Optimization and Algorithms

Operation and Configuration of a Storage Portfolio via Convex Optimization

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***Abstract*—A portfolio of storage devices (batteries) is used to modify a commodity flow (energy) with the purpose of minimizing an average cost function. The batteries have a set of individual parameters that differ according to their size. The utilization of such a portfolio with this goal has two main problems to be considered. First is its real-time response, in terms of charging and discharging of batteries, to variations of price, requested energy, between others. The second problem is finding the best configuration of the portfolio within the possible ones, in which each portfolio has a given cost according to the parameters of the batteries it is constituted of. Both problems are solved using convex optimization.**

Keywords: Convex optimization; Energy management system

I. Introduction

It is proposed the use of a portfolio, constituted by multiple storage devices, that operate in a way that allows us to modify how the flow of a certain commodity is done in order to minimize an average cost function. There are two problems to be considered. The first one is the real time response of the portfolio, in terms of charging and discharging of batteries, when parameters such as the price and requested energy vary along the time. The second one is choosing the best portfolio configuration within the possible ones. It is important to choose a good portfolio configuration since larger ones may have better results but are more expensive. The goal is to find a good trade-off between portfolio cost and average operating cost. The portfolio was operated using a method called receding horizon control (RHC), in which we solve an optimization problem at each time step in order to define a set of actions to be made over a finite time horizon. The methodology presented can be applied to several topics. In this case, it is applied to an Energy System, in which the commodity is energy and the storage devices are batteries. In power systems, batteries are used to produce an energy output flow with certain desired characteristics given an input energy flow and this is used as a solution for many technical aspects as referred in [1]. We emphasize the usefulness of this method in terms of optimizing energy systems operations in order to reduce costs and in terms of promoting a better use of energy, an ever growing and important concern. Other application of this method could be related to the stock market, in which the commodity would be stocks and the storage devices would be the personal financial portfolios, between other possibles.

The paper is organized in the following way. In section II we introduce the system model, the variables in use and formulate the problem. In section III we describe the taken approach to solve this problem. Numerical results are presented in section IV.

II. Problem formulation

A. Modelling Storage System

In this problem is considered a portfolio of n storage devices, as may be seen in figure 1.

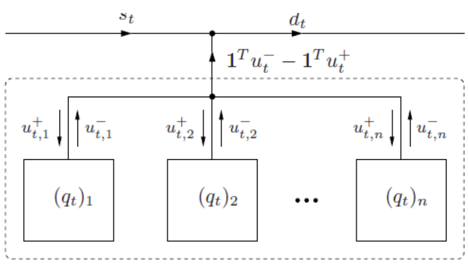


Fig. 1: Portfolio of n storage devices

The whole system dynamics's can be formulated as an equation by

$$q_{t+1} = \eta^l \circ q_t + \eta^c \circ u_t^+ \frac{1}{\eta^d} \circ u_t^- \tag{1}$$

where $q_t \in \mathbb{R}^n$ is the vector of charge levels, $u_t^+ \in \mathbb{R}^n$ is the vector of charging rates (inflows into the storage devices) and $u_t^- \in \mathbb{R}^n$ is the vector of discharging rates (outflows from the storage devices) at time t. The values of η^l, η^c and η^d are constants for each battery that characterize storage leakage, charging and discharging efficiencies, respectively. The range of values for these constants is $[0, 1[$. For simplicity and coherence of results, it is assumed that every battery is completely discharged at beginning, $q_0 = 0$.

The energy balance of the whole portfolio is characterized by applying the *Conservation of Energy* principle, which is defined as a constraint in equation 2,

$$d_t - s_t + \mathbf{1}^T u_t^+ - \mathbf{1}^T u_t^- = 0 \tag{2}$$

where $s_t \in \mathbb{R}^n$ represents the quantity of energy pulled from the source at time t and $d_t \in \mathbb{R}^n$ is the quantity of energy delivered.

