# HOMEWORK 3

# STAT S 520

RAMPRASAD BOMMAGANTY (rbommaga)
4 FEBRUARY 2016

**SOLUTIONS:**

**4.5.10:**

Given that the probability that each guest will accept the invitation = 0.5
The probability that each person who accepts the invitation will attend = 0.8

Also that 7 guests can be accommodated and 12 of them are invited.

$P(Y>7) = 1- P(Y \leq 7) = 1 – pbinom(7,12,0.4) = 0.0573$

**4.5.14:**

Given that there are 25 trials. Also that there are 5 symbols.

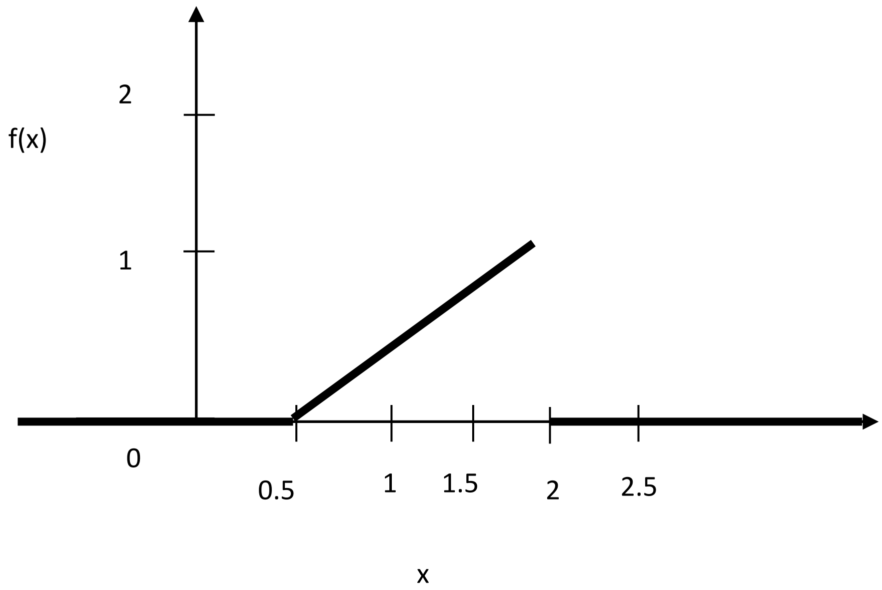
Since all the events are equally likely, the probability = 0.2

a. Number of symbols we can expect the receiver to identify correctly = np = 25 * 0.2 = 5
b. $P(X > 7) = 1 – P(X \leq 7) = 1 – pbinom(7,25,0.2) = 1- 0.890 = 0.109$
c. $P(Y \geq 1) = 1 – P(Y < 1) = 1 – P(Y \leq 0) = 1 – pbinom(0,20,0.2) = 1 – 0.011 = 0.98$

**5.6.2 :**

a.

The graph of f is as follows:




**b** . For f to be a probability density function, $f(x) \geq 0$ condition should be satisfied, which here it does.

Also, the area of the triangle under f(x) should be 1.

Here, Area = (2-1) * 1 = 1

Hence, f is a probability density function.

**c .** $P(1.50<X<1.75) = P(X<1.75) – P(X<1.50)$

Here $P(X < 1.75)$ represents the area under the triangle with base = 0.75 and height = 1.5

Here $P(X < 1.50)$ represents the area under the triangle with base = 0.50 and height = 1

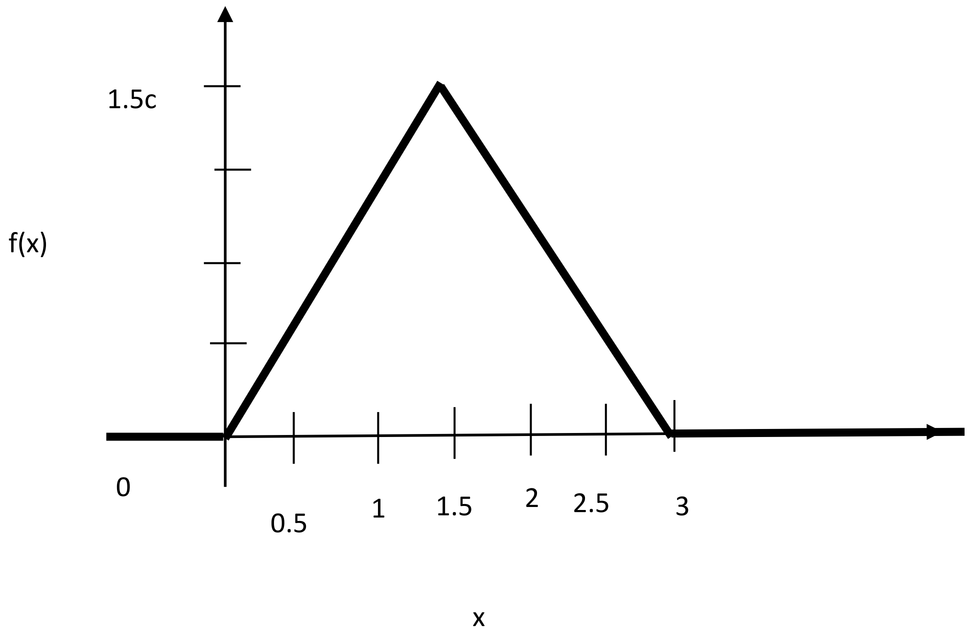
$P(1.50<X<1.75) = (1/2 * 0.75 * 1.5) – (1/2 * 0.5 * 1) = 0.5625-0.25 = 0.3125$

