Deep Space Divergence

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I. Introduction

* In the classical 3-body problem, three masses with initial sets of positions and velocities are placed into a single system. Their subsequent motions, via gravitational forces, can only be modeled numerically
* The system is complicated with the introduction of a fourth body that has slightly different initial conditions with one of the original three. Chaos can then be measured by tracking the gradual divergence between the two similar bodies
* This could be an interesting model for international space programs since they launch multiple satellites at a time into space, making potential divergence between the satellites crucial to understand

Which initial condition of the second satellite will result in the longest and shortest time before a significant divergence?

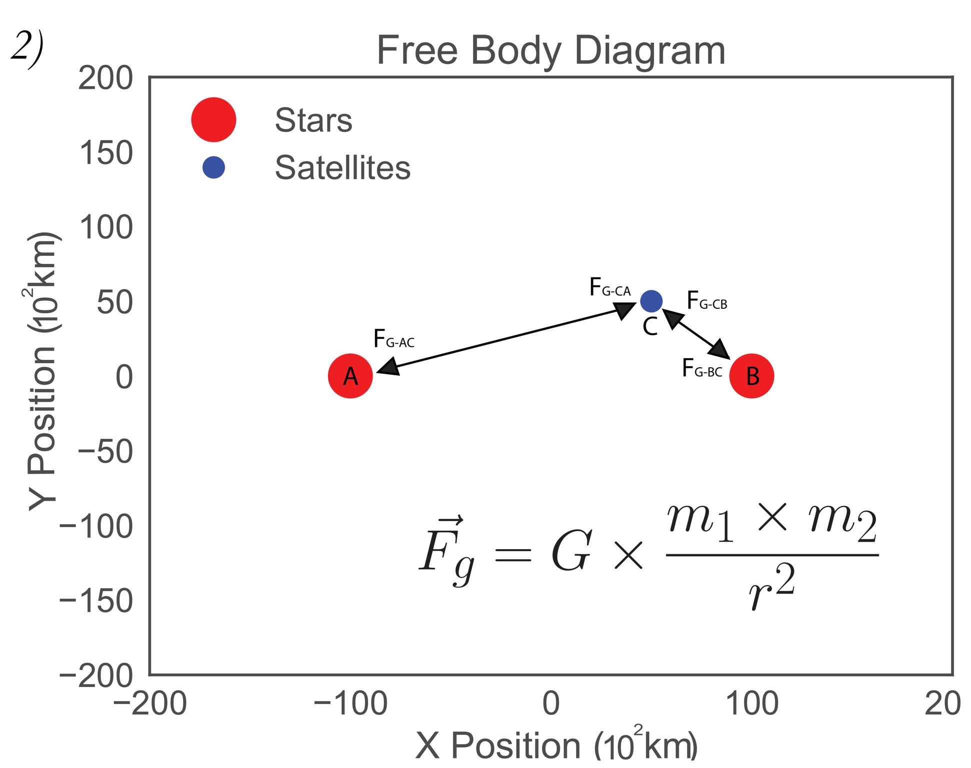
II. Model

Conditions:
* Mass of the stars = 2×10^{30} Kg
* G constant = 6.67×10^{-24} (km)^3/Kg-s^2
* Stars stationary at (10000, 0)km and (-10000, 0)km
* Conditions for control satellite:
    * Initial position (km): (5000, 5000)
    * Initial velocity (km/s): (0, -6500)

Mass of satellite:

