kek et al. (2009)	Staff operation planning: allocation of staff resource and activities	Optimization; heuristics; simulation case study	Minimizing relocation costs	Operator-based	CS

Finally, the relocation start and end positions in FFCS systems are not restricted to CS stations. Thus, relocation recommendations should be defined in more detail on an individual vehicle level including the street name and GPS coordinates of the desired relocation end positions.

Until now, relocation models for the new FFCS systems have been covered by few studies. Barrios and Doig (2014) examined the dependence of the fleet size of FFCS systems on the size of the operating area relative to the desired user walking distance, the expected number of simultaneously used vehicles, the desired level of service and most importantly, the share of relocations. First, they found an analytical fleet size solution for periodic relocations, e.g. at the end of the day. Second, an agent-based simulation provided the minimum fleet sizes for different combinations of city sizes and demand densities and for different relocation scenarios: zero, periodic and continuous relocation. The authors evaluated the trade-offs between the fleet size and different levels of demand or different relocation strategies. For finding a target vehicle distribution, the city was divided into blocks. Each block should have at least one car. This vehicle number per block was not based on a FFCS demand model or historical FFCS booking data. The authors suggested moving vehicles from full blocks with at least p vehicles to empty blocks in order to reach the initial/target vehicle distribution. The study did not integrate an algorithm for conducting the relocations in a most cost efficient way. One important question remained: Which specific vehicles should be moved to which exact destinations in order to minimize relocation travel times etc.? The authors mostly focused on the impact of relocations on the fleet size. The details and the execution of the relocations were not part of the study.

Goeppel and Blumenstock (2012) built a planning and scheduling tool for the maintenance tasks of the FFCS system car2go in Ulm, Germany. Different tasks were considered: exchanging or charging the battery, removing vehicles from restricted areas as well as cleaning, towing and repairing of vehicles. The authors also included relocations into the model but based on a rule that is not preventative but retrospective: vehicles with idle times greater than three days are removed from low demand areas. However, optimal vehicle distributions are not known as the approach does not consider a demand model. Also, the unplugging of completely recharged EVs to unblock charging stations is not integrated into the model. The optimization problem is a mixture of the traveling salesman problem and the vehicle routing problem. It is solved by a sufficiently fast genetic algorithm generating sub-optimal solutions.

Kortum and Machemehl (2012) suggested a method for finding profit maximizing vehicle allocations for a FFCS system given the fleet size. The operating area was divided into zones. The design of the zones was arbitrary and not based on the dynamics of the FFCS system. The study did not focus on how the suggested optimal vehicle allocations are reached by relocations in the most cost efficient way. The operator receives optimal vehicle numbers per zone for the next time period, but does not know which specific vehicles to move. Moreover, the desired destinations of the vehicles are only given on a zone level. The exactness thus strongly depends on the zones' sizes which is bounded below by the computational time. The FFCS demand per zone was not derived based on real FFCS booking data but based on trip-making estimates. When calculating optimal vehicle allocations for the next time period, the authors assume that unmet demand remains for the next time period if it is not met in the current one. However, as moving a vehicle between zones takes some time, potential FFCS customers might have used other means of transport in the meantime. The model suggests all necessary relocations without any prioritization of specific relocations based on their estimated impact on profit.

1.2. An "Ideal" relocation model for FFCS systems

Besides the above mentioned research gaps that were found when analyzing the existing relocation models for FFCS systems, some other additional aspects are important when designing relocation models for FFCS systems.

First, an increasing number of electric vehicles (EVs) will be integrated into free-floating CS fleets. However, CS customers have a certain range anxiety towards EVs, i.e. they do not feel comfortable if their distance is more than 75–80% of the remaining range of the EV (Franke and Krems, 2013). For instance, if an EV has a range of 100 km and the distance of an average FFCS trip is 10 km, EVs with a battery level of less than 13% are rarely booked. As those EVs have to be recharged by the operator at critical times, an increased number of interventions is necessary. This suggests to combine vehicle relocations with the recharging of electric vehicles. Until now, relocation models for CS systems with EVs were only developed for station-based one-way CS systems (Bruglieri et al., 2014) but not for FFCS systems.

Second, as the number of charging stations for EVs is still limited, EVs that are already completely recharged should not block charging stations. Therefore, the fleet manager should unblock the relevant charging stations by moving the EVs to another parking spot. This is especially relevant for EVs in low demand areas as those EVs are relatively unlikely to be taken from the charging stations by the customers. An integrated approach combining FFCS relocations with the refueling of conventional vehicles, the recharging of EVs and the unplugging of completely recharged EVs to unblock charging stations has not been considered in the literature so far.

Third, no relocation model was tested in a real world field trial so far. The case studies were simulation studies only. In FFCS systems, operator-based relocations are very expensive as each automobile has to be moved separately by the operator. Knowledge of the effect of the relocations on the key factors of success are thus crucial. This knowledge provides helpful indication if the FFCS operator should intervene in the system regularly and if an expensive relocation algorithm should be implemented. So far, there is no definitive assessment, if relocation algorithms increase the profit of the CS operator and if they contribute to an improvement of the systems' operation. Moreover, real world field tests give information about the practice-readiness of a relocation model for FFCS systems. Simulation case studies reproduce the reality of a FFCS system to a certain degree based on the historical system behavior. However, they do not consider changing or exceptional circumstances. Testing the algorithm in a real scenario in corporation with the fleet managers of the FFCS system best shows if the model is robust. It also indicates if the model is comprehensible and if the technical output of the model is ready to be transferred to real relocation actions. Real-world field tests are also very important for improving both the model itself as well as the execution of the model output. Real-world field tests should be conducted several times. After each test, a comprehensive analysis of the execution, the effects and the strengths and weaknesses of the model should be conducted. Afterwards, the model should be improved accordingly.

1.3. Outline of this paper

This paper introduces a practice-ready relocation model for FFCS systems with EVs covering most of the above mentioned aspects. It consists of six consecutive steps. The model combines relocations with service trips. Those service trips are the unplugging of completely recharged EVs to unblock charging stations, the recharging of EVs with low battery level and the refueling of conventional vehicles with low fuel state. The first step of the model is the initialization of the model input data. It is applied as needed. A monthly data analysis and zone categorization module enables the calculation of realistic target vehicle distributions. A macroscopic optimization module and two microscopic rule-based modules find profit maximizing relocations leading as far as possible to the target vehicle distribution. An additional module considers service trips that cannot be combined with relocations. Taken as a whole, relocation movements and service trips are suggested on an individual vehicle level with the exact GPS coordinates of the relocation end positions. The authors carried out three real world field tests of the model in October 2013, February 2014 and May 2014 in cooperation with the operator of a FFCS system in Munich, Germany. The tests were conducted for different stages of development and different degrees of automation of the relocation model. Weaknesses of the approach could thus be identified after each test and gave important input for the improvement. In this paper, the effect of the relocations on different measures of success as well as the characteristics of the execution of the relocation movements are depicted for test three. Test three had a high degree of automation and covered steps one to six of the relocation model.

2. Methodology

Relocation algorithms for station-based CS systems find optimal vehicle movements between the stations of the CS system. As mentioned before, FFCS systems can be transformed to station-based CS systems by theoretically defining relocation zones. A zone thus represents an artificial station of the FFCS system. A relocation algorithm finds optimal vehicle movements between those zones. Contrary to stations in station-based systems, those relocation zones do not have strict capacity limits. The capacity limit is the space limit of the zone. Vehicle movements between large zones are too imprecise and do not satisfy specific local demand. On the other hand, vehicle movements between smaller zones are pseudo exact and algorithms might suggest vehicle movements of very small distance. Therefore, our approach combines macroscopic and microscopic zones. The model first tries to reach optimal vehicles numbers on the macroscopic level before detailing the suggested relocations on the microscopic level. The whole integrated relocation model is shown schematically in Fig. 1.

Let us suppose that the pricing model of the FFCS system is composed by a registration fee and a time-dependent usage rate r_u as well as a lower time-dependent rate r_p for parking per minute in $. Some FFCS operators allow trips to the airport of the city. Mostly, an extra airport fee of f_a $ per trip to or from the airport is charged. Let us suppose that the fleet of the FFCS system consists of both vehicles with combustion engine as well as EVs. The EVs can be charged at public charging stations with a specific number of charging points.

2.1. Step 0: (Re-)initialization of the model input data

As-needed, the input data of the considered relocation model is initialized. This step is conducted at least once before the first application of the relocation model. For that purpose, specific files are needed that contain information on the considered FFCS system. The geometry of the operating area of the FFCS system has to be available in form of a Shape-File. Second, in case of a system with EVs, an Excel spreadsheet has to contain the positions of the available charging stations and their corresponding number of charging points. Third, the FFCS operator has to provide an Excel spreadsheet of historical booking data of the previous month (or one of the previous months). This data have to contain the GPS positions of the booking starts of each FFCS booking in the considered time period.

Given this information, the model input data is initialized in several steps that are explained in the following. Afterwards, the initialization is repeated as-needed, e.g. if the operating area has changed, if new charging stations are available or have disappeared or if booking patterns or frequencies have changed to a significant degree.