**5.6.3 :**

Given that $f(x) = \begin{cases} 0 , x < 0 \\ cx, 0<x<1.5 \\ c(3-x), 1.5<x<3 \\ 0, x>3 \end{cases}$

For f to be a pdf the area under f has be equal to 1.

The pdf graph is as follows:




Area of the triangles = (1/2 * 1.5 * 1.5c) + (1/2 * 1.5 * 1.5c) = 2.25c

Now, area = 1

2.25c = 1

c = 0.44

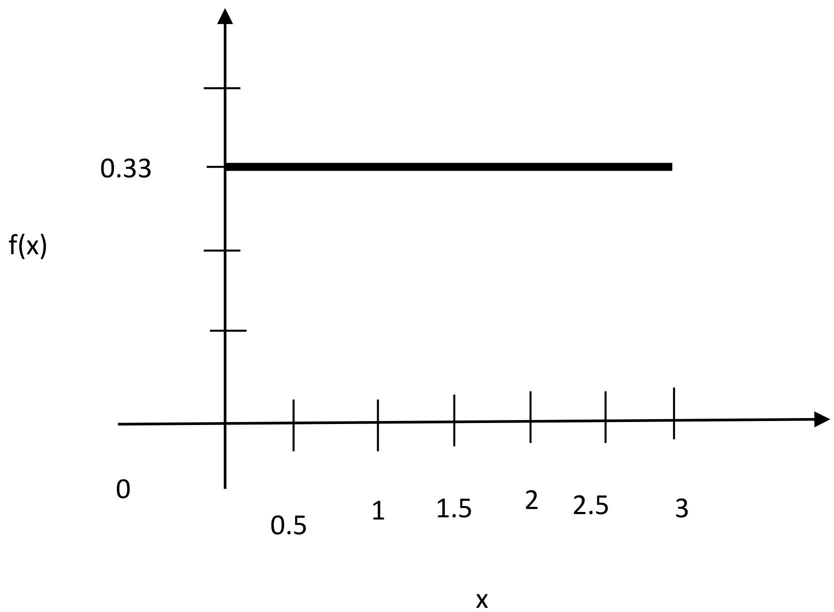
**b .** From the above figure of pdf the expectation can be determined as 1.5

EX = 1.5

**c .** P(X > 2) = The area of the triangle with base = 1 and height = c * (3-2) = 0.44 * 1

P(X > 2) = 0.5 * 0.44 = 0.22

**d .** The pdf of the uniform distribution is as follows:




The height in a uniform distribution is obtained by 1/ b-a = 1 / 3- 0 = 0.33

The expected value of the uniform distribution is b+a/2 where b = 3 and a =0.

Expected value = 1.5

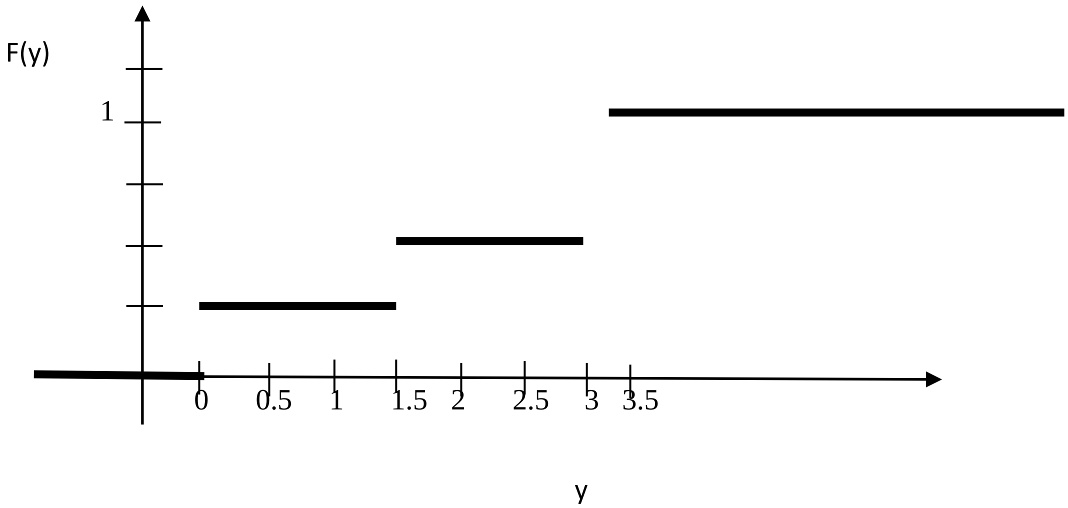
Though the expected values of both X and Y are the same, Y values are more uniformly distributed than X values, hence Y has larger variance.

