HOMEWORK 2

STAT S 520

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SOLUTIONS:

1: The eight tickets given are:

(1, -), (1, 2), (1, 2), (1, 3), (3, 1), (3, 2), (3, -), (3, -)

The condition for independence of two random variables A and B is : P(A,B) = P(A) P(B).

The distribution of the second number will be the same regardless of the first number, so the possible value at the first blank is 1. For the second blank, it can either be 2 or 3. For the third blank, it can be either 2 or 3. Since, the pattern has to repeat but the order is not important.

3.7.8 : Given that,

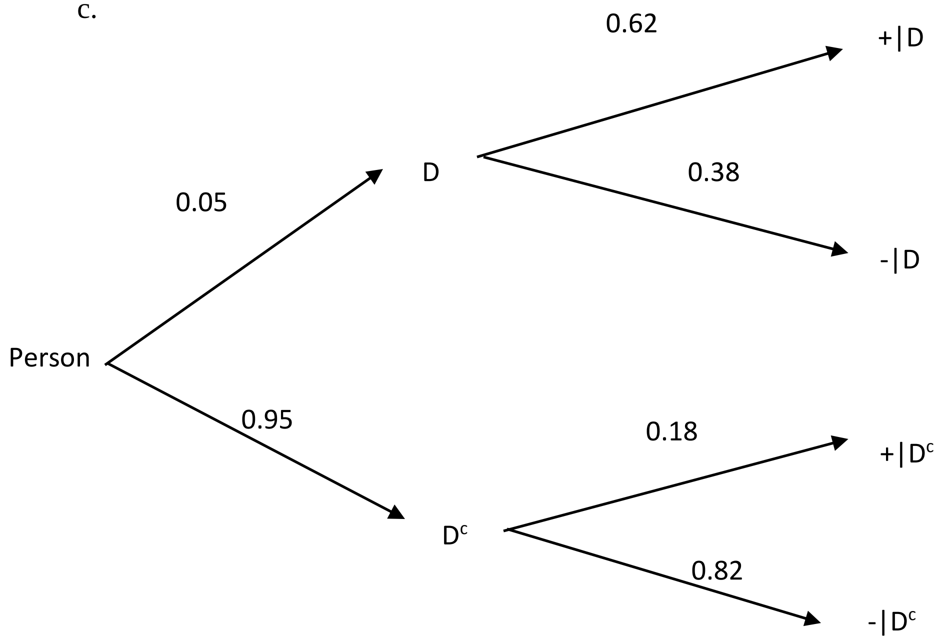
$P(+|D) = 0.62$

$P(-|D^c) = 0.82$

a. $P(+|D^c) = 1 – 0.82 = 0.18$

b. $P(-|D) = 1 -0.62 = 0.38$

c.



d . $P(+) = P(+|D)\, P(D) + P(+|D^{c+})\, P(D^{c})$

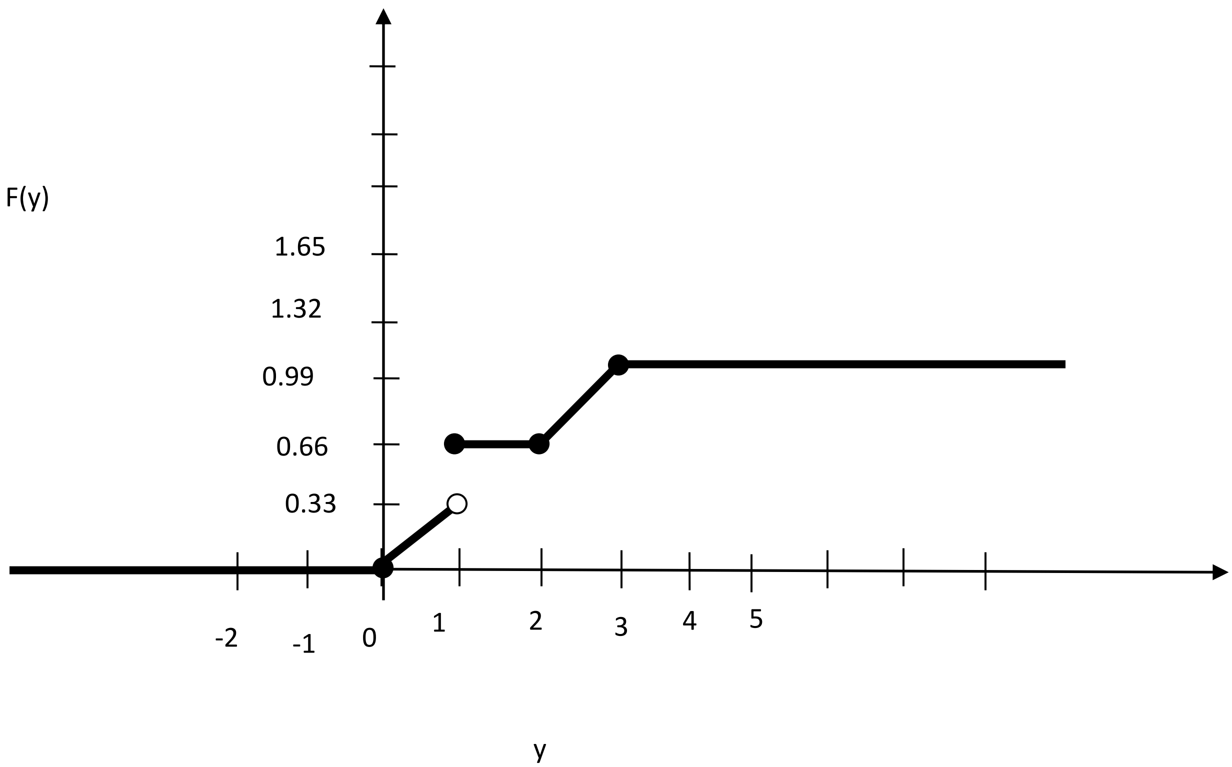
$= 0.62 * 0.05 + 0.18 * 0.95 = 0.031 + 0.171 = 0.202$

e . $P(D|+) = P(D \cap +) / P(+) = P(+|D)\, P(D) / P(+) = 0.62 * 0.05 / 0.202 = 0.031 / 0.202 = 0.1534$