First, the macroscopic zones are defined. The zones have to reflect the dynamics and booking patterns of the FFCS system. This might reduce the re-occurrence of imbalances subsequent to vehicle relocations between the zones. Moreover, the zones should be homogeneous and should cover the whole operating area. As mentioned in the introduction, previous approaches for station-based systems aimed at avoiding vehicle imbalances by optimal station location. This idea is adapted to the macroscopic zones. For finding optimal zones, we use a GIS-based approach similar to the one proposed by García-Palomares et al. (2012) for finding optimal stations for a Bikesharing system. An artificial facility location problem is solved based on the booking distribution of the FFCS system. We use the ArcGIS 10.2 (Environmental Systems Research Institute, ESRI) toolbox "location-allocation" with "minimize impedance (P-Median)", as this approach "covers the whole area relatively uniformly" (García-Palomares et al., 2012) resulting in homogenous zones for the FFCS system. The facilities are located such that the weighted costs between the demand points and the facilities are minimized. The solution facilities (locations) build the centers of the macroscopic zones and are interpreted as artificial vehicle depots (or stations). As they should have minimal travel times to their corresponding bookings and vice versa, the weighted costs for solving "P-Median" are equal to travel times. The macroscopic zones itself are the Thiessen polygons of the facilities. Thiessen Polygons are defined as follows: Each point/booking within a Thiessen polygon is closest to the center of this Thiessen polygon than to any of the other centers. The facility location problem is solved iteratively for different numbers of facilities/zones. Finally, the number of zones is chosen that results in appropriately sized, homogeneous zones. Let nZ be the resulting number of macroscopic zones.

Second, the microscopic zones are defined. According to experts, the maximal distance that a FFCS member is willing to walk to the next vehicle is 500 meters (0.31 mi) (Seign and Bogenberger, 2012). The relocation model suggests specific microscopic zones as end positions for relocation movements. The maximal distance between a FFCS user and a vehicle in the same microscopic zone should thus not exceed the above-mentioned threshold. Ideally, the zones would be circles with a diameter of 500 meters. However, it is not possible to cover the whole operating area with a grid of non-overlapping circles. As hexagons are the closest shape to a circle making this possible, the authors decided to cover the macroscopic zones by hexagons. Let nH be the resulting number of microscopic hexagons.

The macroscopic zones and microscopic hexagons of an example FFCS system are shown schematically in Fig. 2. There is a gap without hexagons. This is a park or an area with forbidden access that is excluded from the operating area.

Third, several model input files are generated based on the defined macroscopic zones and microscopic hexagons. Those are two $nZ \times nZ$ and two $nH \times nH$ matrices of the travel times and distances between the zones and between the hexagons respectively: $tt_{ij}, i,j = 1 : nZ$ or $i,j = 1 : nH$ (in hours) and $dist_{ij}, i,j = 1 : nZ$ or $i,j = 1 : nH$ (in km). The travel times and distances are calculated using the geographic information system software ArcGIS 10.2 Advanced for Desktop. The calculations were based on a street network and historical average speed profiles of a professional traffic data provider. Additionally, an

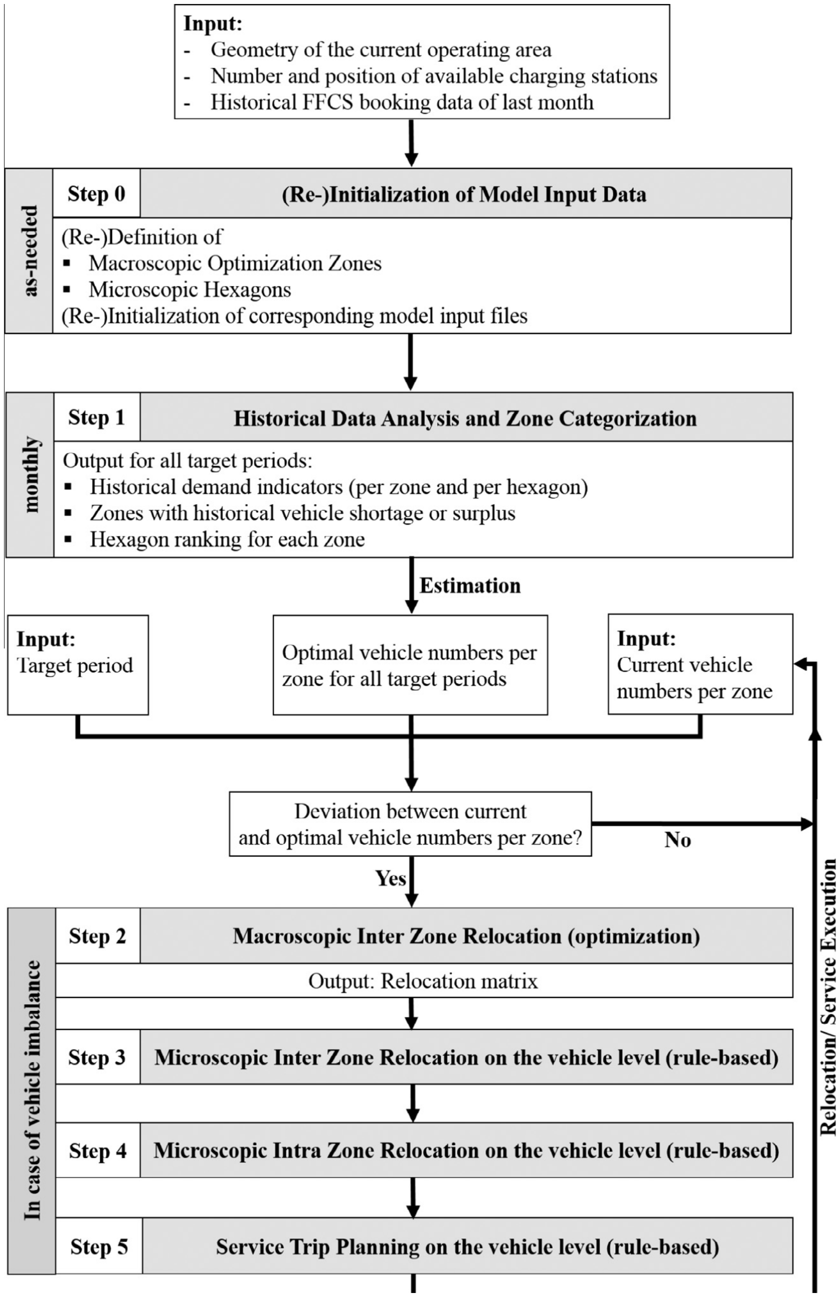

Fig. 1. Structure of the integrated relocation model for free-floating CS systems.

Excel spreadsheet of the hexagons' identification numbers, the GPS coordinates and addresses of their centers and the identification number and name of the corresponding macroscopic zones has to be generated. Finally, an Excel spreadsheet of the GPS coordinates and names of the macroscopic zones has to be available.

2.2. Step 1: Historical data analysis and zone categorization

The first step of the relocation model is a historical data analysis and zone categorization executed periodically, e.g. monthly based on historical booking data of the last month. The input data set consists of the booking start and end times, the GPS positions of the booking starts and ends and the driving and parking durations of each booking in the considered time period. The data analysis is executed for different combinations of target time slices of length l_{tp} (in hours, e.g. 0–3 a.m., 6–9 a.m., etc. for $l_{tp} = 3$) and target days (mean values of all weekdays, all Mondays, all Tuesdays, etc.), called target periods in the following. The length of the time intervals is set according to the dynamics of the FFCS system, e.g. the average

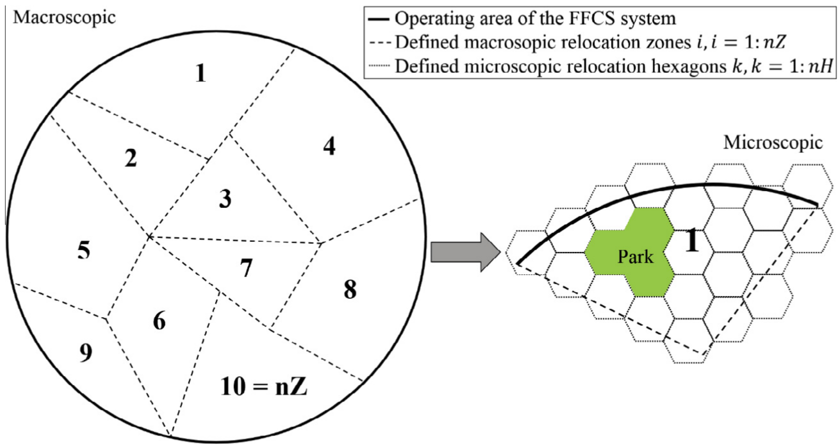

Fig. 2. Operating area of an example FFCS system with macroscopic relocation zones and microscopic relocation hexagons.

time span within which demand patterns are relatively homogeneous. Most of the computations are performed both on a zone and on a hexagon level. The outputs are

- mean historical booking numbers $hb_i^{zone}, i = 1 : nZ$ per zone in the target period,
- mean values for idle times $it_i^{zone}, i = 1 : nZ$ and $it_j^{hex}, j = 1 : nH$ in the target period,
- mean historical vehicle numbers $hv_i^{zone}, i = 1 : nZ$ per zone at the temporal begin of the target period,
- mean sums of driving durations $ds_i^{zone}, i = 1 : nZ$ and $ds_j^{hex}, j = 1 : nH$, parking durations $ps_i^{zone}, i = 1 : nZ$ and $ps_j^{hex}, j = 1 : nH$ and airport trips (to or from the airport) $as_i^{zone}, i = 1 : nZ$ and $as_j^{hex}, j = 1 : nH$ per vehicle on the day starting with the target period.

Based on the output of the historical data analysis, zones with historical vehicle shortage and zones with historical vehicle surplus are identified for each target period. The first indicator for vehicle shortage or surplus in a zone is the mean historical idle time of the zone. If zones have been under-supplied in the past, it is very likely that in general the time between two bookings has been low. However, idle times of vehicles in the same zone might deviate strongly as they are very sensitive to the vehicles' positions, e.g. due to the before-mentioned limited maximal walking distance that customers accept. Outliners might thus falsify this indicator.