**e .**

The CDF of X is as follows:

$$F(y) = \begin{cases} 0 & \text{for } y < 0 \\ \frac{1}{2} * y * c * y = 2/9 * y^2 & \text{for } 0 \leq y \leq 1.5 \\ 1 - c/2\,(3-y)^2 = 1 - 2/9 * (3-y)^2 & 1.5 < y \leq 3 \\ 1 & \text{for } y > 3 \end{cases}$$

The graph is as follows:




**5.** a. $F(x) = \sum f(x) = 1$

b. $EX = \sum x * f(x) = (0.1)* 1 + (0.1)*2+(0.3) * 3 + (0.3) * 4 + (0.1) * 5+ (0.1)*6 = 3.5$

We know that Variance = $EX^2 – (EX)^2$

$EX^2 =$ $(0.1)* 1 + (0.1)*4+(0.3) * 9 + (0.3) * 16 + (0.1) * 25+ (0.1)*36 = 14.1$

$(EX)^2 = (3.5)^2 = 12.25$

Var X = 14.1 – 12.25 = 1.85

c . Given that the die has been rolled 10 times and Y = sum of die rolls.

Let $Y = \phi(x)$

$E(\phi(x)) = \sum \phi(x) f(x)$

$EY = 10 * 1 * 0.1 + 10 * 2 * 0.1 + 10 * 3 * 0.3 + 10* 4 * 0.3 + 10 * 5 * 0.1 + 10 * 6 * 0.1 = 35$

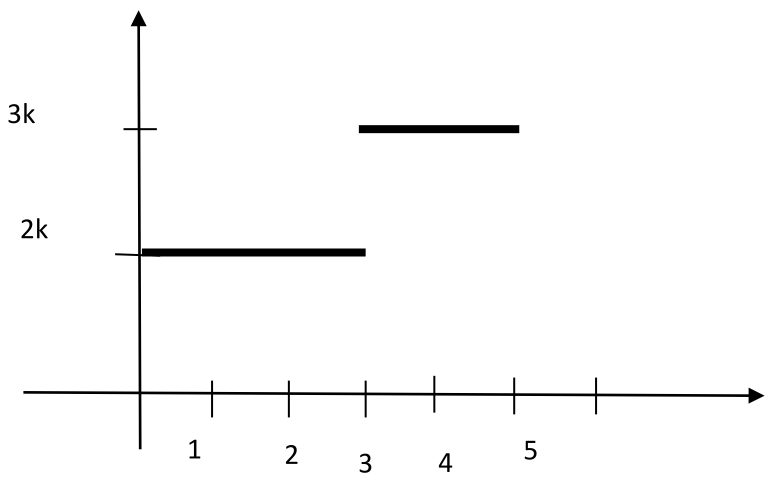
$EY^2 = 10^2 * 0.1 + 20^2 * 0.1 + 30^2 * 0.3 + 40^2 * 0.3 + 50^2 * 0.1 + 60^2 * 0.1 = 1410$

Var Y = $EY^2 – (EY)^2$ = 1410 – 1225 = 185

**6 .**

Given that $f(x) = \begin{cases} 2k & 0 \leq x < 3 \\ 2k & 3 \leq x < 5 \\ 0 & \text{otherwise} \end{cases}$

**a .** The graph of f(x) is as follows:




$P(X = x) = 1 = \sum f(x) \quad = (2k)(3) + (3k)(2) = 12k$

$12k = 1$

$k = 1/12$

**b .**

$P(X \leq 4) = (2k)(3) + (1)(3k) = 2 * 1/12 * 3 + 3 * 1/12 = ½ + ¼ = 3/4$

**c .** For x in [0,3), EX = 1.5 and for x in [3,5), EX = 4

EX = 1.5+4/2 = 2.75