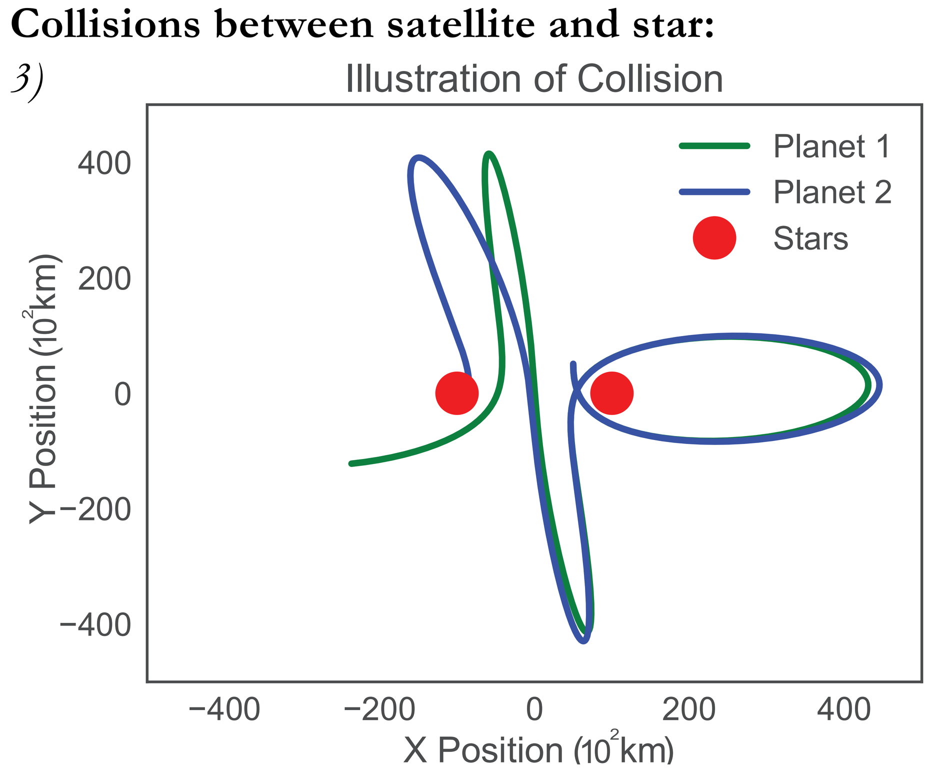
1) $$\vec{V_{new}} = \frac{m_p \cdot \vec{V_{old}} + (G \times \frac{m_1 \times m_2}{r^2})dt}{m_p} \rightarrow \vec{V_{old}} + (G \times \frac{m_2}{r^2})dt$$

**mp cancels in Eq1, so it has no impact on results*

2)


**Free body diagram of 2 stars and 1 satellite. Forces between satellites were not implemented.*

Collisions between satellite and star:

3)


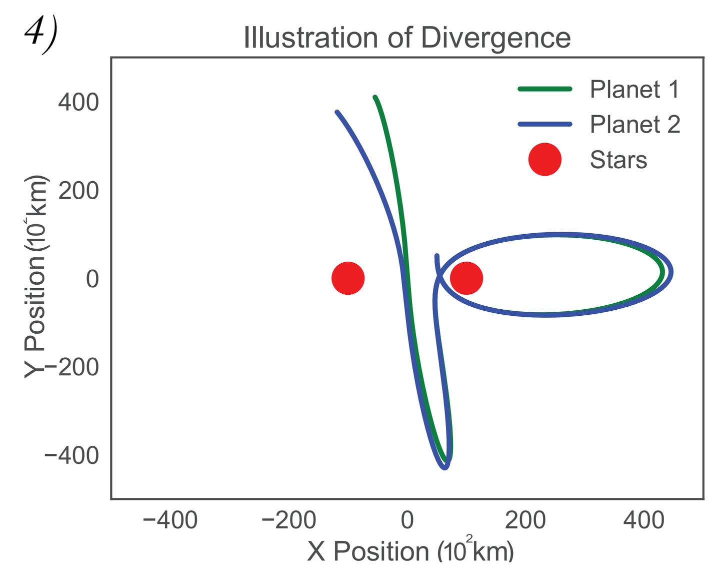
**Collisions would end the simulation*

Abstract

While attempting to build the starship Enterprise, the United Federation of Planets needs to launch two satellites containing crucial materials into a binary star system. Since the satellites do not have any fuel, they enter the system at a given velocity and are only affected by gravitational forces. The materials need to reach the Enterprise before the satellites significantly diverge. Before the launch, the engineers decide to build a model to determine which conditions of the satellites would result in the shortest and longest time before divergence.