Therefore, a second indicator was introduced based on historical vehicle balances. A positive historical vehicle balance means that at the beginning of past target periods the number of vehicles was in average higher than the number of bookings during the target period. That means that more vehicles than needed were available in the zone at the beginning of the target period. Contrary, a negative vehicle balance means that at the beginning of the target period, the number of vehicles in the zone was in average lower than the number of bookings during the period. Some booking requests could thus only be fulfilled because vehicles were entering the zone during the target period.

First, the nZ zones are sorted in ascending order of their mean historical idle times. The classification of the zones is based on the three tertiles of the idle times' distribution, resulting in zones with short, medium and long idle times. Let Z be the set of all nZ macroscopic zones. The set of zones with short idle times is $I_{short} = Q_{\{it_i^{zone}, i=1:nZ\}}(1/3)$ and the set of zones with long idle times is $I_{long} = Z \setminus Q_{\{it_i^{zone}, i=1:nZ\}}(2/3)$ *with* $Q_X(p) := F_X^{-1}(p) := inf\{x \in \mathbb{R} | F_X(x) \geqslant p\}$.

Second, the model computes the historical vehicle balances $bal_i^{zone}, i = 1 : nZ$ as the differences of the historical vehicle numbers per zone hv_i^{zone} and the historical booking numbers per zone hb_i^{zone} : $bal_i^{zone} = hv_i^{zone} - hb_i^{zone}$. The zones are then classified based on the three tertiles of their historical vehicle balances' distribution, resulting in zones with low, medium and high vehicle balances. The zones are sorted by ascending order of their mean historical vehicle balances. The set of zones with low vehicle balance is $B_{low} = Q_{\{bal_i^{zone}, i=1:nZ\}}(1/3)$, the set of zones with high vehicle balance is $B_{high} = Z \setminus Q_{\{bal_i^{zone}, i=1:nZ\}}(2/3)$ and the set of zones with medium vehicle balance is $B_{medium} = Z \setminus (B_{high} \cup B_{low})$.

Based on the two classifications, the zones are grouped into a set of zones with historical vehicle shortages $V^{<} = I_{short} \cap B_{low}$ (short idle time and low vehicle balance) and a set of zones with historical vehicle surplus $V^{>} = I_{long} \cap B_{high}$ (long idle time and high vehicle balance) in the particular target period.

2.3. Step 2: Macroscopic inter zone relocation

In step two, the model gives recommendations for vehicle relocations between the nZ macroscopic zones. The input are the FFCS systems' current vehicle numbers per zone $cv_i^{zone}, i = 1 : nZ$ submitted by the computer system of the FFCS operator.

The user of the relocation model first has to define the target period for which an optimal vehicle distribution should be derived. Based on the results of the historical data analysis and the zone categorization, the optimal vehicle distribution of the currently available vehicles for the target period is estimated. Intuitively, the available vehicles should be distributed over the operating area in proportion to the historical booking distribution of the target period. Therefore, the number of available vehicles in a zone should be at least equal to the zone's share in historical bookings in the target period times the total number of currently available vehicles:

$$\frac{hb_i^{zone}}{\sum_{j=1}^{nZ} hb_j^{zone}} \sum_{j=1}^{nZ} cv_j^{zone}, i = 1 : nZ \tag{1}$$

In the best case, at the beginning of the target period the number of available vehicles in a zone is even equal to the absolute historical booking number hb_i^{zone} of the zone during the whole target period. The preliminary optimal vehicle number per zone is thus first estimated as follows for $i = 1 : nZ$:

$$opt_i^{pre,zone} = max\left(hb_i^{zone}, \frac{hb_i^{zone}}{\sum_{j=1}^{nZ} hb_j^{zone}} \sum_{j=1}^{nZ} cv_j^{zone}\right) \tag{2}$$

For zones in $V^<$ with a historical vehicle shortage in the target period, $opt_i^{pre,zone}$ is still increased until $opt_i^{pre,zone} - hb_i^{zone}$ reaches the medium (balanced) tertile B_{medium} of vehicle balance. On the other hand, for zones in $V^>$ with a historical vehicle surplus $opt_i^{pre,zone}$ is decreased analogously. Possibly resulting negative optimal vehicle numbers are set to zero.

It is possible that the sum of preliminary optimal vehicle numbers $\sum_{i=1}^{nZ} opt_i^{pre,zone}$ is greater than the total number of available vehicles. To avoid this, the final optimal vehicle numbers are set to

$$opt_i^{zone} = \left\lfloor \frac{opt_i^{pre,zone}}{\sum_{i=1}^{nZ} opt_i^{pre,zone}} \sum_{j=1}^{nZ} cv_j + 0.5 \right\rfloor \tag{3}$$

Afterwards, a mathematical optimization model is applied if the current vehicle distribution deviates from the optimal vehicle distribution for the target period. This optimization model is solved on the macroscopic zone level as solving on a hexagon level might result in relocations of very small distance of e.g. 500 meters between neighboring hexagons.

An essential aspect of a relocation model is the integration of local authorities who are thus able to define city-compatible guidelines for the FFCS system. For instance, a maximum number of vehicles per zone can be determined by the city administration. This ensures that FFCS does not compete with public transport in areas of good public transport connection or that the parking situation is not deteriorated in areas that have a shortage of parking spots. On the other hand, a minimum number of vehicles per zone defined by an authority guarantees mobility for people in areas with few public transport who might thus resign their own car. Therefore, an upper threshold $u_i \in \mathbb{N}_{\geqslant 0}$ and a lower threshold $l_i \in \mathbb{N}_{\geqslant 0}$ for the number of vehicles are defined for each zone $i = 1 : nZ$.

If vehicle relocations between macroscopic zones computed by previous relocation runs are not yet finished by the relocation personnel but will presumably be finished before the target period, those are considered in the current run of the model by the vector

$$\mathbf{Y} = (y_i \in \mathbb{N})_{i=1:nZ} \begin{cases} > 0 & \text{if } y_i \text{ vehicle relocations to zone } i \text{ are not yet finished} \\ < 0 & \text{if } y_i \text{ vehicle relocations from zone } i \text{ are not yet finished} \\ = 0 & \text{else} \end{cases} \tag{4}$$

The elements of the vector Y are transmitted to the relocation algorithm before each run manually by the operator. The operator knows which relocations from previous runs still have to be conducted or still have to be finished and if those relocations are about to be finished before the target period of the current run.

Given the current number of vehicles per zone $cv_i^{zone}, i = 1 : nZ$ (the supply), the optimal number of vehicles per zone for the target period opt_i^{zone} (the demand) and the remaining relocations y_i from previous runs, the actual deviation between supply and demand can easily be calculated for each zone by

$$dev_i = \begin{cases} u_i - (cv_i^{zone} + y_i) & \text{if } opt_i^{zone} > u_i \\ l_i - (cv_i^{zone} + y_i) & \text{if } opt_i^{zone} < l_i \\ opt_i^{zone} - (cv_i^{zone} + y_i) & \text{else} \end{cases}, \quad i = 1 : nZ \tag{5}$$

Zones with positive deviation dev_i currently have a shortage of vehicles and are therefore called demand zones. Zones with negative deviation dev_i currently have a surplus of vehicles and are called supply zones. Zones with zero deviation currently have the optimal number of vehicles for the target period. Let dev_i^{pos} be the shortage of vehicles in zone i and let dev_i^{neg} be the surplus of vehicles in zone i.

$$dev_i^{pos} = \begin{cases} dev_i & \text{if } dev_i > 0 \\ 0 & \text{else} \end{cases}, \quad i = 1 : nZ \tag{6}$$

$$dev_i^{neg} = \begin{cases} -dev_i & \text{if } dev_i < 0 \\ 0 & \text{else} \end{cases}, \quad i = 1 : nZ \tag{7}$$

The objective function to be maximized is the profit resulting from vehicle movements between zones. The profit is the difference between the additional sales generated by a better-balanced vehicle distribution and the costs of the relocation movements. The additional sales are the sum of the additional booking durations and the additional airport fees on the subsequent day caused by the relocations between zones. The relocation costs are both time- and distance-dependent (fuel, costs of depreciation, wage for relocation personnel, etc.). An algorithm finds the optimal vehicle movements by maximizing the objective function. The optimization model thus compares additional sales to relocation costs while eliminating the imbalance of supply and demand.

The decision variables of the macroscopic optimization and thus of the objective function are the entries of the so-called relocation matrix R. This matrix indicates the profit maximizing vehicle movements between the zones.

$$R = (r_{ij})_{i,j=1:nZ} \text{ with } r_{ij} \begin{cases} > 0 & \text{if } r_{ij} \text{ vehicles are moved from zone } i \text{ to zone } j \\ = 0 & \text{else} \end{cases} \tag{8}$$

Relocating CS vehicles is connected to costs. The first type of relocation costs are time-dependent relocation costs for the CS personnel that relocate the vehicles. Let c_w be the hourly wage for the relocation personnel in \$ and let tt_{ij} be the travel time between the zones i and j in hours (see step 0), i.e. between the geographic centers of the zones i and j. As each relocation worker first has to approach the CS vehicles by a different means of transport, an average approach time $tt_{approach}$ in hours is added to the pure relocation travel time tt_{ij}. This average approach travel time has to be chosen based on experience. For instance, for a FFCS system in Munich, Germany three real world field tests of vehicle relocations yielded an average approach travel time of 0.45 h. The personnel costs for relocating a vehicle from zone i to zone j are thus equal to $c_w(tt_{approach} * tt_{ij})$.