3.7.14 :

Given that,

$$F(y) = \begin{cases} 0, & y \leq 0 \\ y/3, & y \in [0,1) \\ 2/3, & y \in [1,2] \\ y/3, & y \in [2,3] \\ 1, & y \geq 3 \end{cases}$$



a. We can write $P(X > 0.5)$ as $1 – P(X \leq 0.5)$.

$P(X> 0.5) = 1 – 0.16 = 0.84$ (approx.)

b. $P (2 < X \leq 3)$ = This can be written as $P(X\leq 3) – P(X\leq 2) = 1 – 2/3 = 1/3$

c. $P(0.5 < X \leq 2.5)$ = This can be written as $P(X \leq 2.5) – P(X \leq 0.5) = 0.83 – 0.16 = 0.67$

d. $P(X = 1) = 0.66 – 0.33 = 0.33$

4.5.1:

a. Given that $X(S) = \{1,3,4,6\}$

$P(X=1) = P(X = 6) = 0.1$

$P(X=3) = P(X=4) = 0.4$

$$f(x) = \begin{cases} 0.1, \text{ if } x = 1 \\ 0.4, \text{ if } x = 3 \\ 0.4, \text{ if } x = 4 \\ 0.1, \text{ if } x = 6 \\ 0, \text{ for all other values} \end{cases}$$

b. The cumulative distribution function is the probability that X takes a value less than or equal to x.

$$F(x) = \begin{cases} 0, \text{ if } x < 1 \\ 0.1 \text{ if } 1\leq x < 3 \\ 0.5 \text{ if } 3\leq x < 4 \\ 0.9 \text{ if } 4\leq x < 6 \\ 1 \text{ if } x \geq 6 \end{cases}$$

c . $E[x] = 0.1 * 1 + 0.4 * 3 + 0.4 * 4 + 0.1 * 6 = 0.1 + 1.2+ 1.6 + 0.6 = 3.5$

d . Variance of $X = E[x^2] – (E[x])^2$

$E[x^2] = 0.1 *1 + 0.4 * 9 + 0.4 * 16 + 0.1 * 36= 0.1 + 3.6 + 6.4 + 3.6 = 13.7 – (3.5)^2 = 1.45$

e . Standard deviation of X = sqrt(Variance) = sqrt(1.45) = 1.2041

4.5.2:

Given that, $P(X = x) = (7-x)/20$ for $x = 1,2,3,4,5$ and $P(X=6) = 0$.

a. $f(x) =$

$$f(x) = \begin{cases} 0.3, & \text{if } x = 1 \\ 0.25, & \text{if } x = 2 \\ 0.2, & \text{if } x = 3 \\ 0.15 & \text{if } x = 4 \\ 0.1 & \text{if } x = 5 \\ 0, & \text{for } x = 6 \text{ and all other values} \end{cases}$$

b.

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.3 & \text{if } 1 \le x < 2 \\ 0.55 & \text{if } 2 \le x < 3 \\ 0.75 & \text{if } 3 \le x < 4 \\ 0.90 & \text{if } 4 \le x < 5 \\ 1 & \text{if } x \ge 5 \end{cases}$$

c . $E[x] = 0.3 * 1 + 0.25 * 2 + 0.2 * 3 + 0.15 * 4 + 0.1 * 5 = 0.3 + 0.5 + 0.6 + 0.6 + 0.5 = 2.5$

d . Variance of X $= E[x^2] - (E[x])^2$

$E[x^2] = 0.3 * 1 + 0.25 * 4 + 0.2 * 9 + 0.15 * 16 + 0.1 * 25 = 0.3 + 1 + 1.8 + 2.4 + 2.5 = 8$

$(E[x])^2 = 6.25$

Variance $= 8 - 6.25 = 1.75$

e . Standard deviation = sqrt(1.75) = 1.322

4.5.4 :

a. There are ${}^{25}C_{12}$ ways of selecting 12 persons from a pool of 25 people.

If the two students are selected, then there are 10 people left to choose and they can be selected from the remaining people in ${}^{23}C_{10}$ ways.

Therefore, probability that both students will be selected = ${}^{23}C_{10}$/ ${}^{25}C_{12}$ = 0.22

b. The possible combinations are as follows:

1. 4 employees, 8 retired and 0 other people
2. 3 employees, 6 retired and 3 other people
3. 2 employees, 4 retired and 6 other people

Case 1: The number of ways of selecting 4 employees, 8 retired and 0 others :

$${}^{6}C_{4} * {}^{12}C_{8} * {}^{7}C_{0} = 7425$$

Case 2: The number of ways of selecting 3 employees, 6 retired and 3 other people :

$${}^{6}C_{3} * {}^{12}C_{6} * {}^{7}C_{3} = 20 * 924 * 35 = 646800$$

Case 3 : The number of ways of selecting 2 employees, 4 retired and 6 other people:

$${}^{6}C_{2} * {}^{12}C_{4} * {}^{7}C_{6} = 15 * 495 * 7 = 51975$$

P(jury will contain exactly twice as many retired persons as employees) = 7425 + 646800 + 51975/ 5200300 = 0.1357 .

RESOURCES:

1. An Introduction to Statistical Inference and its Applications with R, Michael W. Trosset.