To Boldy Diverge Where No Man Has Diverged Before

Divergence between satellites:

4)


Divergence found using three conditionals
* Check if position between the two satellites is greater than equation 5
* Check if difference in angle of velocity vectors is greater than 10 degrees
* Check if minimum distance between current position of one and past points of the other is greater than equation 5

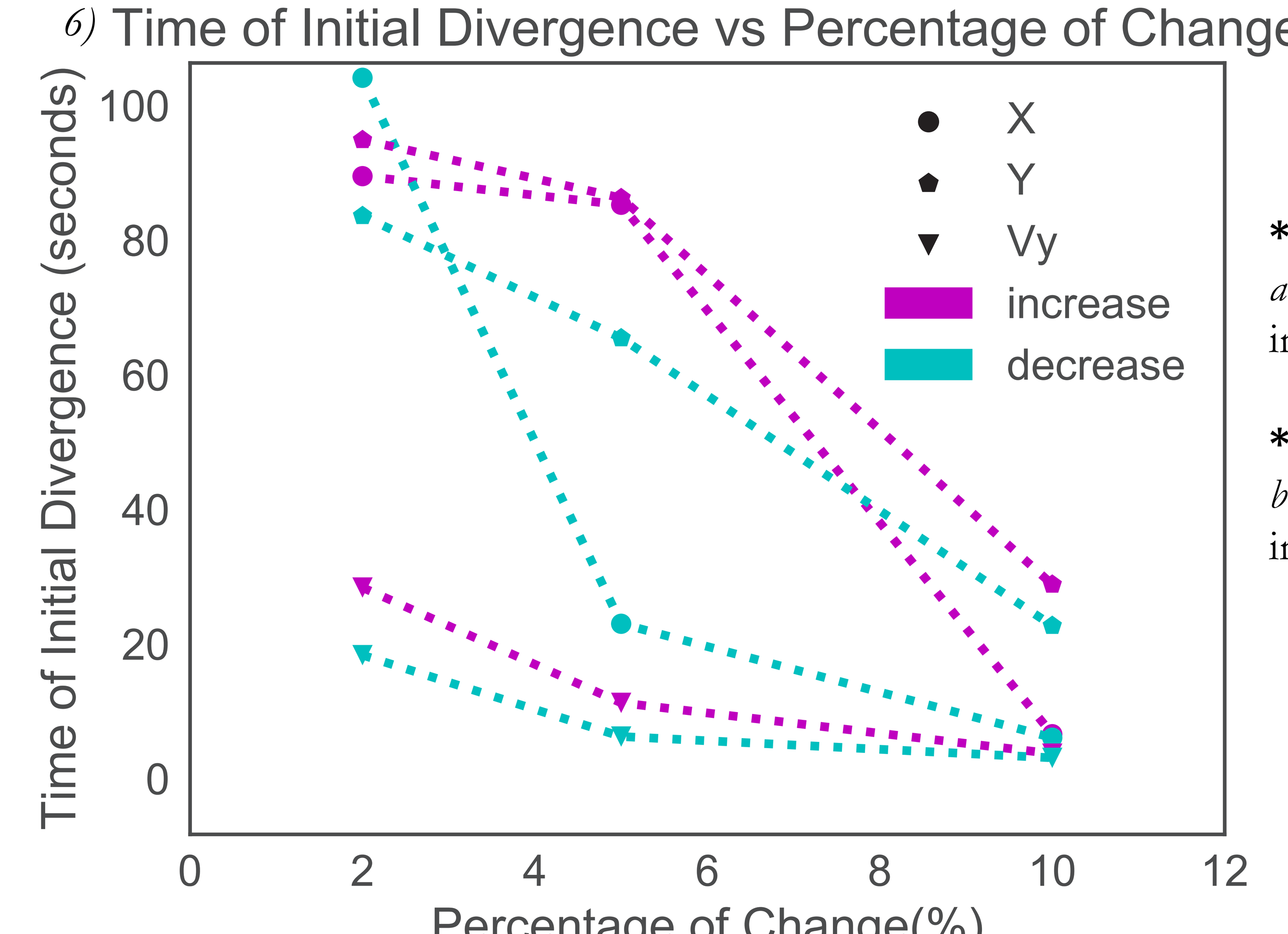
5) $$15 \times (\frac{x_{max}}{400})^2$$