The second type of relocation costs are distance-dependent relocation costs for vehicle movements between zones. Let c_f be the fuel prize per kilometer in \$, let c_d be the costs of depreciation of the vehicle per kilometer in \$ and let $dist_{ij}$ be the distance between two zones i and j in kilometers, i.e. between the geographic centers of the zones. The distance-dependent relocation costs for a vehicle movement between two zones i and j are thus $(c_f + c_d)dist_{ij}$. The distance-dependent costs for the approach to the CS vehicles are neglected, because they strongly depend on the used means of transport. Instead, an upper threshold for the costs for the approach to the CS vehicles is found. Let nw be the number of relocation workers. Let c_a be the maximum total cost per day of one single relocation worker to approach the CS vehicles, e.g. the price of a day ticket for public transport. The approach costs are estimated by $c_a * nw$.

The most important constraint of the optimization problem is the achievement of the optimal number $opt_i^{zone}, i = 1 : nZ$ of available vehicles per zone. However, this is sometimes not possible for all zones. The first reason is the limited total number of available vehicles of the FFCS system. As the values in Eq. (2) are rounded to the next integer, the sum of optimal vehicle numbers is possibly slightly higher than the number of currently available vehicles. The second reason is the limited number of relocations that can be executed due to the limited number of relocation workers and the limited time window. In order to nevertheless guarantee the satisfaction of this constraint, an artificial zone with an infinite number of vehicles is created. Artificial vehicle movements from this artificial zone to zones whose demand cannot be met due to the above-mentioned reasons are represented by the vector $P = (p_i \in \mathbb{N}_{\geqslant 0})_{i=1:nZ}$, where p_i is the shortage of vehicles that remains in zone i despite the application of the relocation model. The costs of those artificial vehicle movements are zero.

The limited number of relocations that can be executed is also the reason for not fully exploited vehicle surplus in specific zones. Those "superfluous" vehicles are stored in a second artificial zone with infinite capacity. Artificial vehicle movements from zones whose vehicle surplus is not fully exploited to this artificial zone are represented by the vector $A = (a_i \in \mathbb{N}_{\geqslant 0})_{i=1:nZ}$ where a_i is the surplus of vehicles that remains in zone i. Again, the costs of those artificial vehicle movements are zero.

Let us now consider a graph $G = (E, V)$ where the vertices V are the nZ macroscopic zones of the FFCS system as well as the two artificial zones. CS vehicles can move via the edges E between the nZ macroscopic zones or from zones to artificial zones and vice versa. The decision variables are the flows r_{ij} of vehicles between the vertices of the network. The costs of the edges are the above-mentioned relocation costs for edges between the vertices of the nZ macroscopic zones and zero costs for edges to or from vertices representing artificial zones. Fig. 3 shows the possible in- and outflows of a non-artificial vertex $i, i = 1 : nZ$ of the graph $G = (E, V)$ for a FFCS system.

During each relocation interval, customers bring vehicles to zone i or take vehicles from zone i. Those user-based in- and outflows are represented in Fig. 3 by $u_{in,i}$ and $u_{out,i}$. The optimization model neglects those user-based vehicle movements as their prediction is uncertain. In order to compensate for this, the system state is monitored within specific time intervals. Whenever the system state deviates to a certain degree from the initial system state at the beginning of the last relocation interval due to user-based vehicle movements, the optimization model is applied again.

A vehicle in zone i generates the following average sales in \$ on the day starting with the target period:

$$s_i^{zone} = ds_i^{zone} * r_u + ps_i^{zone} * r_p + as_i^{zone} * f_a, \quad i = 1 : nZ \tag{9}$$

That means that a relocation from i to j generates in average an increase or decrease of sales on the day starting with the target period of

$$g_{ij}^{zone} = s_j^{zone} - s_i^{zone}, \quad i,j = 1 : nZ \tag{10}$$

The objective function depends on the relocation matrix R and indirectly on the two vectors P and A describing remaining vehicle shortages and surplus. It is the sum of the differences between the increase or decrease in sales caused by a relocation and the total time- and distance-dependent relocation costs.

$$c(R) = \left[\sum_{i=1}^{nZ}\sum_{j=1}^{nZ} r_{ij}[g_{ij}^{zone} - dist_{ij}(c_f + c_d) - (tt_{approach} + tt_{ij})c_w]\right] - nw * c_a \tag{11}$$

The optimization model has several constraints to be satisfied.

$$r_{ij} \geqslant 0 \quad \forall\, i,j = 1 : nZ \tag{12}$$

$$r_{ii} = 0 \quad \forall\, i = 1 : nZ \tag{13}$$

$$r_{ij} = 0 \quad \forall\, i \in (X_1 \cup X_3) \vee j \in (X_2 \cup X_3) \tag{14}$$

$$\sum_{i=1}^{nZ}\sum_{j=1}^{nZ} r_{ij} \leqslant max^r \tag{15}$$

$$\sum_{i=1}^{nZ}\sum_{j=1}^{nZ} r_{ij} \geqslant \min\left(max^r, p^{min}\sum_{i=1}^{nZ} dev_i^{pos}\right) \tag{16}$$

$$\sum_{j=1}^{nZ} r_{ji} + p_i = dev_i^{pos} \quad \forall\, i = 1 : nZ \tag{17}$$

$$\sum_{j=1}^{nZ} r_{ij} + a_i = dev_i^{neg} \quad \forall\, i = 1 : nZ \tag{18}$$

$$z_i^A, z_i^E \leqslant 1 \quad \forall\, i = 1 : nZ \tag{19}$$

$$z_i^A + z_i^E \leqslant 1 \quad \forall\, i = 1 : nZ \tag{20}$$

$$\frac{1}{max^r}\sum_{j=1}^{nZ} r_{ij} \leqslant z_i^A \quad \forall i = 1 : nZ \tag{21}$$

$$\frac{1}{max^r}\sum_{j=1}^{nZ} r_{ji} \leqslant z_i^E \quad \forall i = 1 : nZ \tag{22}$$

$$p_i, a_i \geqslant 0 \quad \forall i = 1 : nZ \tag{23}$$

$$r_{ij}, p_i, a_i, z_i^A, z_i^E \in \mathbb{N} \quad \forall i,j = 1 : nZ \tag{24}$$

First, constraint 12 ensures that the entries of the relocation matrix are positive.

Second, the start and the end zone of a relocation movement must be different. This is guaranteed by constraint 13.

Third, if zones are e.g. temporarily not accessible due to street works, road closures or major events, they should be excluded from assigning vehicle movements to them. Also, it is possible that the CS operator himself excludes some zones from vehicle movements for various reasons. Let X_1 be the set of zones that are excluded from putting vehicles into them, let X_2 be the set of zones that are excluded from taking vehicles from them and let X_3 be the set of zones that are excluded from

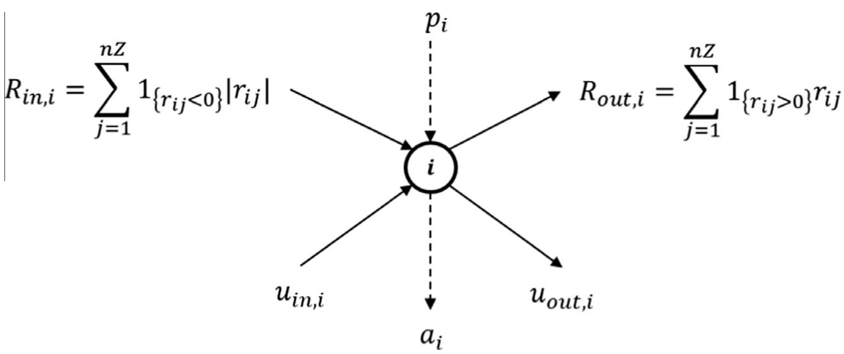

Fig. 3. A non-artificial vertex $i, i = 1 : nZ$ of the relocation graph G = (E,V) with the in- and outflows per relocation period.

both taking vehicles from them and putting vehicles into them. Constraint 14 ensures that the corresponding entries of the relocation matrix of those zones are set to zero.

As mentioned above, the number of relocations that can be executed is limited by the number of relocation workers *nw* and by the time window θ that is set for the relocations. Let tt_{rel} be the average duration (in hours) of a pure relocation movement (moving a CS vehicle between two zones). This average duration is derived from real world field tests of the relocation model. The approximate maximal number of relocations is thus $max_r = \lfloor \frac{\theta}{tt_{app}+tt_{rel}} * nw \rfloor$. Constraint 15 ensures that no more than max_r vehicle movements are suggested by the model.

The optimization algorithm naturally only suggests vehicle movements with average increase of sales greater than relocation costs. This is the only way to increase the objective function. Considering averages of sales for each zone delivers good indications for whether a relocation is profitable or not. However, the additional sales produced by a relocation might be much higher when considering zones microscopically, i.e. if vehicles are moved from cold spots of a zone to hot spots of another zone like realized in the following steps three and four. Second, if also non-monetary sales by additional customer satisfaction are considered, the increase in the sales is even higher. It would be unsatisfactory if the macroscopic step neglected relocations that produce negative profit on the macroscopic level but positive profit on the microscopic hexagon level. Therefore, the authors decided to add constraint 16 that guarantees that a certain number of relocations is suggested even if those relocations generate negative profit on the macroscopic level. The CS operator is able to define a certain percentage p^{min} of the total vehicle shortage $\sum_{i=1}^{nZ} dev_i^{pos}$ that must be eliminated by the relocations. Constraint 16 guarantees that all the relocations that are necessary for this purpose are found but no more than max^r relocations.