B. Defining Physical Constraints

Every battery has physical limits when it comes to storing charge. For the grid in which the power flows there are some physical limitations as well. The ranges of values allowed by the batteries for some of the parameters aforementioned are formulated as mathematical constraints in equation 3.

$$0 \leq q_t \leq Q,\ 0 \leq u_t^+ \leq C,\ 0 \leq u_t^- \leq D, \quad (3)$$

t= 0, 1, ...,T

where $Q \in \mathbb{R}^n$ is the vector of storage capacities, $C \in \mathbb{R}^n$ is the vector of maximum charging rates and $D \in \mathbb{R}^n$ is the vector of maximum discharging rates. It is worth noting that this values may differ for each kind of battery. It is also worth noting that for matters of optimization, (1) is also being treated as a constraint.

C. Cost Function

The main objective is to minimize the operation cost of portfolio of *n* batteries. That cost can be described by the sum of the cost of energy pulled from the source, $\phi_t^{sr}(s_t)$, and of a cost that depends on the amount of delivered energy, $\phi_t^{de}(d_t)$. This can be formulated as

$$l_t(s_t, d_t) = \phi_t^{sr}(s_t) + \phi_t^{de}(d_t). \quad (4)$$

Regarding this two parameters it's worth specifying more about them.

In the case of $\phi_t^{sr}(s_t)$, it was for a first situation defined as

$$\phi_t^{sr}(s_t) = \begin{cases} p_t\, s_t, & 0 \leq s_t \leq S^{\max} \\ +\infty, & \text{otherwise} \end{cases} \quad (5)$$

and for a second situation defined as

$$\phi_t^{sr}(s_t) = \begin{cases} p_t\, s_t, & 0 \leq s_t \leq 0.7 \\ 2\, p_t\, s_t - 0.7\, p_t & 0.7 < s_t \leq S^{\max} \\ +\infty, & \text{otherwise} \end{cases} \quad (6)$$

where we have defined $S^{\max}$ as the maximum value of energy that can be pulled from the source at an instance *t*.

In the case of $\phi_t^{de}(d_t)$, this was defined as

$$\phi_t^{de}(d_t) = \alpha(r_t - d_t)_+ \quad (7)$$

where r_t is the amount of energy requested at time *t* and α works as a penalty for not meeting the required demand of energy.

The formulation of the problem is, now, completed, and it can be mathematically described by

$$\begin{array}{ll} \underset{q_t, s_t, d_t, u_t^+, u_t^-}{\text{minimize}} & \frac{1}{T}\sum_{t=0}^{T} l_t(s_t, d_t) \\ \textit{subject to} & q_0 = 0 \\ & 0 \leq q_t \leq Q,\ 0 \leq u_t^+ \leq C \\ & 0 \leq u_t^- \leq D,\ t = 0, ..., T \\ & d_t - s_t + \mathbf{1}^T u_t^+ - \mathbf{1}^T u_t^- = 0 \\ & q_{t+1} = \eta^l \circ q_t + \eta^c \circ u_t^+ - (1/\eta^d) \circ u_t^- \end{array} \quad (8)$$

III. Approach

The formulated problem is convex, so, it can be efficiently solved and implemented in MATLAB using "CVX: Matlab Software for Disciplined Convex Programming" [2].

IV. Numerical results

In order to build our portfolios of storage devices we will assume that there are three kinds of batteries, which one with different characteristics. Those storage devices will be referred by their storage capacity as small (S), medium (M) and large (L). We'll be using the same values that were used by [1]. The parameter values for each kind of battery can be seen in the table I

device	Q	C	D	η	η_c	η_d	J^{cap}/unit
S	1	0.5	0.5	0.995	1	1	2
M	2	0.5	0.5	0.99	0.9	0.9	3
L	5	0.75	0.75	0.98	0.8	0.8	5

TABLE I: Storage devices parameters

A interval of time from 0 to 120 hours was used, with a step of 30 minutes, corresponding to five days. We have defined r_t and p_t as proposed by [1] as

$$\begin{aligned} r_t &= \exp(0.2 + 0.4\cos(2\pi\, t/48 - 5\pi/4)) \\ p_t &= \exp(0.15 + 0.4\cos(2\pi\, t/48 - 3\pi/2)) \end{aligned} \quad (9)$$