IV. Results

Parameters
* Y Position of Satellite Two (10^2km)
* X Position of Satellite Two (10^2km)
* Downward Velocity (Vy) of Satellite Two (10^2km/sec)

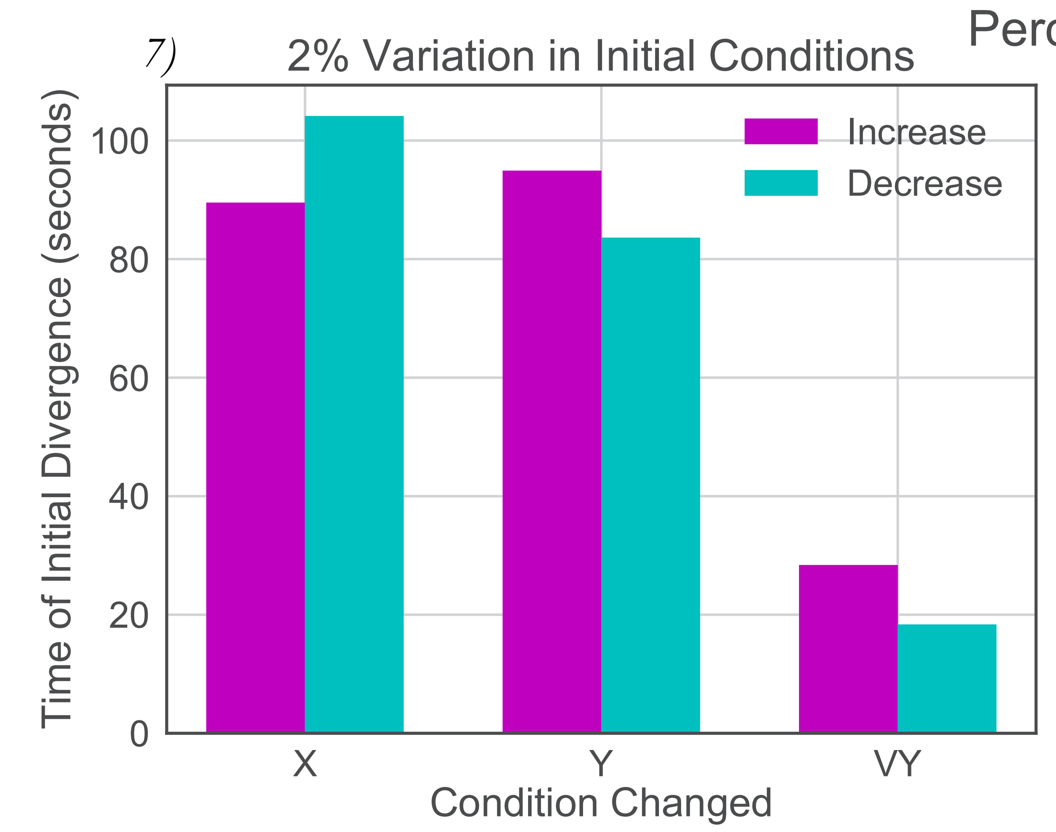
Metric
* Time of Initial Divergence (seconds)

6) Time of Initial Divergence vs Percentage of Change



* **Increase** = vary parameter *above* satellite one's absolute initial conditions

* **Decrease** = vary parameter *below* satellite one's absolute initial conditions

7)


**Bar graph of 2% Variation Data*

The Search for Divergence...