All demand should (artificially) be met by relocating the vehicles according to the relocation matrix and by considering the artificial vehicle movements for shortages that cannot be eliminated and the artificial vehicle movements for surplus that cannot be exploited. That means that the total "virtual" inflow of a zone consisting of the real inflow derived by vehicle relocations and the artificial vehicle movements for remaining vehicle shortage is equal to the total initial vehicle shortage dev_i^{pos} of a zone (see constraint 17). As the entries of *P* are non-negative (constraint 23), the number of vehicles entering a zone cannot exceed the shortage of vehicles in this zone. Analogous, constraint 17 guarantees that the total "virtual" outflow of a zone consisting of the real outflow derived by vehicle relocations and the artificial vehicle movements for remaining surplus vehicles is equal to the total initial vehicle surplus dev_i^{neg} of the zone. As the entries of *A* are non-negative (constraint 23), the number of vehicles leaving a zone does not exceed the surplus of vehicles in this zone. Constraints 17 and 18 also guarantee together with the non-negativity of the elements of the relocation matrix (constraint 12) that a_i and p_i are not greater than zero at the same time, i.e. that a zone cannot have vehicle shortage or surplus at the same time.

Relocations should not conflict with each other. If the relocation model suggests a vehicle movement from zone *i* to zone *j*, the model output should not contain relocations in the opposite direction. Constraints 19, 20 as well as 21 and 22 account for this fact.

Finally, the elements of the artificial vectors *P* and *A* have to be natural numbers greater or equal to zero (see constraint 23 and 24), because remaining vehicle shortages and surplus move in one direction only (from the artificial zone to the microscopic zones for *P* and vice versa for *A*). Constraint 24 also ensures that the elements of the relocation matrix *R* and the help variables z^A and z^E are natural numbers.

2.4. Step 3: Microscopic inter zone relocation on the vehicle level

The third step of the relocation model is rule-based and acts on the *nH* microscopic hexagons. It details the macroscopic optimal zone to zone movements from step two on a microscopic level. In order to combine relocations with service trips, the model considers different conditions when choosing vehicles from hexagons based on the relocation matrix.

First, vehicles with high idle times are preferred as those are likely to be located in hexagons with low demand. Second, if a combination of the relocations with refueling of conventional vehicles or recharging of EVs is desired, vehicles with low but still sufficient fuel or battery level are taken first (e.g. fuel or battery level $< 25\%$). Recharging has priority over refueling due to its complexity. Conventional vehicles are refueled during the relocation movement whereas EVs are brought to the closest charging station in the macroscopic end zone of the relocation. If there are no charging stations in the macroscopic end zone of the relocation, recharging is neglected and other vehicles are preferred. Third, completely recharged EVs at charging stations in macroscopic start zones are preferred over other vehicles in order to unblock charging stations. That means that specific vehicle groups are preferred when choosing individual vehicles to be relocated.

The system operator is able to decide if vehicle relocations should be combined with several service tasks. Those adjustments are made by setting the values of the parameters *unplug*, *recharge* and *refuel* appropriately.

$$unplug = \begin{cases} 1 & \text{if the unplugging of completely recharged EVs should be considered} \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

$$recharge = \begin{cases} 1 & \text{if the recharging of EVs with low battery level should be considered} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

$$refuel = \begin{cases} 1 & \text{if the refueling of conventional vehicles with low fuel state should be considered} \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

Let $soc_{>}$ be the lower bound for the battery level so that an EV is still considered as completely recharged. Let $soc_{<}$ be the upper bound for the battery level so that an EV is still regarded as requiring being brought to a charging station. Finally, let $sof_{<}$ be the fuel level that a conventional vehicle has to fall below so that it is considered as being in need of refueling. Then a vehicle group index $vi_l, l = 1 : \sum_{i=1}^{nZ} cv_i$ is defined indicating which characteristics vehicle l has.

$$vi_l = \begin{cases} 1 & \text{if vehicle } l \text{ is an EV with charging state } > soc^{>} \text{ currently connected to a charging station} \\ 2 & \text{if vehicle } l \text{ is an EV with charging state } < soc^{<} \text{ currently not connected to a charging station} \\ 3 & \text{if vehicle } l \text{ is a conventional vehicle with fuel state } < sof^{<} \\ 4 & \text{else} \end{cases} \tag{28}$$

The first input of step three are the vehicle flows recommended by the relocation matrix R. Those flows are detailed on the hexagon level such that the operator's profit is maximized. Vehicles should be taken from those hexagons of the proposed start zone that have the lowest booking frequency in the target period compared to the other hexagons of the zone. They should be brought to those hexagons in the proposed end zone that have the highest booking frequency in the target period. Low historical idle times of a hexagon indicate that the historical booking frequency in the hexagon was high and that there was a shortage of vehicles whereas high historical idle times indicate the opposite. Therefore, the hexagons' average idle times are the first criterion for choosing start and end hexagons for the specific relocations. As relocations are expensive, the operator is not only interested in reaching low idle times following the relocations. The overall sales that a relocated vehicle generates on the day following the relocation is also of interest. Those overall sales strongly depend on the end position of the first booking following the relocation and all the subsequent bookings. Therefore, the average historical sales per hexagon and vehicle on the day starting with the target period are the second criterion for choosing start and end hexagons for the relocations. For each zone, a hexagon ranking is developed based on those tow criteria.

Let $M_i = \{m_k, k = 1 : nh_i\}, i = 1 : nZ$ be the set of the nh_i hexagons in zone i. First, for each zone i the historical idle times per hexagon $it_k^{hex}, k = 1 : nh_i$ are considered. The hexagons are sorted by their idle times in ascending order resulting in a rank $rank_{1k}, k = 1 : nh_i \in [1, nh_i]$ for each hexagon. Second, the average sales per hexagon and vehicle $s_k^{hex}, k = 1 : nh_i$ on the day starting with the target period are considered. Those sales are calculated analogous to Eq. (9). The hexagons are sorted by their average sales in descending order resulting in a rank $rank_{2k}, k = 1 : nh_i \in [1, nh_i]$ for each hexagon. The total rank of a hexagon is $rank_k = rank_{1k} + rank_{2k}, k = 1 : nh_i \in [2, 2nh_i]$. "Cold" hexagons are the hexagons whose total rank $rank_k$ is in the worst (third) tertile of the rank values' distribution. "Hot" hexagons are in the best (first) tertile of the rank values distribution: $C_i = S_i \setminus Q\{rank_k, k = 1 : nh_i\}(2/3)$ and $H_i = Q\{rank_k, k = 1 : nh_i\}(1/3)$ with $Q_X(p) := F_X^{-1}(p) := inf\{x \in \mathbb{R} | F_X(x) \geqslant p\}$. C_i and H_i are sorted sets beginning with the worst and best ranked hexagon respectively. The general rule of this rule-based step three is

"*For all pairs* $(i,j), i,j = 1 : nZ$ *with* $r_{ij} > 0$ *: Take* r_{ij} *vehicles from the cold hexagons in the set* C_i *of the macroscopic start zone i. If* $vi_l \in [1;3;4]$, *move them to the hot hexagons in the set* H_j *of the macroscopic end zone j. If* $vi_l = 2$, *move them to the closest charging station in the macroscopic end zone j*."

In this step of the model, it is possible to define a maximum total vehicle number *maxNoVeh* per end hexagon to ensure a homogeneous vehicle distribution in the macroscopic end zone. Each hot hexagon in an end zone is filled with vehicles until the maximum number of vehicles is reached. Afterwards, the next best hot hexagon is chosen. If all of the hot hexagons have reached the maximum number of vehicles, the process starts again with the best hexagon and a maximum number of additional vehicles per hexagon of one. The process stops when the number of vehicles to be moved according to the relocation matrix of step two is reached.

A run of the whole relocation model naturally has no information on the exact relocation starts and ends of a previous run. In order to prevent contradicting vehicle movements it has to be avoided that the relocation starts of all the previous runs are relocation ends in subsequent runs and vice versa. Therefore, the exact starts and ends of a run are submitted to the subsequent run. Neighboring or identical hexagons to any start hexagon of a previous run are excluded from being chosen as end hexagons. On the other hand, neighboring or identical hexagons to any end hexagon of a previous run are excluded from being chosen as start hexagons.

The output of this step is a detailed list of recommended vehicle relocations between zones. For each relocation, the list contains the vehicle ID, the GPS position and address of the vehicle, the corresponding start hexagon and start zone, an information on whether the vehicle has to be recharged or refueled as well as the recommended end hexagon and end zone of the relocation. The GPS coordinates and the addresses of the centers of the end hexagons calculated in step 0 are also named in the list as desired end position of the vehicles. In case of recharging, the end positions are the positions of the closest charging station in the end zone. The list also contains the estimated additional profit, additional sales, additional costs, distance and total travel time of each relocation.

2.5. Step 4: Microscopic intra zone relocation on the vehicle level

The main objective of step two was to find the most profitable vehicle flows between zones that eliminate as far as possible vehicle shortages and surplus of the *nZ* macroscopic zones. The profit of the vehicle flows was estimated on a macroscopic level, i.e. based on average values of the zones. The rule-based step three detailed each vehicle flow by finding the most profitable relocations between the specific zones on a microscopic hexagon level. However, the above mentioned steps do not consider the fact that the vehicle distribution within a zone can also be disadvantageous. That means that vehicles are possibly located in hexagons with low booking probabilities and have bad preconditions for their overall sales on the day following the target period. This rule-based step four gives the FFCS operator the possibility to improve (to a certain degree) the vehicle distribution within macroscopic zones. It operates analogous to step three. The hexagon ranking is the same as before.

