

STUDENT'S ID NO: _____ SIGNATURE: _____



UNIVERSITY OF GHANA

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DEPARTMENT OF TEACHER EDUCATION

SCHOOL OF EDUCATION AND LEADERSHIP

COLLEGES OF EDUCATION

END OF SEMESTER ONE EXAMINATIONS FOR LEVEL 200, 2021/2022

B.ED. PROGRAMME

COURSE CODE: **TEJS 245**

COURSE TITLE: **LEARNING, TEACHING AND APPLYING FURTHER ALGEBRA**

Instruction: Answer all questions in Section A and any three questions in Section B.

Time: 2 hours

SECTION A

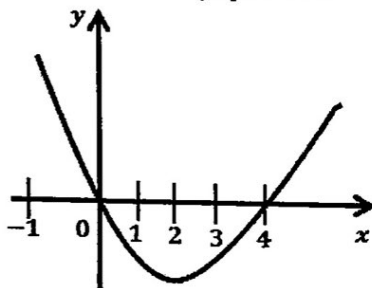
[25 Marks]

Answer all the questions in this section.

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1. A binary operation $\%$ is defined over the set \mathbb{R} of real numbers by $a \% b = a + b + ab$. Find the identity element under the operation $\%$.
- A. 0
 - B. 1
 - C. 2
 - D. 3

2. Find the zeros of the graph below:



- A. -1 and 4
 - B. 0 and -4
 - C. 2 and 0
 - D. 0 and 4
3. The first four terms of a linear sequence are 5, 9, 13, 17. What is the tenth term of the sequence?
- A. 29
 - B. 30
 - C. 41
 - D. 45
4. Find the n th term of the Geometric Progression $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$
- A. 2^{n-1}
 - B. 2^n
 - C. $\frac{1}{2^n}$
 - D. $\frac{1}{2^{n-1}}$

5. Calculate the product of the third and fourth terms of the sequence

$$T_n = 2(2 + 3^{(n-1)}).$$

- A. 1616
- B. 1352
- C. 1276
- D. 1160

6. What is the binomial term $(p - q)^4$ in expanded form?

- A. $P^4 - 4P^3q + 6p^2q^2 - 4pq^3 + q^4$
- B. $q^4 - 4P^3q + 6p^2q^2 - 4pq^3 + q^4$
- C. $P^4 + 4P^3q + 6p^2q^2 - 4pq^3 + q^4$
- D. $P^4 - 4P^3q + 6p^2q^2 - 4pq^3 + q^4$

7. Find the value of m if $(x - 1)$ and $(x + 2)$ are the factors of $x^2 + x + m$.

- A. 2
- B. $\frac{1}{2}$
- C. $-\frac{1}{2}$
- D. -2

8. A particle moving uniformly covers a distance S at time t seconds, given by $S = 24t - 6t^2$. Find the value of t when the particle covers a distance of 24metres.

- A. 4 seconds
- B. 3 seconds
- C. 2 seconds
- D. 1 second

9. If $(x-2)$ and $(x+1)$ are factors of $x^2 + bx + c = 0$. Find the value of $(b + c)$.

- A. -3
- B. -2
- C. 3
- D. 2

10. Find the quadratic equation whose roots are $\frac{1}{2}$ and $-\frac{3}{2}$.

- A. $X^2 + 4X - 3 = 0$
- B. $4X^2 + X - 3 = 0$
- C. $X^2 - 4X - 3 = 0$
- D. $4X^2 + 4X - 3 = 0$

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11. Find the minimum value of $f(x) = 3x^3 - 2x + 1$.

