

STAT S 520
HOMEWORK 11
RAMPRASAD BOMMAGANTY(rbommaga)

Exercise 13.4 Problem 2:

Solution:

The observed counts are as follows :

| Brown | Yellow | Red | Blue | Orange | Green |
|-------|--------|-----|------|--------|-------|
| 121 | 84 | 118 | 226 | 226 | 123 |

The expected counts are:

| Brown | Yellow | Red | Blue | Orange | Green |
|--------|--------|--------|--------|--------|--------|
| 116.74 | 125.72 | 116.74 | 215.52 | 179.60 | 143.68 |

The R code for the computation is as follows:

```
> observed= c(121,84,118,226,226,123)
> total <- sum(observed)
> expected<-c(0.13*total,0.14*total,0.13*total,0.24*total,0.20*total,0.16*total)
> G2 <- 2* sum(observed*log(observed/expected))
> 1-pchisq(G2,df=5)
[1] 1.141029e-05
```

Since the p-value is small compared to conventional significance level values, we can reject the null hypothesis that the values are credible and conclude that the claimed proportions are not credible in light of the data.

Exercise 13.4 Problem 5:

Solution:

- a. To check that the given function is indeed a pmf we can use the following code:

```

> x1<- log10(1+(1/1))
> x2<- log10(1+(1/2))
> x3<- log10(1+(1/3))
> x4<- log10(1+(1/4))
> x5<- log10(1+(1/5))
> x6<- log10(1+(1/6))
> x7<- log10(1+(1/7))
> x8<- log10(1+(1/8))
> x9<- log10(1+(1/9))
> sum(x1,x2,x3,x4,x5,x6,x7,x8,x9)
[1] 1

```

Since the sum totals to 1, the given function is indeed a pmf.

b. Given observed count values:

| Leading digit | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|-----|----|----|----|----|----|----|----|----|
| Number of towns | 107 | 55 | 39 | 22 | 13 | 18 | 13 | 23 | 15 |

Using the null hypothesis that the leading digits of town populations follow Benford's law, we proceed.

The expected counts are as follows:

| Leading digit | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Number of towns | 91.81 | 53.70 | 38.10 | 29.55 | 24.15 | 20.41 | 17.68 | 15.60 | 13.95 |

The R code for the computation of the p-value is as follows:

```

> o <-c(107,55,39,22,13,18,13,23,15)

> e <-c(91.81,53.70,38.10,29.55,24.15,20.41,17.68,15.60,13.95)

> G2 <- 2*sum(o*log(o/e))

> 1-pchisq(G2,df=8)
[1] 0.04767116

```

Since, the p value is less than conventional values of significance level, we can reject our null hypothesis and conclude that it is not plausible that leading digits of town populations follow Benford's law.

Exercise 13.4 Problem 11:

Solution: Given $n = 538$

The observed counts are as follows:

| | Positive | Partial | None |
|----|----------|---------|------|
| LP | 74 | 18 | 12 |
| NS | 68 | 16 | 12 |
| MC | 154 | 54 | 58 |
| LD | 18 | 10 | 44 |

Using the formula $e_{ij} = o_{i+}o_{+j} / n$, we compute the expected counts as follows:

| | Positive | Partial | None |
|----|----------|---------|-------|
| LP | 60.69 | 18.94 | 24.35 |
| NS | 56.02 | 17.48 | 22.48 |
| MC | 155.24 | 48.45 | 62.29 |
| LD | 42.02 | 13.11 | 16.86 |

The G^2 value is computed by doing $o_{ij} * \log(o_{ij} / e_{ij})$ for each cell and adding all of them and multiplying by 2.

The computed G^2 value is 68.48.

The R code to compute p-value is :

```
> 1-pchisq(68.48,df=6)
```

```
[1] 8.377743e-13
```

The df value is taken as 6, since $r = 4$ and $c = 3$, hence $(4-1)*(3-1) = 6$.

Since the p-value is quite low, we can reject our null hypothesis and conclude that the treatment to Hodgkin's disease does vary by histological type.

Problem 4:

a.