In order to facilitate the combination of inter and intra zone relocations, this step is only conducted for the start and end zones of the macroscopic step two that are visited anyway. For each inter zone vehicle movement, the responsible relocation employee is allowed to conduct add_1 additional intra zone relocations in the start and in the end zone of the macroscopic relocation. The number add_1 is chosen individually by the operator depending on his priorities. The larger add_1, the larger the focus on the rule-based intra zone relocations and the time invested in those relocations. The consequence is, that possibly not all of the inter zone relocations can be conducted within the given specific time window and with the given number of relocation workers. The lower add_1, the larger the priority is on generating the optimal vehicle numbers per zone and thus on equally satisfying customers in all zones. Like in step three, it is possible to exclude zones from intra zone relocations. When choosing vehicles from hexagons, the same conditions concerning idle times, fuel and battery level and completely recharged vehicles as in step three are applied. Again, additional rules are integrated that prevent that start hexagons of previous runs are neighboring or identical to end hexagons of the current run and vice versa. The rule of this step is

"*For all pairs* $(i,j), i,j = 1 : nZ$ *with* $r_{ij} > 0$: *Take at a maximum* $add_1 * r_{ij}$ *vehicles from the cold hexagons in the set* C_i *of the considered macroscopic zone i. If* $vi_l \in [1;3;4]$, *move them to the hot hexagons in the set* H_i *of the same macroscopic zone i. If* $vi_l = 2$, *move them to the closest charging station in the same macroscopic zone i. Proceed analogously for zone j.*"

In order to reach a homogeneous vehicle distribution, each hot hexagon in H_i is again filled with vehicles until the maximum number *maxNoVeh* of vehicles is reached. Afterwards, the next best hexagon is chosen. If all of the hot hexagons have reached the maximum number of vehicles, the process starts again with the best hexagon and a maximum number of additional vehicles per hexagon of one. The process stops when $add_1 * r_{ij}$ vehicles have been chosen or if there are no more vehicles in the cold hexagons of the considered zone. It has to be mentioned, that intra zone relocations do not generate optimal vehicle numbers per hexagon. However, they shift vehicles from those hexagons with the lowest demand in the target period to those hexagons with the highest demand. They therefore improve the vehicle availability in high demand areas while increasing the profit of the operator.

The output of this step is a detailed list of recommended vehicle relocations within zones. For each relocation, it contains the same information as the output list of step 4.

2.6. Step 5: Service trip planning on the vehicle level

Step 5 considers the remaining pure service trips that are not handled so far. Vehicles that have to be unplugged, recharged or refueled are partly considered when detailing the macroscopic inter and intra zone relocation movements in steps three and four. However, there might be other vehicles that require service trips. Again, for practical reasons this step is only conducted for the start and end zones of the macroscopic step two that are visited anyway.

For each inter zone vehicle movement, the responsible relocation employee is allowed to conduct add_2 additional pure service trips in the start zone and in the end zone of the macroscopic relocation. The number add_2 is chosen individually by the operator depending on his priorities. The larger add_2, the larger the focus on service trips and the time invested in those. Like in step four, the consequence is, that possibly not all of the inter zone relocations can be conducted within the given specific time window and with the given number of relocation workers. For example, if the operator has separate relocation staff that is responsible for the service trips, add_2 can bet set to zero. Analogous to the other steps, the FFCS operator is able to exclude zones from service trips. The service trips are - with descending priority - the unplugging of completely recharged EVs to unblock charging stations, recharging trips or refueling trips.

The output of this step is a detailed list of recommended service trips. For each service trip, it contains the vehicle ID, the GPS position and address of the vehicle, the corresponding start hexagon and start zone, an information on whether the vehicle has to be unplugged, recharged or refueled as well as the GPS coordinates and address of the recommended charging station in case of recharging. The list also contains the estimated costs for the service trip and the distance and total travel time in case of recharging.

2.7. Output of the model

Finally, the results of the steps three to five are combined. If an end hexagon of an inter/intra zone relocation or service trip is neighboring or identical to the start hexagon of another inter/intra zone relocation, the relevant service trip or relocation and the second relocation are combined to one vehicle movement. However, this is only possible for specific combinations of vehicle types. For instance, a service trip or relocation of a vehicle of type 2 cannot be combined with another relocation as the vehicle has to be brought to a charging station. The final output is a detailed list of all the recommended inter and intra zone relocations and service trips containing all of the before-mentioned informations.

Fig. 4 shows an example for the combination of inter zone relocations with intra zone relocations and pure service trips. For each inter zone relocation, one additional intra zone relocation and one additional service trip are conducted if necessary.

2.8. Software implementation

The whole integrated relocation model is implemented in the software MATLAB. The mathematical optimization problem of step two is a linear mixed integer program (MIP). The MIP is modeled using the TOMLAB modeling class TomSym. It is a well-defined "transportation problem" from the network optimization literature. Therefore, it can be solved despite integer variables by appropriate optimization solvers in polynomial time. The optimization problem is solved by means of the TOMLAB optimization solver package CPLEX. CPLEX solves mixed-integer linear and quadratic programs and linear and quadratic programs with simplex or barrier solvers. Quadratic constraints are also supported. For the solution of the above mentioned problem, a Branch-and-Cut MIP solver is used. The CPU time for solving the optimization problem is very low, for instance 0.6 s for a FFCS system with 15 macroscopic zones.

3. Field trial results

Until now, three real world field tests have taken place in different development phases of the model for a FFCS system in Munich, Germany. The operating area of the system is shown in Fig. 5. It comprises an area of approximately 79 km^2 (30.5 mi^2) with a main area in the inner city of Munich and five external areas. Because of parking restrictions, the historic city center has been excluded. The pricing model is composed by a registration fee and a time-dependent usage rate (0.31 Euros) as well as a lower time-dependent rate for parking (0.15 Euros). For trips to the airport of Munich, the CS operator charges an extra airport fee of 12 Euros. The fleet consists of approximately 390 vehicles with combustion engine and 20 EVs. The EVs can be charged at 29 public charging stations with two charging points in the greater area of Munich where 13 charging stations lie within the operating area. The CS operator also possesses one private charging garage with eight charging points, mainly used for recharging overnight. Relocations are currently conducted reactively, i.e. if idle times exceed certain thresholds or based on experience.

The macroscopic steps act on 15 zones with an average area of 5.3 km^2 (2 mi^2). The microscopic steps act on 478 hexagons with an area of approximately 0.16 km^2 (0.06 mi^2). The zones and hexagons are shown in Fig. 5. The before mentioned facility location problem was solved for the main operating area iteratively with different pre-defined numbers of facilities. Finally, the number of ten facilities was chosen, because it resulted in appropriately sized homogeneous zones. The five other macroscopic zones are the five external areas of the operating area.

As mentioned in the introduction, Weikl and Bogenberger (2014) detected imbalances between vehicle supply and demand for the considered FFCS system on Monday mornings. Therefore, the three tests were conducted during nights from Sunday to Monday in October 2013, February 2014 and May 2014.

Prior to each test, the historical data analysis of step one was conducted for CS booking data of the particular previous month for the target period "Monday, 6 to 9 a.m.".

The three test exhibited different degrees of automation. In a first field test in October 2013, step two of the model was tested. As steps three to five were not implemented yet, those were realized manually by experts. The trial showed that an implementation of the microscopic steps three to five is necessary.

In February 2013, the second field test was conducted. As the 15 macroscopic zones of step two seemed to be too large in the first test, the authors decided to conduct a modified version of step two with microscopic hexagons. Like before, the steps three to five were not implemented yet. Those were neglected, as they were considered almost redundant due to the increased degree of detail of step two. However, experts still had to select the specific vehicles in the start hexagons recommended by step two. In this second test, microscopic hexagons were used in step two. On the one hand, this enabled the automatic output of the exact end positions of the relocations (centers of the hexagons). On the other hand, the smaller zones together with a defined minimum distance led to illogical vehicle movements. This showed that a combination of macroscopic and microscopic steps like in the current version of the relocation model makes sense.

For the third field test in May 2013, the implementation of the steps three to five was finished. Pure service trips were not considered and the relocations were not combined with refueling due to the conflict between the late time window of the test and the opening hours of gas stations in Munich. However, recharging and unplugging of completely recharged vehicles was possible. As test three represents the final version of the relocation model, its results are introduced in detail. In total, 36 vehicles were relocated between 10 p.m. and 3:30 a.m. (see Fig. 6).

Two fleet manager were responsible for the coordination of six student service workers. The workers approached the CS vehicles by different means of transport: mainly public transport, car (driven by another worker), by foot, folding bikes and BS. There were two runs of the whole relocation model (steps two to five), one at 8:52 p.m. with 22 relocations and one at

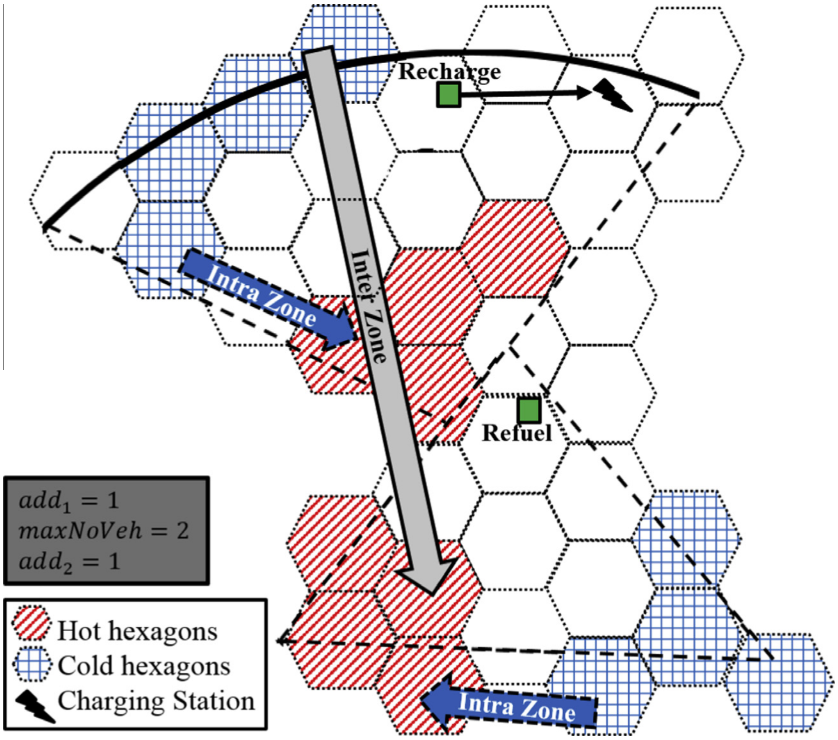

Fig. 4. Example of the combination of microscopic rule-based inter and intra zone relocations and service trips on the vehicle level.