- A. $\frac{2}{3}$
- B. $\frac{2}{3}$
- C. $\frac{1}{3}$
- D. $\frac{1}{2}$

12. Given $f(x) = 3x^3 + 4x^2 - 2x + 8$ and $g(x) = 2x^3 + 3x^2 + 5x + 2$, find $f(x) + g(x)$.

- A. $3x^3 + 2x^2 + 3x + 9$
- B. $3x^3 + 4x^2 - 2x + 8$
- C. $3x^3 - 4x^2 - 2x + 8$
- D. $3x^3 + 4x^2 - 2x + 9$

13. Find the remainder when $x^2 + 2x - 8$ is divided by $x + 3$.

- A. 5
- B. 8
- C. -5
- D. -8

14. What is the value of m if $4 \times 3^m = 324$?

- A. 3
- B. 4
- C. 27
- D. 81

15. Given that $M = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$, $N = \begin{pmatrix} m & x \\ n & y \end{pmatrix}$ and $MN = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$. Find the matrix N .

- A. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- B. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
- C. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- D. $\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$

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16. The matrices A and B are given by $A = \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -5 \\ -7 & 9 \end{pmatrix}$ find $A + B$.

- A. $\begin{pmatrix} 5 & 1 \\ 3 & 17 \end{pmatrix}$
- B. $\begin{pmatrix} 5 & 1 \\ -3 & 17 \end{pmatrix}$
- C. $\begin{pmatrix} -5 & 1 \\ -3 & 17 \end{pmatrix}$
- D. $\begin{pmatrix} 5 & 1 \\ -3 & -17 \end{pmatrix}$

17. If $2^a = \sqrt{64}$ and $\frac{b}{a} = 3$, evaluate $a^2 + b^2$.

- A. 48
- B. 90
- C. 160
- D. 25

18. Expand $(2x + 3)^3$ using the binomial theorem.

- A. $2x^3 + 36x^2 + 54x + 27$
- B. $2x^3 + 18x^2 + 54x + 27$
- C. $8x^3 + 27x^2 + 54x + 9$
- D. $8x^3 + 36x^2 + 54x + 27$

19. A binary operation is defined by $a * b = a^2 - b^2 + ab$, where a and b are real numbers.

Evaluate $\sqrt{2} * \sqrt{3}$

- A. $1 - \sqrt{6}$
- B. $\sqrt{6} - 1$
- C. $\sqrt{6}$
- D. $\sqrt{6} + 1$

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20. The area of a triangle with base length $(x + 3y)$ is $(x^2 + 2xy - 3y^2)$. Determine the height of the triangle in terms of x and y .

- A. $2x - y$
- B. $x - 2y$
- C. $y - 2x$
- D. $2(x - y)$

21. Find the value of k such that the determinant of the matrix $\begin{pmatrix} 4 & 6 \\ -2 & k \end{pmatrix}$ is 24.

- A. -9
- B. -3
- C. 3
- D. 9

22. A binary operation Δ , is defined on the set \mathbb{R} , of real numbers by $a \Delta b = \frac{a}{b} - \frac{b}{a}$ where $a, b \in \mathbb{R}$ and $a, b \neq 0$, Evaluate $2 \Delta \sqrt{2}$

- A. $\frac{1}{\sqrt{2}}$
- B. $\frac{2}{\sqrt{2}}$
- C. $\frac{\sqrt{2}}{4}$
- D. $\frac{\sqrt{2}}{2}$

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23. Given the matrices $A = \begin{pmatrix} -2 & 5 \\ 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -3 \\ -6 & 5 \end{pmatrix}$. Find AB

A. $\begin{pmatrix} -38 & 31 \\ -2 & 3 \end{pmatrix}$

B. $\begin{pmatrix} -20 & -11 \\ 32 & -15 \end{pmatrix}$

C. $\begin{pmatrix} -20 & 11 \\ 32 & -15 \end{pmatrix}$

D. $\begin{pmatrix} -20 & -11 \\ 32 & -15 \end{pmatrix}$

24. Solve for x in the logarithmic equation, $\log_2(25 - x) = 4$.

A. 8

B. 9

C. 17

D. 19

25. If $\log_3 5 = 1.465$, find the value of $\log_3 25 + \log_3 15$.

A. 3.930

B. 4.395

C. 4.930

D. 5.395

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SECTION B

[75 Marks]

Answer any three questions in this section.