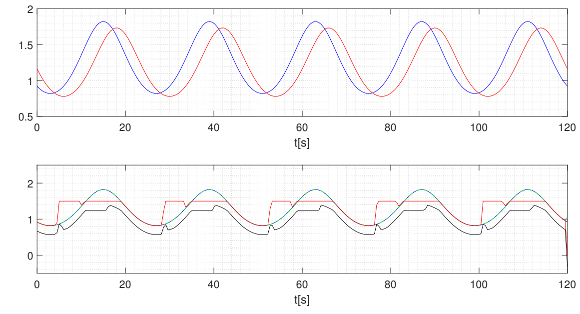
where it is tried to replicate a somewhat real behaviour of the energy demand. Additionally, α was also set to 20, a value expected to be much bigger than p_t and the maximum amount of energy than be pulled from the source at each instant of time *t*, $S^{\max}$ is set to 1.5.

Using the approach presented in III, there were conducted simulations for two important aspects, the time evolution of the system behaviour, including the charge state and choosing the optimal portfolio.

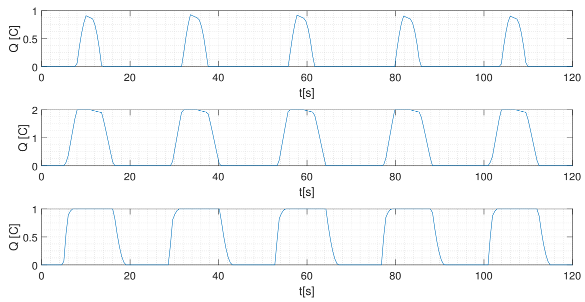
A. Time evolution of charge and costs for a standard portfolio

Using a basic portfolio with $n = 3$ constituted by one small battery, one medium battery and one large battery, simulations were made to see the time evolution of the charge of batteries, as well as the evolution of the demand and price of energy, for the two situations of $\phi_t^{sr}(s_t)$ mentioned.

Figure 2 shows the evolution with time, for a period of 5 days, of several parameters for two proposed situations of $\phi_t^{sr}(s_t)$ in section II according to equation 5. The simulation for the situation with equation 6 was similar and it's not shown. As it can be seen in figure 2 (a) the requested energy was always delivered. We observe that the energy delivered without using batteries is more relevant than the energy delivered from storage. The fact that there are no excessive charging/discharging penalties or an energy storage cost, encourages the system to buy energy when it has a low price, only limited by the the total amount of energy it can pull from the source at each instant *t* and the amount of energy that it can hold. It is worth noting that when p_t reaches a certain price, the amount of energy pulled from the source lowers and

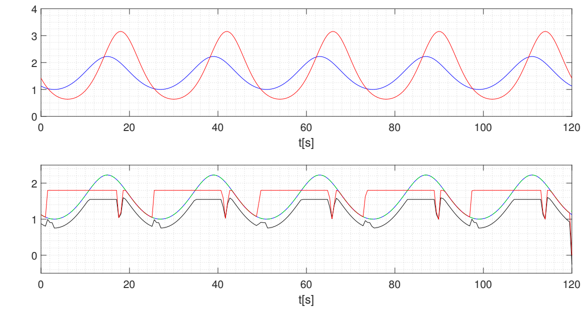


(a) Top: Demand (blue), price (red), Bottom: Demand (blue), energy delivered (green), energy delivered without using stored energy (black), energy pulled from source (red)



(b) Storage device charges profiles for basic portfolio. Top: $(q_t)_L$, Middle: $(q_t)_M$, Bottom: $(q_t)_S$.

Fig. 2: Parameters time evolution according to equation 5.



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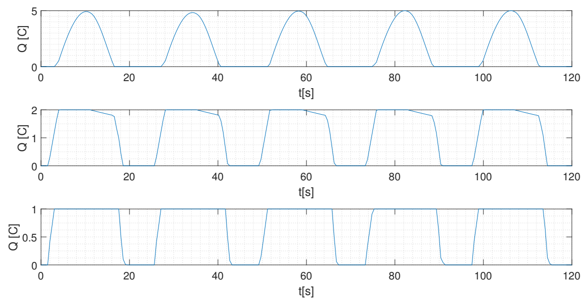


(b) Storage device charges profiles for basic portfolio. Top: $(q_t)_L$, Middle: $(q_t)_M$, Bottom: $(q_t)_S$.