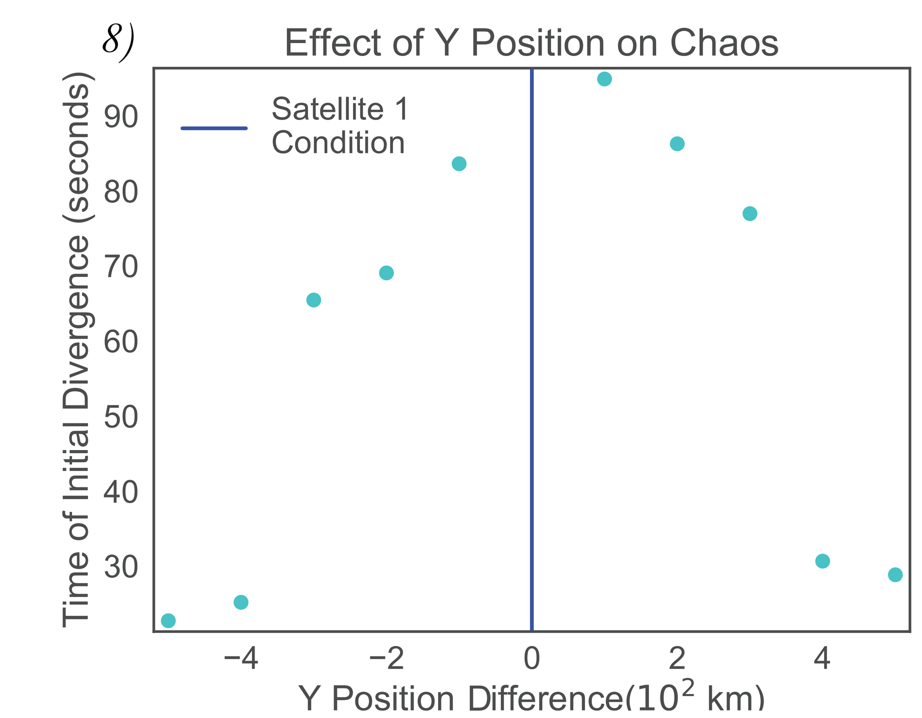
2% bar graph
* Quickest divergence = Vy decrease (18.35 seconds)
* Slowest Divergence = X decrease (104.14 seconds)

5% bar graph
* Quickest divergence= Vy decrease (6.26 seconds)
* Slowest Divergence = Y increase (86.29 seconds)

10% bar graph
* Quickest divergence = Vy decrease (3.09 seconds)
* Slowest Divergence = Y increase (28.85 seconds)

V. Interpretation and Validation

* Across all three percentages of change, a decrease in the absolute numerical value of **Vy** would always result in the shortest time before a significant divergence between the two satellites
* The satellite with the faster initial speed starts with a higher momentum, which allows it to resist the gravitational pull from the stars more than the other satellite, causing a greater impact on the satellites' paths
* The parameter resulting in the longest time before a significant divergence was the same for the 5% and 10% changes (Y increase) but different for the 2% change (X decrease)
* The parameter, position X, had a dramatic drop in time before divergence from 2% to 5%
* A greater difference between the satellites' conditions results in a shorter time of divergence (Fig. 8)

8)


Model Validation

**When the conditions of the satellites are the same, they do not diverge during the simulation.*

VI. Limitations

* Stationary binary stars (reference frame that is rotating along with stars) reduce the complexity of the computationally intensive model significantly
* Unrealistic gravitational constant
* Model only applies to this specific set of conditions
* Inaccuracy caused by run_odeint( )

VII. Conclusion and Future Work

A decrease in the absolute value of the downward velocity resulted in the shortest time before divergence, while an increase in Y position and decrease in X position were found to cause the longest time before divergence

Ideas to improve model
* Test and generalize model with more diverse initial conditions
* Implement rotating binary stars
* Test change in velocity in X direction
* Increase number of satellites

References

* Allain, Rhett. "This Is the Only Way to Solve the Three-Body Problem." Wired, Conde Nast, 3 June 2017, www.wired.com/2016/06/way-solve-three-body-problem/
* 3-Body Gravitational Problem, faraday.physics.utoronto.ca/PVB/Harrison/Flash/Chaos/ThreeBody/ThreeBody.html