11:10 p.m. with 14 relocations. The 36 movements were composed by 24 inter zone and 12 intra zone relocations. Step five suggested the unplugging of two completely recharged EVs in zone 6 in order to unblock the charging station (see Fig. 6).

Subsequent to the test, the impact of the relocations on different measures of success was evaluated. Therefore, historical booking data of the test CS system for the last four Mondays before the test were compared to booking data for the particular Monday following the test. For the comparison, the mean values of the previous Mondays were calculated.

First, the results are summarized in the diagram in Fig. 6. It has to be mentioned that a higher number of airport trips per vehicle is positive for the CS operator as airport trips generate higher-than-average booking durations and an airport fee is charged. The most important measure result is the change in net profit resulting from the relocations. We estimated the relocation costs that would have occurred if the operator conducted the relocations by himself. The time-dependent costs were calculated by multiplying the relocation durations (for approaching plus moving the CS vehicles from A to B) with the estimated hourly wage for a relocation worker (15 Euros per hour). The distance-dependent costs were calculated by multiplying the distances of the relocations with the estimated fuel price per km (0.08 Euros) and the estimated cost of depreciation (0.1 Euros/km). The cost for the approach to the CS vehicles were set to their maximum value of 6 Euros per relocation worker per day. This is the daily fee for public transport in Munich. Other means of transport (folding bike, Bikesharing, car driven by another worker, by foot) have lower total costs per worker. The benefit of the relocations is calculated by comparing the average earnings of the previous Mondays with the earnings of the Monday after the relocation test minus the estimated relocation costs. This results in a net profit increase of 5.8% for test three showing that the applied relocation model leads to the satisfaction of otherwise unmet demand.

Second, the start times of the first customer bookings following the relocations were analyzed. Customers used almost all relocated vehicles (80.6%) before 9 a.m. on the following Monday, meaning that almost all relocations were suitable for the target period.

We also compared the idle times of the relocated vehicles with the idle times of nearby vehicles (within a buffer of radius 1 km). As the end times of the relocations are mostly later than the last booking end times of the comparable vehicles, the mean idle times starting from the end time of the test were calculated. Test three resulted in a reduction of 31% of the mean idle time of the relocated vehicles compared to the not relocated vehicles in the neighborhood.

A worker was able to finish in average 1.1 relocations per hour due to the high degree of automation. The mean duration of a pure relocation trip (worker moves CS vehicle from A to B) was 18 min. The mean duration of the approach to the relocated CS vehicles was 30 min. The different means of transport for the approach exhibited different travel speeds. The fastest mode of transport was the shared bike with a speed (air distance km per hour) of 16 followed by the car (13.5), the folding bike (9.0), public transport (7.0) and by foot (5.0). The mean total duration of a relocation (approach to CS vehicle plus moving CS vehicle from A to B) was 48 min.

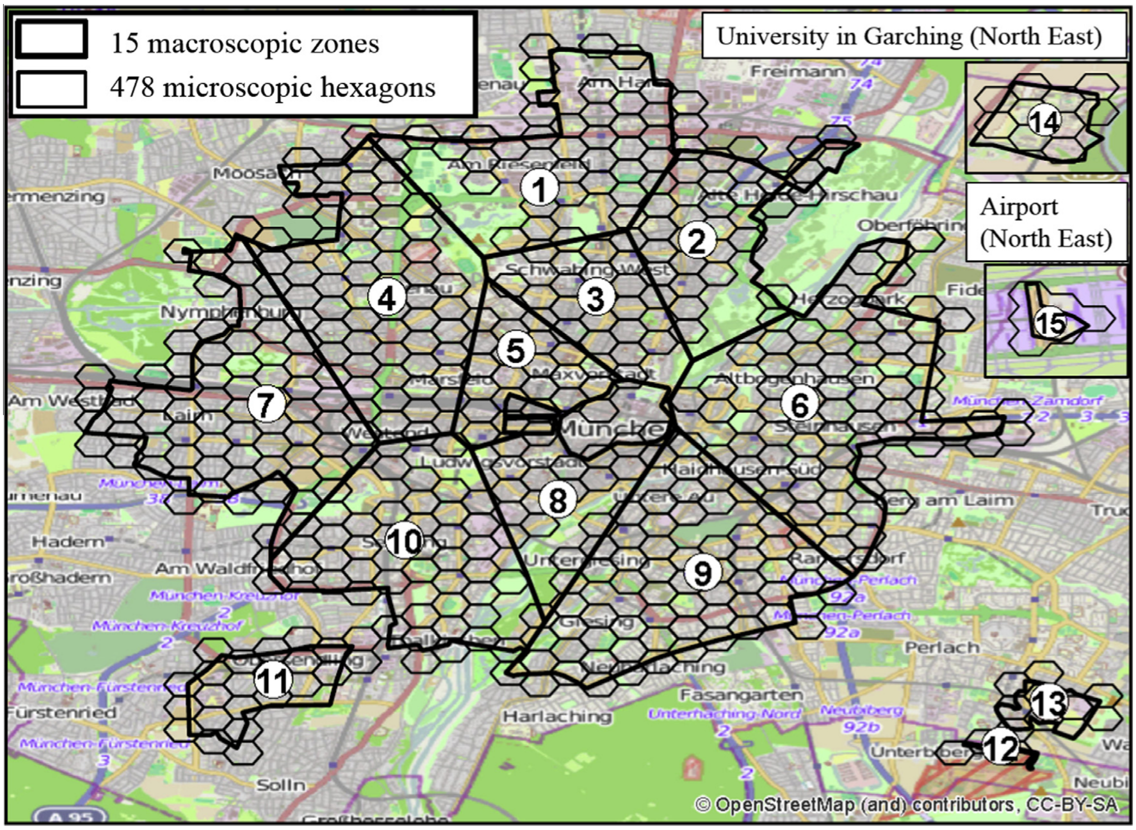

Fig. 5. Operating area of the considered FFCS system in Munich, Germany with the defined macroscopic zones and microscopic hexagons.

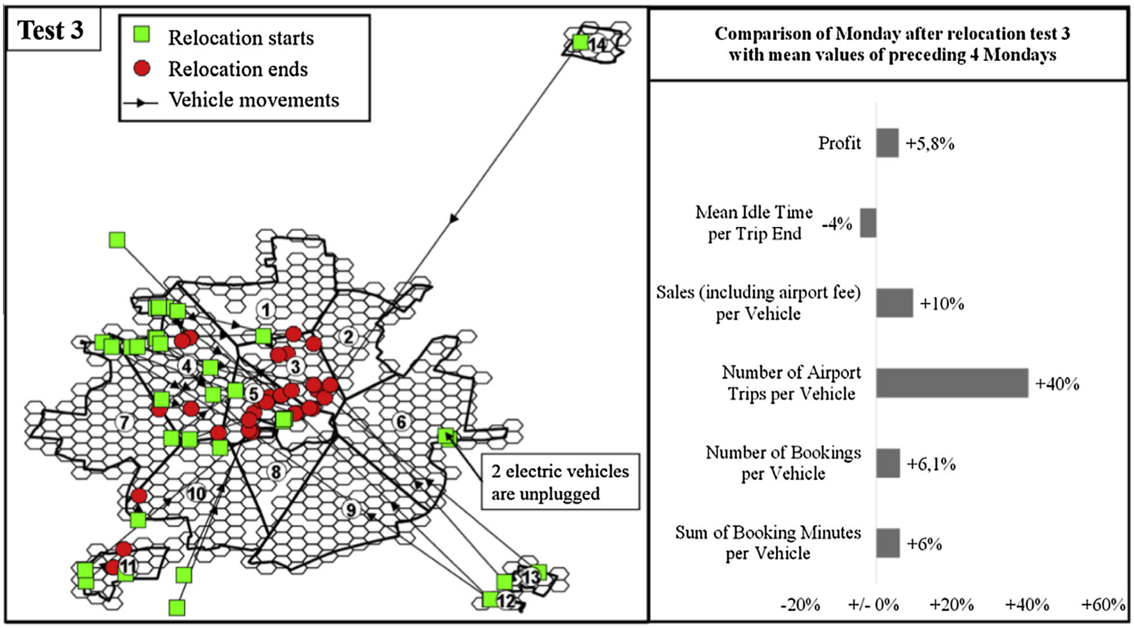

Fig. 6. The realized 36 relocation movements and service trips of the third field test of the integrated relocation model for free-floating CS systems in May 2014.

The mean distance of the conducted vehicle movements was 4.6 km, composed by 6.2 km for inter zone and 1.6 km for intra zone relocations. The model suggested exact addresses as ends of the relocations. Those are the centers of the hexagons. As parking pressure is high in some areas, the relocation workers are not always able to find a parking spot at the suggested address. However, data analysis showed that all actual ends except one lay within an air distance of 400 meters to the suggested address. This is very positive, as CS members are willing to walk a distance of approximately 500 meters to the next CS vehicle as mentioned before.

