STUDENT'S ID NO:______________________________ SIGNATURE: ______________

UNIVERSITY OF GHANA

DEPARTMENT OF TEACHER EDUCATION
SCHOOL OF EDUCATION AND LEADERSHIP
COLLEGES OF EDUCATION
END OF SEMESTER ONE EXAMINATIONS FOR LEVEL 300, 2022/2023
B.ED. PROGRAMME

COURSE CODE: **TEJS 323**
COURSE TITLE: **LEARNING, TEACHING, AND APPLYING CALCULUS**

Instruction: Answer **all** questions in Section A and any **three** questions in Section B.
Time: 2 hours

SECTION A
[25 Marks]
Answer all the questions in this section.

1. Which of the following constitute the problems of the main branches of calculus?
   A. Limits and continuity problem.
   B. Minimum and maximum problem.
   C. Movement and volume problem.
   D. Tangent and area problem.

2. Find the first derivative of $y = x^2 - \frac{1}{x^3}$

   A. $\frac{dy}{dx} = 2x - \frac{3}{x^2}$

   B. $\frac{dy}{dx} = x^2 - \frac{1}{x^3}$

   C. $\frac{dy}{dx} = 2x + \frac{3}{x^4}$

   D. $\frac{dy}{dx} = 2x - \frac{3}{x^4}$

3. The limit of a function $f(x)$ is said to exist If ________

   A. $\lim_{x\to+\infty} f(x) = \infty$

   B. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = \infty$

   C. $\lim_{x\to a^-} f(x) = L$ (Where L is the limiting value)

   D. $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$ (Where L is the limiting value)

4. Find $f(x)$ if $\lim_{x\to 2} f(x) = 1776$

   A. $+\infty$

   B. $-\infty$

   C. 888

   D. 1776

5. Evaluate $\lim_{x\to 1} \frac{x^2-1}{x^2+3x-4}$

   A. $1/4$
   B. $2/5$
   C. 0
   D. -3

6. Which of the following functions is NOT continuous everywhere?
   A. $f(x) = \frac{x^2-4}{x+2}$
   B. $f(x) = (x+3)^4$
   C. $f(x) = 1066$
   D. $f(x) = mx + b$

7. For what values of x is there a discontinuity in the graph of $y = \frac{x^2-9}{x^2-5x+6}$
   A. $x = 3, x = -3$
   B. $x = 3, x = -2$
   C. $x = 3, x = 2$
   D. $x = -2, x = 5$

8. Solve $\lim_{x\to 9} \frac{x-9}{3x-\sqrt{x}}$
   A. 0
   B. -3
   C. -6
   D. 6

9. What is the derivative with respect to x of $(x + 1)^3 - x^3$?

   A. $3x + 6$

   B. $3x - 6$

   C. $6x + 3$

   D. $3x^2 + 6x + 3$

10. Implicitly, find $\frac{dy}{dx}$ of the equation $xy = 1$

   A. $-\frac{x}{y}$

   B. $\frac{x}{y}$

   C. $-\frac{y}{x}$

   D. $\frac{y}{x}$

11. Evaluate the $\lim_{x \to 6}(3x^2 + 7x - 16)$.

   A. -6
   B. 44
   C. 62
   D. 134

12. Evaluate $\int_1^3 (3x^2 + 2x + 4)dx$

   A. 28
   B. 42
   C. 46
   D. 54

13. Find the velocity when $t = 2$, for the falling body for which $s = 5t^3$.

   A. 15
   B. 30
   C. 40
   D. 60

14. Given that $\frac{dy}{dx} = 2x + 1$, find the equation of the tangent at the point $(0,1)$.

   A. $y = x - 1$
   B. $y = 1 - x$
   C. $y = 2x + 1$
   D. $y = x + 1$

15. A particle moves along a straight line such that its distance, S, from the origin at any time, t seconds, is given by $S = t^3 - 2t^2 + 5t + 3$. Find the velocity of the particle at the end of the 3rd second.
   A. $12ms^{-1}$
   B. $15ms^{-1}$
   C. $20ms^{-1}$
   D. $27ms^{-1}$

16. The distance, $S\,m$, which a particle covers in $t\,sec$ is given as $S = \frac{3}{2}t^2$. Find the acceleration of the particle.
   A. $2ms^{-2}$
   B. $3ms^{-2}$
   C. $4ms^{-2}$
   D. $5ms^{-2}$