Fig. 3: Time evolution of several parameters for a stricter scenario according to equation 5.

it begins rising again when the price is lower. It is also worth noting, that since storage leakage is to be taken in account, the system also tries to charge the batteries as close as possible as the time requested energy starts to increase.

Analyzing figure 2 (b) it can be observed that when the price is lower and we are approaching a peak of request energy, batteries tend to be fully charged or close to it, minus the large battery, which is normal, since it can hold more energy than the others and has worst η parameters. We can also observe that the discharging periods occur close to the moments in which the energy pulled from the source has an higher price.

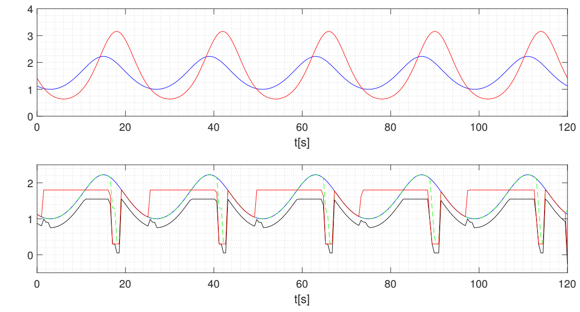
The price evolution with time affected the behaviour of the portfolio, but has not reached a value that obliged its response to be more drastic. It was therefore decided to do another simulation, for the same portfolio, but with stricter constraints. We have changed the $S^{\max}$ to 1.8, increased the requested energy to $r_t = \exp(0.4 + 0.4\cos(2\pi\, t/48 - 5\pi/4)$, the price of energy to $p_t = \exp(0.35 + 0.8\cos(2\pi\, t/48 - 3\pi/2)$ and define $\phi_t^{sr}(s_t)$ as

$$\phi_t^{sr}(s_t) = \begin{cases} p_t\, s_t, & 0 \leq s_t \leq 0.3 \\ 6.5\, p_t\, s_t - 1.65\, p_t, & 0.3 < s_t \leq S^{\max} \\ +\infty. & \text{otherwise} \end{cases} \tag{10}$$

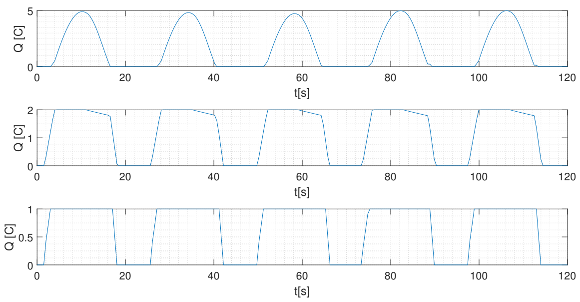
With this scenario there are lot of more conclusions to be drawn and the behaviour is clearly different. It can be seen that in the situation of figure 3 (a) the demand for energy is fulfilled but not in the case of figure 4 (a). Not that the system could not pull energy from the source, but because the penalty $\phi_t^{de}(d_t)$ from not meeting the demanded energy was lower than the cost of pulling energy from the source. It can also be seen that the system is pulling energy from the source at its allowed maximum for longer periods of time, since it has to fulfill a slightly higher demand and has to keep energy stored for the price peaks, now more relevant. Another thing that can be noticed is that in figures 3 (b) and 4 (b), batteries used for longer periods of time, and respond more accordingly to price variations, since price has now a value high enough to provoke drastic behaviour responses by the portfolio.

As expected too, in this cases the average cost of operation is a lot higher. The situation of figure 4 had a cost of 14.7 and the situation of figure 3 a cost of 2.68. As a measure of comparison, figure 2 had an average operation cost of 1.59 and a case in the same scenario with equation 6 applied had 2.33. This were situations in a more favorable scenario, so this was expected.

Another scenarios would be interesting to simulate, such as a situation in which $S^{\max}$ is severely limited, an hypothetical situation in which price and requested energy have a bigger difference of phase between them, seeing how a change in the parameter α affects the ratio of requested energy that is delivered, and see if the behaviour of the model goes accordingly to our intuition, but unfortunately there is no opportunity to present all of them here.



