

Chapter 4

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(4.A)

(i)

We will assume that there is an $a \cdot b = 0$ such that $a, b \in R$ and that $b \neq 0$ as is the definition of zero divisor. We then show that

$$\begin{aligned}x \cdot y &= 0 \\x' \cdot x \cdot y &= 0 \\1 \cdot y &= 0 \\y &= 0\end{aligned}$$

thus we have arrived a contradiction because we said that $y \neq 0$.

0.1 (ii)

We will show that

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

is a unit matrix by showing that there exists some $A' \in R$ such that $A' \cdot A = 1$. Since we can show that

$$\begin{aligned}A' \cdot A &= 1 \\ \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix} \cdot A &= 1\end{aligned}$$

It cannot be a zero divisor by our theorem in (i).

0.2 (iii)

We will begin by analyzing the nilpotent elements of $\mathbb{Z}/8\mathbb{Z}$