4. Conclusions and outlook

This paper presented a practice-ready relocation model for FFCS systems with both conventional and electric vehicles. The model was tested within a real world field trial in cooperation with a FFCS system operator in Munich, Germany.

The approach uses macroscopic zones of the operating area that are defined based on a facility location problem and microscopic hexagons with a diameter of 500 meters. The first step is the (re-) initialization of the model input data that is executed once before the first application of the model and later on as-needed. The second step a historical data analysis and zone categorization module that generates the input for the calculation of a target vehicle distribution for different target periods. If vehicle supply and demand deviate from each other, five macroscopic or microscopic steps using optimization and rule-based methods are applied. Step two solves a mixed integer program for finding macroscopic profit maximizing zone to zone vehicle relocations that as far as possible lead to optimal vehicle numbers per zone. Step three details the macroscopic relocations microscopically. Based on rules, it choses individual vehicles to be relocated and recommends end hexagons for the vehicle movements. Specific vehicles are prioritized so that relocations are immediately combined with service trips like the unplugging of EVs to unblock charging stations, the recharging of EVs and the refueling of conventional vehicles. As vehicle imbalances also occur within a zone, step four suggests vehicle movements within a zone analogously to step three. Remaining pure service trips that are not handled by previous steps are found in step five. Intra zone relocations and service trips are chosen such that they can be best combined with inter zone relocations. Compared to other approaches, this relocation model is practice-ready with low computational time. It combines different important aspects like demand prediction, elimination of vehicle imbalances and integration of service trips. The recommended vehicle relocations and service trips have a high degree of detail.

Most importantly, the model was applied to a FFCS system in Munich, Germany within three real world field tests. The tests were conducted for different stages of development of the model and thus exhibited different degrees of automation. Test three represents the final version of the model and its results were therefore depicted in detail. Test three had positive impacts on the key measures of success underlining that vehicle relocations make sense for FFCS systems. Summing up, test three succeeded in joining a high degree of automation, efficiency of the relocations and promising overall results especially for the net profit resulting from the relocations, the airport trips and the mean idle time of the relocated vehicles.

The model however still has some weaknesses. First, it has to be mentioned that until now the number of vehicles entering the zones during the target period is not added when calculating the vehicle balance bal_i^{zone} used in the second part of the zone categorization. The authors wanted to avoid unidentified vehicle shortages or overrated vehicle surplus caused by wrong estimates of this parameter. However, large user-based inflows to a zone during the target period might eliminate vehicle shortage or increase vehicle surplus indicated by the current version of bal_i^{zone}. Therefore, user-based zone inflows have to be considered in future versions of the model. The impact of those changes to the model has to be analyzed and evaluated.

In the future, the model should be extended by a sub-microscopic order execution planning module. The objective of this step is further minimizing the total duration of the relocations and service trips by assigning specific trips to each worker and generating schedules. Consequently, more relocations and service trips can be fulfilled in the given time window and with the available staff size. This step has to be modeled and implemented in detail. Afterwards, further field tests of the whole model should take place. The results should be evaluated and possibly arising weaknesses should be eliminated.

Furthermore, the calculation of the optimal number of vehicles per zone uses historical FFCS booking data only. Future work should also integrate a real demand model.

Finally, the question remains on how sensitive the results of the model are to changes in the operating area definitions, especially on the definition of the macroscopic zones. Simulations should be conducted with macroscopic zones of different size and the results should be compared.

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References

Barrios, J.A., Doig, J.C., 2014. Fleet sizing for flexible carsharing systems: a simulation-based approach. In: Proc. Transportation Research Board 93rd Annual Meeting.

Barth, M., Todd, M., 1999. Simulation model performance analysis of a multiple station shared vehicle system. Transport. Res. Part C: Emerg. Technol. 7, 237–259.

Barth, M., Todd, M., Xue, L., 2004. User-based vehicle relocation techniques for multiple-station shared-use vehicle systems. In: Proc. Transportation Research Board 83rd Annual Meeting.

Bruglieri, M., Colorni, A., Luè, 2014. The vehicle relocation problem for the one-way electric vehicle sharing: an application to the milan case. Procedia – Social Behav. Sci. 111, 18–27.

Bundesverband CarSharing e. V., 2014. Datenblatt CarSharing in Deutschland.

Cepolina, E.M., Farina, A., 2012. A new shared vehicle system for urban areas. Transport. Res. Part C: Emerg. Technol. 21, 230–243.

Chemla, D., Meunier, F., Wolfler Calvo, R., 2013. Bike sharing systems: solving the static rebalancing problem. Discr. Optim. 10, 120–146.

Clemente, M., Fanti, M., Mangini, A., Ukovich, W., 2013. The vehicle relocation problem in car sharing systems: modeling and simulation in a petri net framework. In: Colom, J.M., Desel, J. (Eds.), Application and Theory of Petri Nets and Concurrency, Lecture Notes in Computer Science, vol. 7927. Springer, Berlin, Heidelberg, pp. 250–269.

Correia, G., Jorge, D.R., Antunes, D.M., 2014. The added value of accounting for users' flexibility and information on the potential of a station-based one-way car-sharing system: an application in Lisbon, Portugal. J. Intell. Transport. Syst. 18, 299–308.

Di Febbraro, A., Sacco, N., Saeednia, M., 2012. One-way car-sharing: solving the relocation problem. In: Proc. Transportation Research Board 91st Annual Meeting.

Fan, W., Xu, Y., 2013. Managing dynamic vehicle allocation for car sharing systems: a stochastic programming approach. In: Proc. Transportation Research Board 92nd Annual Meeting.

Firnkorn, J., Mueller, M., 2011. What will be the environmental effects of new free-floating car-sharing systems? The case of car2go in Ulm. Ecol. Econ. 70, 1519–1528.

Franke, T., Krems, J.F., 2013. Interacting with limited mobility resources: psychological range levels in electric vehicle use. Transport. Res. Part A: Policy Pract. 48, 109–122.

García-Palomares, J.C., Gutiérrez, J., Latorre, M., 2012. Optimizing the location of stations in bike-sharing programs: a GIS approach. Appl. Geogr. 35, 235–246.

Goeppel, M., Blumenstock, A., 2012. Automatisierte Servicedisposition zur Flotteninstandhaltung am Beispiel car2go. At – Automatisierungstechnik 60, 102–112.

Jorge, D., Correia, G., 2013. Carsharing systems demand estimation and defined operations: a literature review. EJTIR 13, 201–220.

Jorge, D., Correia, G.H.A.d., Barnhart, C., 2013. Comparing optimal relocation operations with simulated relocation policies in one-way carsharing systems. In: Proc. Transportation Research Board 92nd Annual Meeting.

Kek, A.G.H., Cheu, R.L., Meng, Q., Fung, C.H., 2009. A decision support system for vehicle relocation operations in carsharing systems. Transport. Res. Part E: Logist. Transport. Rev. 45, 149–158.

Kortum, K., Machemehl, R.B., 2012. Free-floating Carsharing Systems: Innovations in Membership Prediction, Mode Share, and Vehicle Allocation Optimization Methodologies.

Lenz, B., Bogenberger, K., 2014. WiMobil – Wirkung von E-CarSharing-Systemen auf Mobilitaet und Umwelt in urbanen Raeumen: Halbzeitkonferenz zur Nutzung von E-Carsharing-Systemen am Beispiel von car2go, DriveNow und Flinkster. <http://www.erneuerbar-mobil.de/de/events/pdf-dateien/folien-wimobil.pdf>.

Martinez, L.M., Caetano, L., Eiró, Cruz, F., 2012. An optimisation algorithm to establish the location of stations of a mixed fleet biking system: an application to the city of Lisbon. Procedia – Social Behav. Sci. 54, 513–524.

Miller-Hooks, E., Nair, R., 2010. Fleet Management for Vehicle Sharing Operations. <http://www.mautc.psu.edu/docs/UMD-2008-02.pdf>.

Morency, C., Trépanier, M., Godefroy, F., 2011. Insights into Montreal's bikesharing system. in: Proc. Transportation Research Board 90th Annual Meeting.

Nourinejad, M., Roorda, M.J., 2014. A dynamic carsharing decision support system. Transport. Res. Part E: Logist. Transport. Rev. 66, 36–50.

Sayarshad, H., Tavassoli, S., Zhao, F., 2012. A multi-periodic optimization formulation for bike planning and bike utilization. Appl. Math. Modell. 36, 4944–4951.

Seign, R., Bogenberger, K., 2012. Prescriptions for the successful diffusion of carsharing with electric vehicles. In: Proc. Conference on Future Automotive Technology.

Shaheen, S.A., Mallery, M.A., Kingsley, K.J., 2012. Personal vehicle sharing services in North America. Res. Transport. Business Management 3, 71–81.

The Straits Times, 2008. Honda car-sharing scheme in Singapore to end. <http://wildsingaporenews.blogspot.de/2008/03/honda-car-sharing-scheme-in-singapore.html>.

Waserhole, A., Jost, V., 2013. Vehicle Sharing System Pricing Regulation: A Fluid Approximation. <http://hal.archives-ouvertes.fr/hal-00727041/>.

Weikl, S., Bogenberger, K., 2014. Nachfragephaenomene von free floating car sharing-systemen – Raeumlich-Zeitliche Angebots-Nachfrage-Asymmetrie. In: Forschungsgesellschaft fuer Strassen- und Verkehrswesen (Eds.), Proceedings of Heureka 2014: Optimierung in Verkehr und Transport, pp. 591–606.