(a) Top: Demand (blue), price (red), Bottom: Demand (blue), energy delivered (green), energy delivered without using stored energy (black), energy pulled from source (red)



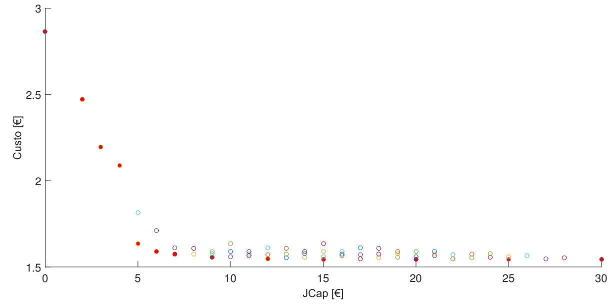
(b) Storage device charges profiles for basic portfolio. Top: $(q_t)_L$, Middle: $(q_t)_M$, Bottom: $(q_t)_S$.

Fig. 4: Time evolution of several parameters for a stricter scenario according to a modified equation 5.

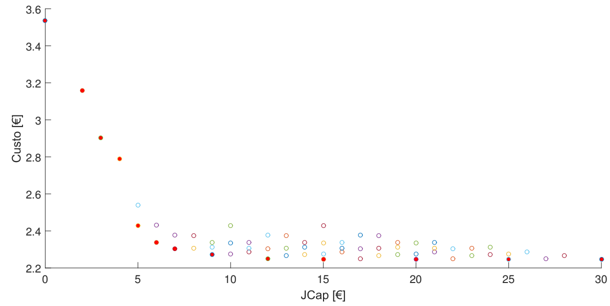
B. *Finding optimal amount of storage capacity*

In figure 5 it is represented the system's 5-day operating cost evolution for all 64 candidate portfolios with the increase of capital cost. Notice that this value is obtained since we can choose 0, 1, 2 or 3 batteries of each kind to form a portfolio. The definition of Pareto efficiency was used to obtain the optimal configurations, which are highlighted in red.

Analyzing with more depth the case of the figure 5 (a), of the five leftmost Pareto points, with the lowest values of capital cost, J^{cap}, only the fifth can deliver the requested energy at all times. This means that the first four ones are operating at their limits and are not being able to deliver the required energy. It's worth noting that portfolios with higher capacity are able to deliver more of the requested energy, which results in less penalties caused by being unable to deliver the requested amount of energy. But, since most of the portfolios are able to match the requested energy, we get a small dispersion of results of average cost of operations for the several capital costs as we get into portfolios with more storing capability. Focusing now on the three rightmost portfolio configurations they all have an average operating cost of 1.475. This means that we've reached a saturation point, in which adding more storing capacity to the system will not be beneficial and on the contrary, will only increase the capital costs without benefiting us. So the best portfolio we could use in this situation, if we wanted minimum average operating costs with the lowest possible capital costs, would be a portfolio of 3 small batteries,



(a) Capital vs average operating costs, ϕ_t^{sr} according to equation 5.



(b) Capital vs average operating costs, ϕ_t^{sr} according to equation 6.

Fig. 5

3 medium batteries and one 1 large battery.

Applying similar logic to the case of the figure 5 (b), the observations will all be the same, with the slight change that average operating costs will be higher due to the fact of the cost of pulling energy from the source being higher in this case. The three rightmost points will all have average operating costs of 2.11, and by the same reasons mentioned to the previous case, the same portfolio will be the optimal one.

This kind of simulation can be equally useful to show which configuration is the optimal one for a given possible capital cost a client would be willing to invest.

V. Conclusions

In this paper were presented methods, using convex optimization, that allow us to decide how to operate and choose a portfolio of n energy storage devices requiring the minimum average operating cost while having the lowest capital cost. Numerical results show that for a given set of conditions it's possible to find an optimal portfolio and that from a certain point, adding more storage capacity to the system is not economically viable.

References

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[2] Michael C. Grant, Stephen P. Boyd, *The CVX Users Guide* available at `http://cvxr.com/cvx/doc/CVX.pdf`, 2016