1. a. If $(x - 2)$ is a factor of the polynomial function $g(x) = x^3 - ax^2 + 17x + b$ and a remainder of -36 when divided by $(x + 1)$, find the values of a and b .
(5marks)
 - b. Given the matrix $M = \begin{pmatrix} k & 7 \\ 8 & 2 \end{pmatrix}$, find the value of k for which M has no inverse.
(5marks)
 - c. Use the expansion of $(1 - 5x)^5$ to estimate the value of 0.995 , correct to 3 significant figures.
(5marks)
 - c. Find the greatest value of the function $f(x) = 1 - 6x - x^2$.
(3marks)
 - d. At a poultry farm, six hens and one duck cost GHC40.00 while four hens and three ducks cost GHC36.00. What is the cost of each type of bird?
(5marks)
-
2. a. The 3rd and 7th terms of an exponential sequence are 81 and 16 respectively. Find the 1st and the 5th terms, when $r > 0$.
(5marks)
 - b. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, find the value of x .
(5marks)
 - c. Solve $37x^2 - 36x - 7 = 0$ using the method of completing of squares.
(5 marks)
 - d. A marathon runner begins his first day of training by running 2km. He then increases this distance by $\frac{1}{2}$ km more each day from the distance he ran the previous day.
 - i. On what day would he first cover 15km?
(3marks)
 - ii. What is the total distance covered after 30 days?
(3marks)
 - e. Solve the equation $2^{2x} - 9(2^x) + 20 = 0$
(4 marks)

3. a. A binary operation Δ is defined on the set $S = \{0, 1, 2, 3\}$ by:

$$a\Delta b = a + b - ab, \text{ where } a, b \in S.$$

- Determine whether S is closed or not. (3marks)
- Show that Δ is commutative. (3marks)
- Find the neutral element, e , of S . (3marks)
- Find the inverse of the element 3. (3marks)

- b) (i) Find the vertex and the axis of symmetry of the graph of the function

$$f(x) = x^2 - x - 6. \quad (7\text{marks})$$

- (ii) Find the x - and y -intercepts of the graph and sketch the graph. (6marks)

4. a. The base of a triangle is 4cm longer than its height. If the area of the triangle is 48cm^2 , find its base and height. (5marks)

- b. Using binomial theorem, find the value of $(0.99)^{15}$ up to six decimal places.

(5marks)

- c. A factory produced 360 tons of plastic goods in 2001. It then increased its production by 10% every year.

- (i) Write down the number of tons produced for the next three years. (3 marks)

- (ii) Determine the number of tons produced in 2008. (5 marks)

- (iii) Calculate the total production of the factory from 2001 to 2010 (5 marks)

- d. Divide $x^3 + 6x - 20$ by $x - 2$. (2marks)

5. (a) Given that $Z = \begin{pmatrix} 3 & -2 & 0 \\ 1 & -4 & -5 \\ 2 & 1 & -3 \end{pmatrix}$ and $F = \begin{pmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \\ 3 & -2 & 7 \end{pmatrix}$

Evaluate

- (i) ZF (5 marks)

- (ii) FZ (5 marks)

- b. (iii) What conclusion can you draw from the result in (i) and (ii) above? (2 marks)

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c. Solve the system of linear equations using matrix approach and the Cramer's rule.

$$\begin{cases} 5x - 3y = 28 \\ -2x + 4y = 14 \end{cases} \quad (8\text{marks})$$

d. The accumulated value, A , of a sum of money with principal, P , after t years at annual percentage rate r compound n times a year is given by: $A = P \left(1 + \frac{r}{n}\right)^{nt}$. How long will it take GH¢25,000.00 to grow to GH¢500,000 at 9% annual interest compounded monthly? (5 marks)