17. Differentiate $y = \sin\sqrt{x}$.
   A. $\cos\sqrt{x}$
   B. $\frac{1}{2}\cos\sqrt{x}$
   C. $\frac{\cos\sqrt{x}}{2}$
   D. $\frac{\cos\sqrt{x}}{2\sqrt{x}}$

18. Evaluate: $\lim_{x\to 1}\frac{1-\cos x}{2x}$
   A. 0
   B. $\frac{-1}{2}$
   C. 2
   D. $\frac{1}{2}$

19. If $y = (t^2 + 2)^2$ and $t = x^{1/2}$, determine dy/dx
   A. $3/2$
   B. $(2x^2 + 2x) / 3$
   C. $2(x + 2)$
   D. $x^{5/2} + x^{1/2}$

20. Find $\frac{dy}{dx}$ if $y = 3e^x$

A. $3e^x$

B. $-3e^x$

C. $3\ln e^x$

D. $-3\ln e^x$

21. In which interval is the following function continuous?

$$f(x) = \frac{x^2}{2x + 4}$$

A. $[-2, 2]$
B. $(-\infty, -2) \cup (-2, +\infty)$
C. $(-\infty, 2) \cup (2, +\infty)$
D. $(-2, 2)$

22. What is the derivative of $Sin(4x^2)$?
A. $8xCos(4x^2)$
B. $-8\,xCos(4x^2)$
C. $8xSin(4x^2)$
D. $-8xSin(4x^2)$

23. If $y = \ln x$, find $\frac{dy}{dx}$

A. $\frac{dy}{dx} = \frac{\ln x}{x}$

B. $\frac{dy}{dx} = \ln x$

C. $x\ln x$

D. $\frac{dy}{dx} = \frac{1}{x}$

24. If the derivative of sinx is cosx; then the integration of $cosx$ is

A. $-$Cosx

B. $-$Sinx

C. Sinx

D. Tan x

25. Evaluate $\int_0^2 (x^2 + 3)dx$

A. $\frac{26}{3}$

B. $\frac{24}{3}$

C. $\frac{25}{3}$

D. $\frac{3}{26}$

SECTION B

[75 Marks]

Answer any three questions in this section.

1a) Consider the piecewise function; **[10marks]**

$$g(x) = \begin{cases} 5x+3, & x<2 \\ 2x^2+5, & 2 \le x < 4 \\ x^3-5x+3, & x \ge 4 \end{cases}$$

Determine whether or not $g(x)$ is continuous at

i. $x = 2$

ii. $x = 4$

1b) Use implicit differentiation to find $\frac{dy}{dx}$ for the Folium of Descartes $x^3 + y^3 = 3xy$. **[8marks]**

1c) From the first principles, find the derivative of x of the function **[7marks]**

y = 5x

2a) State L'Hospital's Rule and use it to evaluate $\lim_{x\to 0} \frac{\sin(2x)+7x^2-2x}{x^2(x+1)^2}$. **[7marks]**

2b) State the conditions for the continuity of a function and determine if the following function is continuous at $x = -2$ **[10marks]**

$g(x) = \frac{x^2+2x}{x^2-4}$

2c) Evaluate $\lim_{x\to\infty} \frac{3x^2-5x-1}{7+2x-4x^2}$ **[8marks]**

3a) Sketch the curve of $y = 3x^2 - x^3$. **[7marks]**

3b. A particle P moving along a straight line passes through a point O with a speed of $3ms^{-1}$. The acceleration at time t seconds after passing through O is $(6t+8)ms^{-2}$. Calculate **[18marks]**

i. The velocity of P when $t = 3seconds$

ii. The distance covered by P between instances when t = 2seconds and t = 5 seconds

iii. Average velocity between t = 2seconds and t = 5 seconds

4a) Find the limit of the function $f(x) = \frac{x^2+4x-12}{x^2-2x}$ at $x = 2$. **[7marks]**

4b) The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres? **[10marks]**

4c) Determine the area below $f(x) = 3 + 2x - x^2$ and above the x – axis. **[8marks]**

5a) Using $n = 8$ approximate the value of $\int_0^4 \cos(1 + \sqrt{x})\,dx$ using Simpson's rule **[10marks]**

5b) Evaluate $\int_{-2}^{2} \left(\frac{x^3-x^2+1}{x^2}\right) dx$ **[5marks]**

5c) Given that $\lim_{x\to 0} \frac{sinx}{x} = 1$ and $\lim_{x\to 0} cosx = 1$, show that $\lim_{x\to 0} \frac{Tanx}{x} = 1$ **[10marks]**