Given $n = 8474$

The observed values are as follows:

| | Heart Disease | No Heart Disease |
|----------------|---------------|------------------|
| Low Anger | 53 | 3057 |
| Moderate Anger | 110 | 4621 |
| High Anger | 27 | 606 |

Using the formula $e_{ij} = o_{i+}o_{+j} / n$, we compute the expected counts as follows:

| | Heart Disease | No Heart Disease |
|----------------|---------------|------------------|
| Low Anger | 69.73 | 3040.26 |
| Moderate Anger | 106.07 | 4624.92 |
| High Anger | 14.19 | 618.80 |

The X^2 value is computed by doing $(o_{ij}-e_{ij})^2/e_{ij}$ for each cell and adding all of them.

The computed X^2 value is 16.06

The degrees of freedom is : $(3-1) * (2-1) = 2$

The R code for p-value is as follows:

```
> 1-pchisq(16.06,df=2)
[1] 0.0003255482
```

Using conventional significance levels, it can be seen that the p-value is quite low. Hence, we can reject our null hypothesis and conclude that anger is associated with heart disease.

b.

No, this analysis alone does not prove that anger affects the chance of heart disease. Heart disease can occur due to multiple factors apart from anger like obesity, unhealthy lifestyle, stress levels etc. to name a few. While the above test does prove that anger is associated with heart disease, it can be said anger alone is not the cause for heart disease.

Problem 5:

Solution:

a. The R code is as follows:

```
> EPL201415 <- read.csv("http://www.football-data.co.uk/mmz4281/1415/E0.csv")
> home <- EPL201415$FTHG
> away <- EPL201415$FTAG
> N = c(home, away)
> maximum = max(home)
> observed = vector()
> for (i in 0:maximum) {observed = c(observed,length(home[home == i]))}
> observed
[1] 92 119 102 46 12 5 3 0 1
> games = sum(observed)
> goals = sum((0:maximum) * observed)
> average= goals/games
> e = games * dpois(0:20, average)
> o = c(observed[1:5],sum(observed[6:9]))
> e = rep(NA,6)
> e[1:5] = games * dpois(0:4, average)
> e[6] = games * (1-ppois(4,average))
> X2 = sum((o-e)^2/e)
> 1-pchisq(X2, df= 4)
[1] 0.4131955
```

Since the p value is much greater than conventional significance level, we can accept our null hypothesis that the home goals are a fit for the Poisson model.

b.

The R code is as follows:

```
> EPL201415 <- read.csv("http://www.football-data.co.uk/mmz4281/1415/E0.csv")
> away <- EPL201415$FTAG
> maximum = max(away)
> observed=vector()
> for(i in 0:maximum){ observed = c(observed,length(away[away == i]))}
> games = sum(observed)
> goals = sum((0:maximum) * observed)
> average = goals/games
> e = games * dpois(0:20, average)
> o = c(observed[1:4],sum(observed[5:7]))
> e = rep(NA,5)
> e[1:4] = games * dpois(0:3, average)
> e[5] = games * (1-ppois(3,average))
> X2 = sum((o-e)^2/e)
> 1-pchisq(X2, df= 3)
[1] 0.7566041
```

Since the p value is much greater than conventional significance level, we can accept our null hypothesis that the away goals are a fit for the Poisson model.

c.

The R code is as follows:

```
> total = c(home,away)
> maximum = max(total)
> observed=vector()
> for(i in 0:maximum){observed = c(observed,length(total[total == i]))}
> games = sum(observed)
> goals = sum((0:maximum) * observed)
> avg = goals/games
> e = games * dpois(0:20, avg)
> o = c(observed[1:5],sum(observed[6:8]))
>
> e = rep(NA,6)
```

```
> e[1:5] = games * dpois(0:4, avg)
> e[6] = games * (1-ppois(4,avg))
> X2 = sum((o-e)^2/e)
> 1-pchisq(X2, df= 4)
[1] 0.4312175
```

Since the p value is much greater than conventional significance level, we can accept our null hypothesis that the total goals are a fit for the Poisson model.

Problem 6:

